5. Given that

$$y = \frac{5x^2 - 10x + 9}{(x-1)^2} \qquad x \neq 1$$

show that $\frac{dy}{dx} = \frac{k}{(x-1)^3}$, where k is a constant to be found.

(6)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

7. The point P lies on the curve with equation

$$x = (4y - \sin 2y)^2$$

Given that P has (x, y) coordinates $\left(p, \frac{\pi}{2} \right)$, where p is a constant,

(a) find the exact value of p

(1)

The tangent to the curve at P cuts the y-axis at the point A.

(b) Use calculus to find the coordinates of A.

(6)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

4. (i) $$f(x) = \frac{(2x+5)^2}{x-3} \qquad x \neq 3$$

(a) Find $f'(x)$ in the form $\frac{P(x)}{Q(x)}$ where $P(x)$ and $Q(x)$ are fully factorised quadratic expressions.

(b) Hence find the range of values of x for which $f(x)$ is increasing.

(6)

(ii) $$g(x) = x\sqrt{\sin 4x} \qquad 0 \leqslant x < \frac{\pi}{4}$$

The curve with equation $y = g(x)$ has a maximum at the point M.

Show that the x coordinate of M satisfies the equation

$$\tan 4x + kx = 0$$

where k is a constant to be found.

(5)

3.

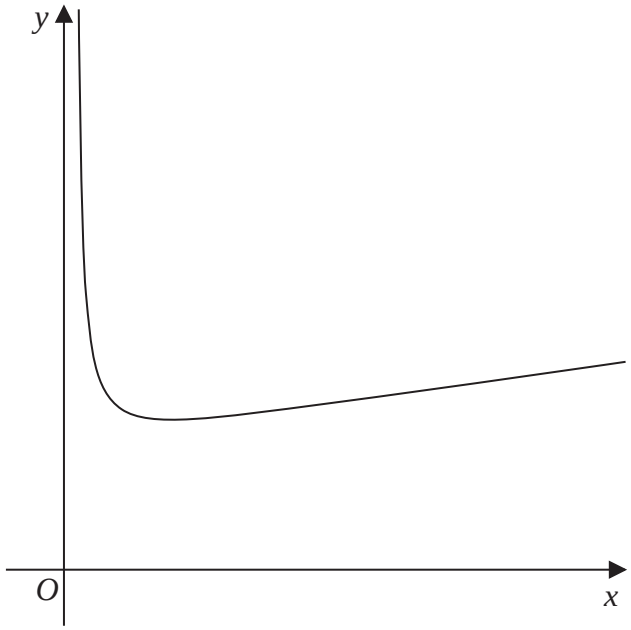


Figure 1

Figure 1 shows a sketch of a curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{2x + 3}{\sqrt{4x - 1}} \qquad x > \frac{1}{4}$$

(a) Find, in simplest form, $\mathrm{f}'(x)$.

(4)

(b) Hence find the range of f.

(3)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

8. (i) The curve C has equation $y = g(x)$ where

$$g(x) = e^{3x} \sec 2x \qquad -\frac{\pi}{4} < x < \frac{\pi}{4}$$

(a) Find $g'(x)$

(2)

(b) Hence find the x coordinate of the stationary point of C.

(3)

(ii) A different curve has equation

$$x = \ln(\sin y) \qquad 0 < y < \frac{\pi}{2}$$

Show that

$$\frac{dy}{dx} = \frac{e^x}{f(x)}$$

where $f(x)$ is a function of e^x that should be found.

(4)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

9. (a) Given that

$$\frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} \equiv x^2 + P + \frac{Q}{x - 4} \qquad x > -3$$

find the value of the constant P and show that $Q = 5$

(4)

The curve C has equation $y = g(x)$, where

$$g(x) = \frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} \qquad -3 < x < 3.5 \qquad x \in \mathbb{R}$$

(b) Find the equation of the tangent to C at the point where $x = 2$
Give your answer in the form $y = mx + c$, where m and c are constants to be found.

(5)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

6.

$$f(x) = x\cos\left(\frac{x}{3}\right) \qquad x > 0$$

(a) Find $f'(x)$

(2)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

10. The curve C has equation

$$x = 3\sec^2 2y \qquad x > 3 \qquad 0 < y < \frac{\pi}{4}$$

(a) Find $\frac{dx}{dy}$ in terms of y.

(2)

(b) Hence show that

$$\frac{dy}{dx} = \frac{p}{qx\sqrt{x-3}}$$

where p is irrational and q is an integer, stating the values of p and q.

(3)

(c) Find the equation of the normal to C at the point where $y = \frac{\pi}{12}$, giving your answer in the form $y = mx + c$, giving m and c as exact irrational numbers.

(5)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

June 2021

1. The curve C has equation

$$y = x^2 \cos\left(\frac{1}{2}x\right) \qquad 0 < x \leqslant \pi$$

The curve has a stationary point at the point P.

(a) Show, using calculus, that the x coordinate of P is a solution of the equation

$$x = 2\arctan\left(\frac{4}{x}\right)$$

(4)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

7. Given that

$$x = 6\sin^2 2y \qquad 0 < y < \frac{\pi}{4}$$

show that

$$\frac{dy}{dx} = \frac{1}{A\sqrt{(Bx - x^2)}}$$

where A and B are integers to be found.

(5)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

October 2021

1. The function f is defined by

$$f(x) = \frac{5x}{x^2 + 7x + 12} + \frac{5x}{x + 4} \qquad x > 0$$

(a) Show that $f(x) = \frac{5x}{x + 3}$

(3)

(b) Find f^{-1}

(3)

(c) (i) Find, in simplest form, $f'(x)$.

(ii) Hence, state whether f is an increasing or a decreasing function, giving a reason for your answer.

(3)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

6. (i) The curve C_1 has equation

$$y = 3\ln(x^2 - 5) - 4x^2 + 15 \qquad x > \sqrt{5}$$

Show that C_1 has a stationary point at $x = \frac{\sqrt{p}}{2}$ where p is a constant to be found.

(4)

(ii) A different curve C_2 has equation

$$y = 4x - 12\sin^2 x$$

(a) Show that, for this curve,

$$\frac{dy}{dx} = A + B\sin 2x$$

where A and B are constants to be found.

(b) Hence, state the maximum gradient of this curve.

(4)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

8. A curve C has equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = \arcsin\left(\frac{1}{2}x\right) \qquad -2 \leqslant x \leqslant 2 \qquad -\frac{\pi}{2} \leqslant y \leqslant \frac{\pi}{2}$$

(a) Sketch C.

(1)

(b) Given $x = 2\sin y$, show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sqrt{A - x^2}}$$

where A is a constant to be found.

(3)

The point P lies on C and has y coordinate $\frac{\pi}{4}$

(c) Find the equation of the tangent to C at P. Write your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

9.

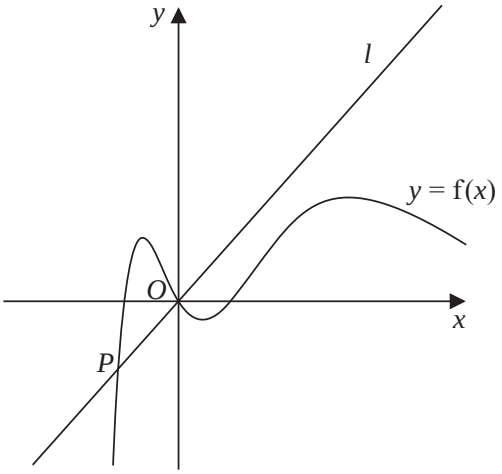


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = x(x^2 - 4)e^{-\frac{1}{2}x}$$

(a) Find $f'(x)$.

(2)

The line l is the normal to the curve at O and meets the curve again at the point P.

The point P lies in the 3rd quadrant, as shown in Figure 3.

(b) Show that the x coordinate of P is a solution of the equation

$$x = -\frac{1}{2}\sqrt{16 + e^{\frac{1}{2}x}}$$

(4)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

January 2022

1. Find, using calculus, the x coordinate of the stationary point on the curve with equation

$$y = (2x + 5)e^{3x}$$

(4)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

10.

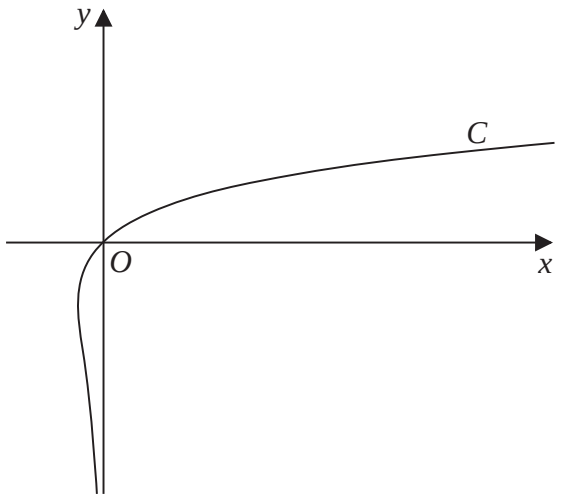


Figure 3

Figure 3 shows a sketch of the curve C with equation

$$x = ye^{2y} \qquad y \in \mathbb{R}$$

(a) Show that

$$\frac{dy}{dx} = \frac{y}{x(1+2y)}$$

(4)

Given that the straight line with equation $x = k$, where k is a constant, cuts C at exactly two points,

(b) find the range of possible values for k.

(3)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

June 2022

1. The curve C has equation

$$y = (3x - 2)^6$$

(a) Find $\dfrac{dy}{dx}$

(2)

Given that the point $P\left(\dfrac{1}{3}, 1\right)$ lies on C,

(b) find the equation of the normal to C at P. Write your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(4)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

6.

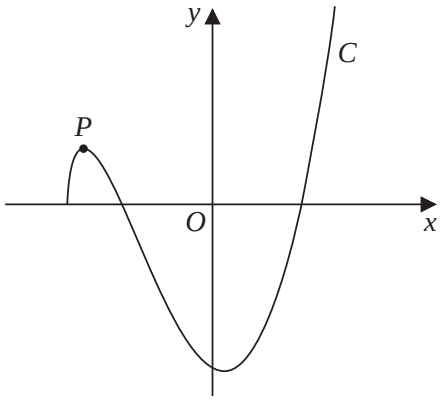


Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

The function f is defined by

$$f(x) = 5(x^2 - 2)(4x + 9)^{\frac{1}{2}} \qquad x \geqslant -\frac{9}{4}$$

(a) Show that

$$f'(x) = \frac{k(5x^2 + 9x - 2)}{(4x + 9)^{\frac{1}{2}}}$$

where k is an integer to be found.

(4)

(b) Hence, find the values of x for which $f'(x) = 0$

(1)

Figure 3 shows a sketch of the curve C with equation $y = f(x)$.

The curve has a local maximum at the point P

(c) Find the exact coordinates of P

(2)

The function g is defined by

$$g(x) = 2f(x) + 4 \qquad -\frac{9}{4} \leqslant x \leqslant 0$$

(d) Find the range of g

(3)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

9.

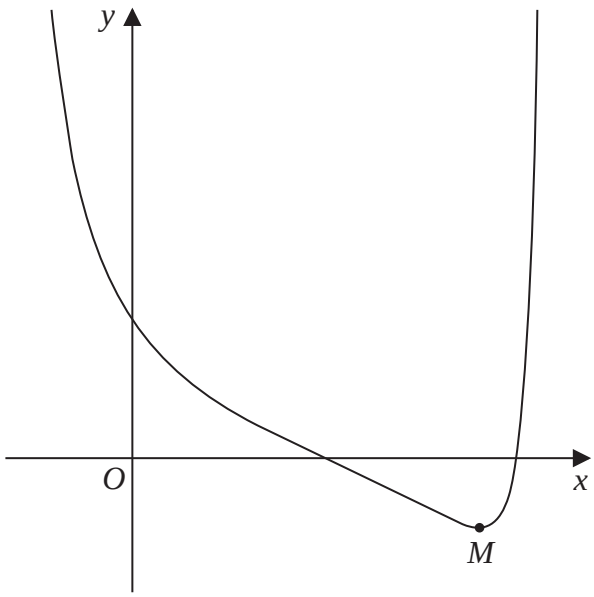


Figure 5

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 5 shows the curve with equation

$$y = \frac{1 + 2\cos x}{1 + \sin x} \qquad -\frac{\pi}{2} < x < \frac{3\pi}{2}$$

The point M, shown in Figure 5, is the minimum point on the curve.

(a) Show that the x coordinate of M is a solution of the equation

$$2\sin x + \cos x = -2$$

(4)

(b) Hence find, to 3 significant figures, the x coordinate of M.

(5)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

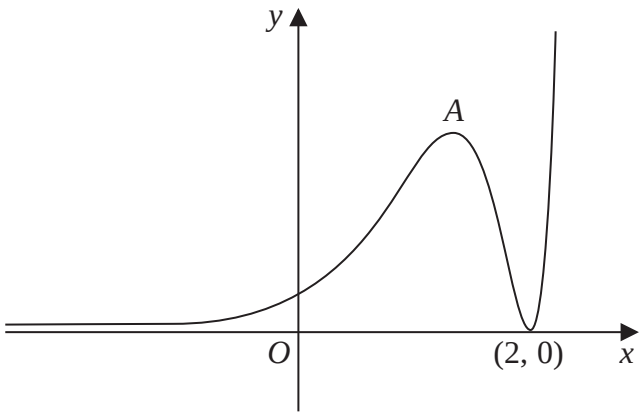


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = (x - 2)^2 \mathrm{e}^{3x} \qquad x \in \mathbb{R}$$

The curve has a maximum turning point at A and a minimum turning point at $(2, 0)$

(a) Use calculus to find the exact coordinates of A.

(5)

Given that the equation $\mathrm{f}(x) = k$, where k is a constant, has **at least** two distinct roots,

(b) state the range of possible values for k.

(2)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

October 2022

4. $y = \log_{10}(2x + 1)$

(a) Express x in terms of y.

(2)

(b) Hence, giving your answer in terms of x, find $\frac{dy}{dx}$

(3)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

6.

$$y = \frac{2 + 3\sin x}{\cos x + \sin x}$$

Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{a\tan x + b\sec x + c}{\sec x + 2\sin x}$$

where a, b and c are integers to be found.

(6)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

7. The curve C has equation

$$x = 3\tan\left(y - \frac{\pi}{6}\right) \qquad x \in \mathbb{R} \quad -\frac{\pi}{3} < y < \frac{2\pi}{3}$$

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{a}{x^2 + b}$$

where a and b are integers to be found.

(4)

The point P with y coordinate $\frac{\pi}{3}$ lies on C.

Given that the tangent to C at P crosses the x-axis at the point Q.

(b) find, in simplest form, the exact x coordinate of Q.

(5)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

9.

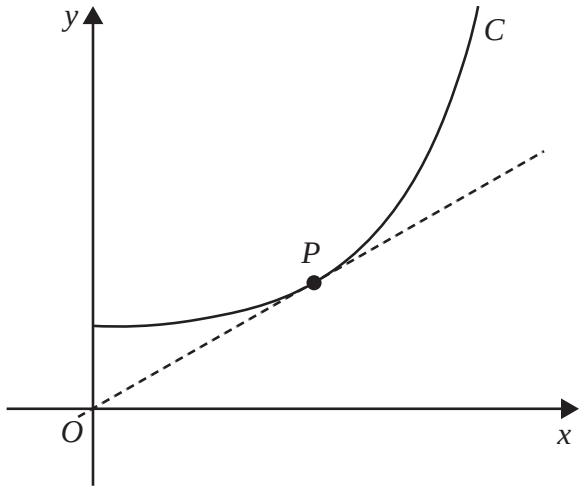


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = \sqrt{3 + 4e^{x^2}} \qquad x \geqslant 0$$

(a) Find $\dfrac{dy}{dx}$, giving your answer in simplest form.

(2)

The point P with x coordinate α lies on C.

Given that the tangent to C at P passes through the origin, as shown in Figure 3,

(b) show that $x = \alpha$ is a solution of the equation

$$4x^2e^{x^2} - 4e^{x^2} - 3 = 0$$

(3)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

3. (i) Find $\frac{\mathrm{d}}{\mathrm{d}x}\ln\left(\sin^2 3x\right)$ writing your answer in simplest form.

(2)

(ii) (a) Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(3x^2 - 4\right)^6$

(2)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

8.

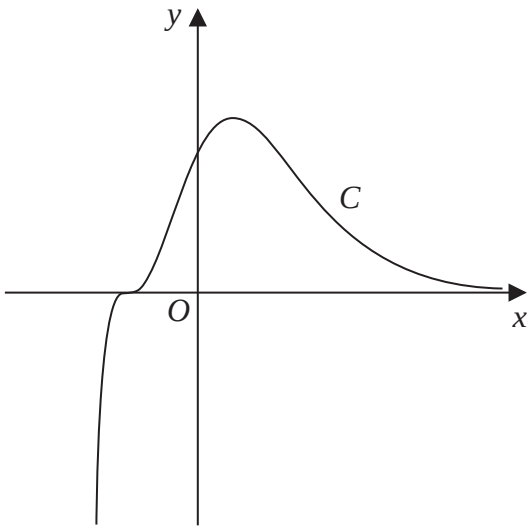


Figure 3

Figure 3 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (2x + 1)^3 \mathrm{e}^{-4x}$$

(a) Show that

$$\mathrm{f}'(x) = A(2x + 1)^2 (1 - 4x)\, \mathrm{e}^{-4x}$$

where A is a constant to be found.

(4)

(b) Hence find the exact coordinates of the two stationary points on C.

(3)

The function g is defined by

$$\mathrm{g}(x) = 8\,\mathrm{f}(x - 2)$$

(c) Find the coordinates of the maximum stationary point on the curve with equation $y = \mathrm{g}(x)$.

(2)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

10.

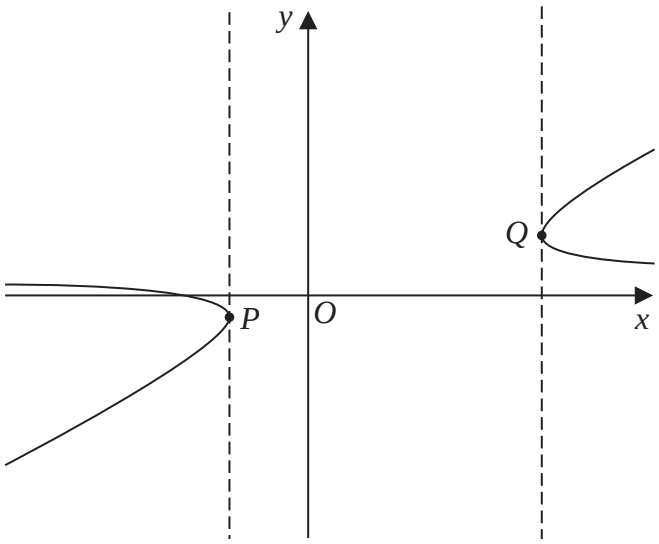


Figure 4

Figure 4 shows a sketch of the curve with equation

$$x = \frac{2y^2 + 6}{3y - 3}$$

(a) Find $\frac{dx}{dy}$ giving your answer as a fully simplified fraction.

(4)

The tangents at points P and Q on the curve are parallel to the y-axis, as shown in Figure 4.

(b) Use the answer to part (a) to find the equations of these two tangents.

(4)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

5. The curve C has equation

$$y = \frac{\ln(x^2 + k)}{x^2 + k} \qquad x \in \mathbb{R}$$

where k is a positive constant.

(a) Show that

$$\frac{dy}{dx} = \frac{Ax(B - \ln(x^2 + k))}{(x^2 + k)^2}$$

where A and B are constants to be found.

(3)

Given that C has exactly three turning points,

(b) find the x coordinate of each of these points. Give your answer in terms of k where appropriate.

(3)

(c) find the upper limit to the value for k.

(1)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

October 2023

7.

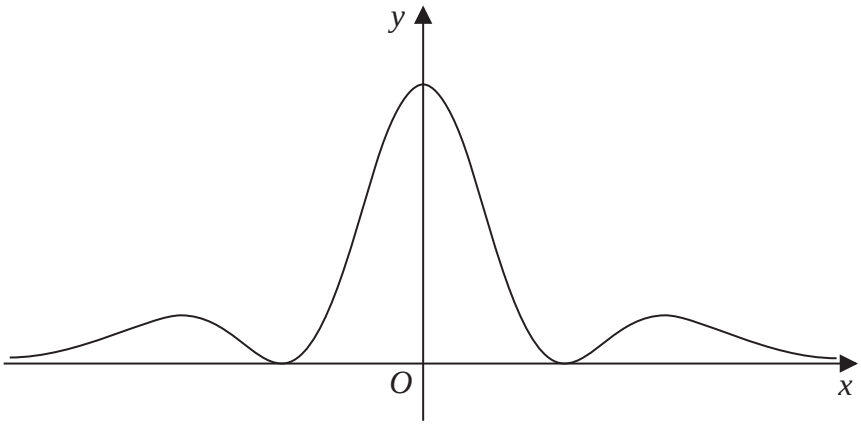


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \mathrm{e}^{-x^2}\left(2x^2 - 3\right)^2$$

(a) Find the range of f

(2)

(b) Show that

$$\mathrm{f}'(x) = 2x\left(2x^2 - 3\right)\mathrm{e}^{-x^2}\left(A - Bx^2\right)$$

where A and B are constants to be found.

(4)

Given that the line $y = k$, where k is a constant, $k > 0$, intersects the curve at exactly two distinct points,

(c) find the exact range of values of k

(4)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

10. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve C has equation

$$x = \sin^2 4y \qquad 0 \leqslant y \leqslant \frac{\pi}{8} \qquad 0 \leqslant x \leqslant 1$$

The point P with x coordinate $\frac{1}{4}$ lies on C

(a) Find the exact y coordinate of P

(2)

(b) Find $\frac{\mathrm{d}x}{\mathrm{d}y}$

(2)

(c) Hence show that $\frac{\mathrm{d}y}{\mathrm{d}x}$ can be written in the form

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\sqrt{q + r(x + s)^2}}$$

where q, r and s are constants to be found.

(3)

Using the answer to part (c),

(d) (i) state the x coordinate of the point where the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ is a minimum,

(ii) state the value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at this point.

(2)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

6.

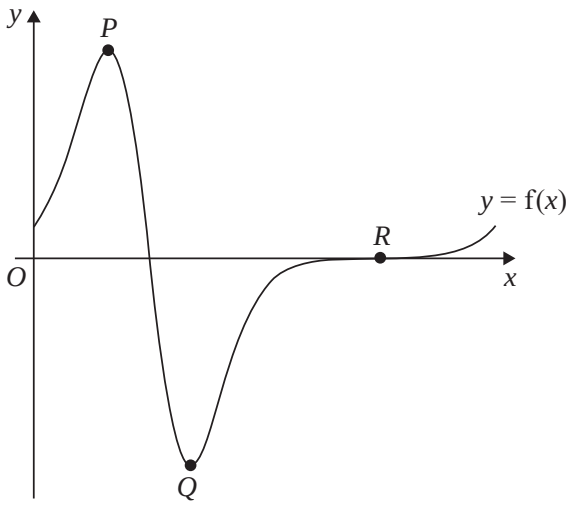


Figure 1

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 2\mathrm{e}^{3\sin x}\cos x \qquad 0 \leqslant x \leqslant 2\pi$$

The curve intersects the x-axis at point R, as shown in Figure 1.

(a) State the coordinates of R

(1)

The curve has two turning points, at point P and point Q, also shown in Figure 1.

(b) Show that, at points P and Q,

$$a\sin^2 x + b\sin x + c = 0$$

where a, b and c are integers to be found.

(4)

(c) Hence find the x coordinate of point Q, giving your answer to 3 decimal places.

(2)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$y = \frac{16}{9(3x - k)} \qquad x \neq \frac{k}{3}$$

where k is a positive constant not equal to 3

(a) Find $\frac{dy}{dx}$ giving your answer in simplest form in terms of k.

(2)

The point P with x coordinate 1 lies on C.

Given that the gradient of the curve at P is -12

(b) find the two possible values of k

(3)

Given also that $k < 3$

(c) find the equation of the normal to C at P, writing your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(3)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

6.

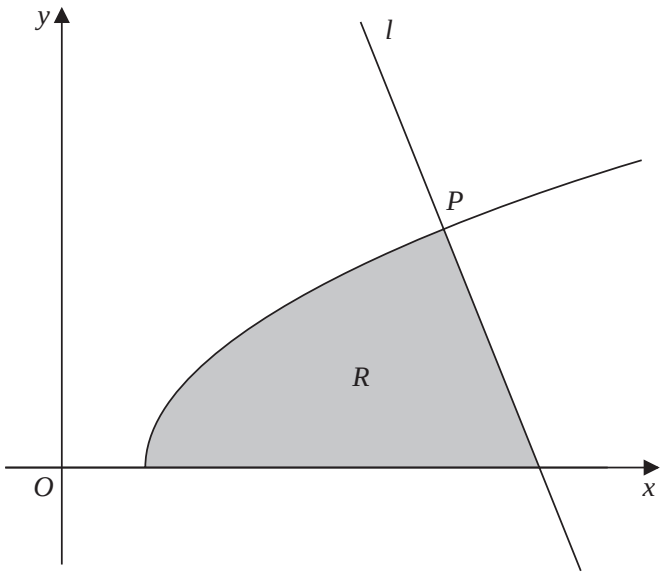


Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = \sqrt{4x - 7}$$

The line l, shown in Figure 3, is the normal to the curve at the point $P(8, 5)$

(a) Use calculus to show that an equation of l is

$$5x + 2y - 50 = 0$$

(5)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

9.

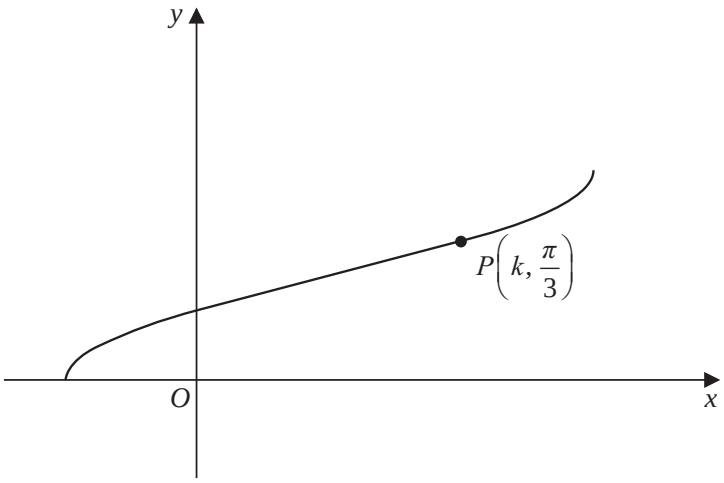


Figure 5

The curve shown in Figure 5 has equation

$$x = 4\sin^2 y - 1 \qquad 0 \leqslant y \leqslant \frac{\pi}{2}$$

The point $P\left(k, \frac{\pi}{3}\right)$ lies on the curve.

(a) Verify that $k = 2$

(1)

(b) (i) Find $\frac{dx}{dy}$ in terms of y

(ii) Hence show that $\frac{dy}{dx} = \frac{1}{2\sqrt{x+1}\sqrt{3-x}}$

(6)

The normal to the curve at P cuts the x-axis at the point N.

(c) Find the exact area of triangle OPN, where O is the origin.

Give your answer in the form $a\pi + b\pi^2$ where a and b are constants.

(3)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

8.

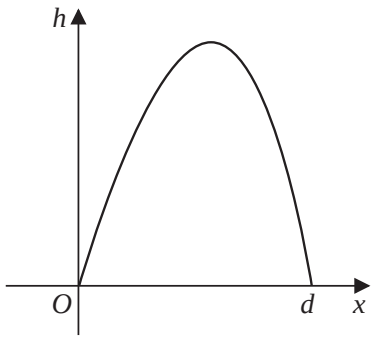


Figure 4

Figure 4 is a graph showing the path of a golf ball after the ball has been hit until it first hits the ground.

The vertical height, h metres, of the ball above the ground has been plotted against the horizontal distance travelled, x metres, measured from where the ball was hit.

The ball travels a horizontal distance of d metres before it first hits the ground.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

The path of the ball is modelled by the equation

$$h = 1.5x - 0.5x\,e^{0.02x} \qquad 0 \leqslant x \leqslant d$$

Use the model to answer parts (a), (b) and (c).

(a) Find the value of d, giving your answer to 2 decimal places.

(*Solutions relying entirely on calculator technology are not acceptable*.)

(3)

(b) Show that the maximum value of h occurs when

$$x = 50\ln\left(\frac{150}{x + 50}\right)$$

(4)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

2.

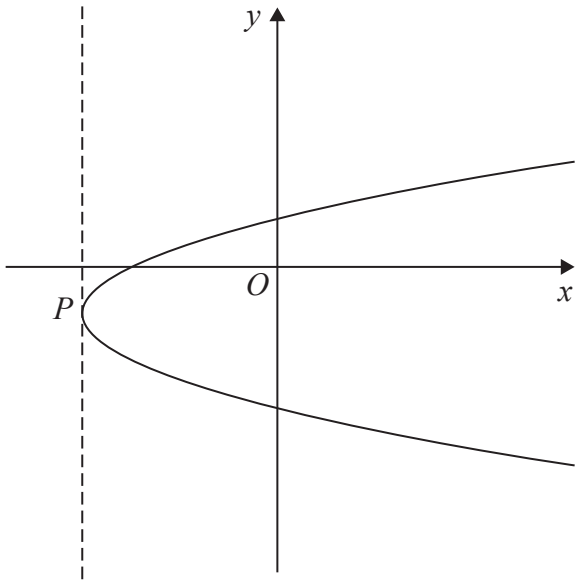


Figure 1

Figure 1 shows a sketch of the curve with equation

$$x = 2y^2 + 5y - 6$$

(a) Find $\frac{dy}{dx}$ in terms of y.

(2)

The point P lies on the curve and is shown in Figure 1.

Given that the tangent to the curve at P is parallel to the y-axis,

(b) find the coordinates of P.

(3)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

7.

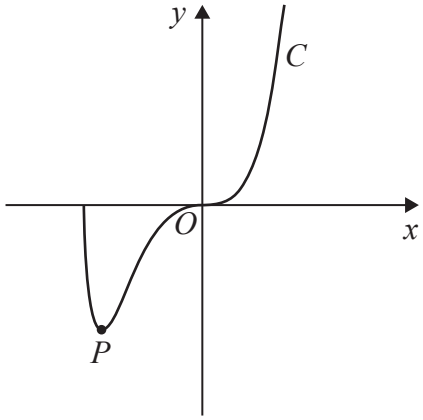


Figure 3

The curve C has equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = x^3\sqrt{4x+7} \qquad x \geqslant -\frac{7}{4}$$

(a) Show that

$$\mathrm{f}'(x) = \frac{kx^2(2x+3)}{\sqrt{4x+7}}$$

where k is a constant to be found.

(4)

The point P, shown in Figure 3, is the minimum turning point on C.

(b) Find the coordinates of P.

(2)

(c) Hence find the range of the function g defined by

$$\mathrm{g}(x) = -4\,\mathrm{f}(x) \qquad x \geqslant -\frac{7}{4}$$

(2)

The point Q with coordinates $\left(\frac{1}{2}, \frac{3}{8}\right)$ lies on C.

(d) Find the coordinates of the point to which Q is mapped when C is transformed to the curve with equation

$$y = 40\,\mathrm{f}\left(x - \frac{3}{2}\right) - 8$$

(2)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

3. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$y = \frac{4x + 1}{(x + 3)^2} \qquad x \neq -3 \qquad x \in \mathbb{R}$$

Use calculus to find the range of values of x for which y is increasing.

(6)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

9.

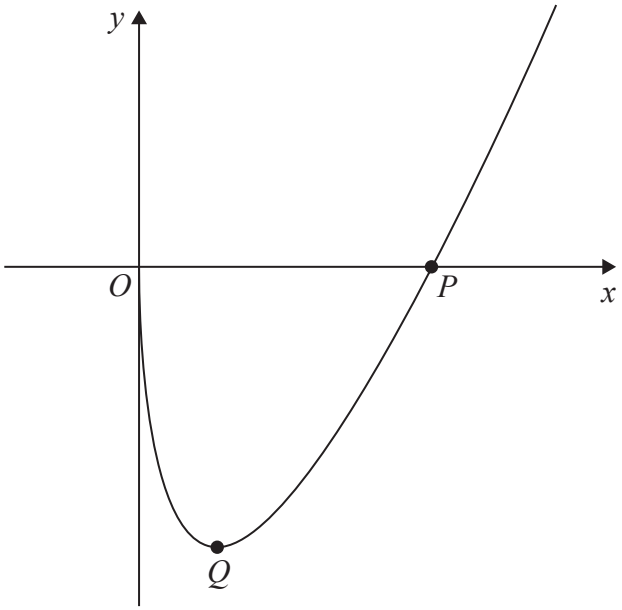


Figure 1

Figure 1 shows a sketch of part of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = 6\sqrt{x}\ln(4x) \qquad x > 0$$

The curve cuts the x-axis at point P

(a) State the x coordinate of P

(1)

The point Q, shown in Figure 1, is the stationary point on C

(b) Use calculus to find the exact coordinates of Q

(5)

(c) Hence find the range of the function $\mathrm{g}(x)$ where

$$\mathrm{g}(x) = -2\,\mathrm{f}(x)$$

(2)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

10.

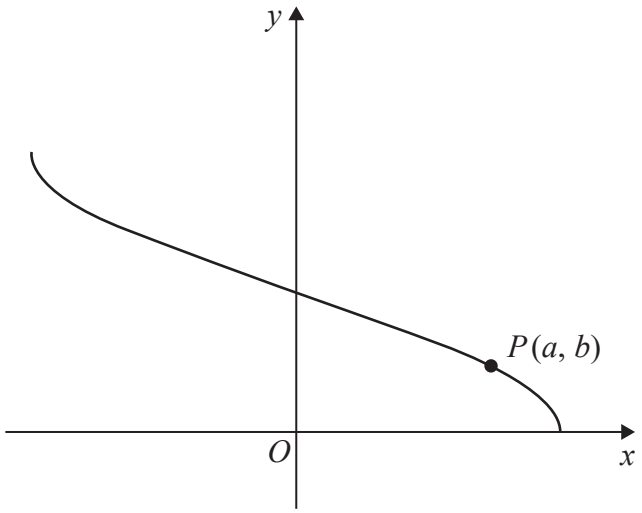


Figure 2

Figure 2 shows a sketch of the curve with equation

$$x = 3\cos 2y \qquad -3 \leqslant x \leqslant 3 \qquad 0 \leqslant y \leqslant \frac{\pi}{2}$$

(a) Find $\frac{\mathrm{d}x}{\mathrm{d}y}$ in terms of y.

(2)

(b) Hence show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{k}{\sqrt{9 - x^2}}$$

where k is a constant to be found.

(3)

The point $P(a, b)$ lies on the curve and is shown in Figure 2.

Given that

- the gradient of the curve at P is $-\frac{1}{4}$
- both a and b are positive

(c) find the exact values of a and b.

(4)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

9:

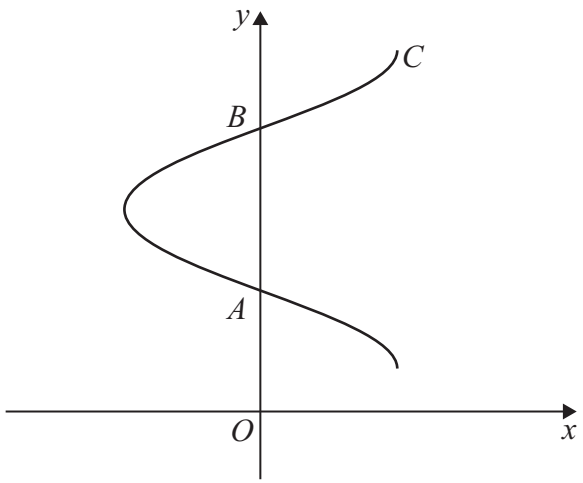


Figure 2

Figure 2 shows a sketch of the curve C with equation

$$x = \frac{2}{3}\sin\left(3y + \frac{\pi}{4}\right) \qquad \frac{\pi}{12} < y < \frac{3\pi}{4}$$

The curve intersects the y-axis at the points A and B as shown.

(a) Find the exact value of the y coordinate of

- point A
- point B

(3)

(b) Show that

$$\left(\frac{dy}{dx}\right)^2 = \frac{1}{p - qx^2}$$

where p and q are integers to be found.

(4)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

The **normal** to C at A and the **tangent** to C at B intersect at the point D.

Using

- the answer to part (b)
- the sketch of curve C in Figure 2

(c) find, in simplest form, the exact x coordinate of D.

(4)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

3: The curve C has equation

$$x = \frac{4y^2 - 3}{2y + 1} \qquad y \neq -\frac{1}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of y, giving the answer in simplest form.

(4)

A point P lies on C.

Given that

- the gradient of the tangent to C at P is $\frac{1}{3}$
- the point P lies above the x-axis

(b) find the coordinates of P.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

5: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

A curve C has equation

$$y = 5\cos 2x - 12\sin 2x \qquad -\pi < x < \pi$$

The point A with x coordinate $\frac{\pi}{3}$ lies on C.

(a) Use algebraic differentiation to find the gradient of the tangent to C at A.
Give the answer in simplest form.

(3)

(b) Express $5\cos 2x - 12\sin 2x$ in the form

$$R\cos(2x + \alpha)$$

where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

Find the exact value of R and find the value of α to 3 decimal places.

(3)

A point P lies on C.

Given that the gradient of the tangent to C at P is 6

(c) find the greatest possible value for the x coordinate of P.
Give the answer to 2 decimal places.

(4)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

6:

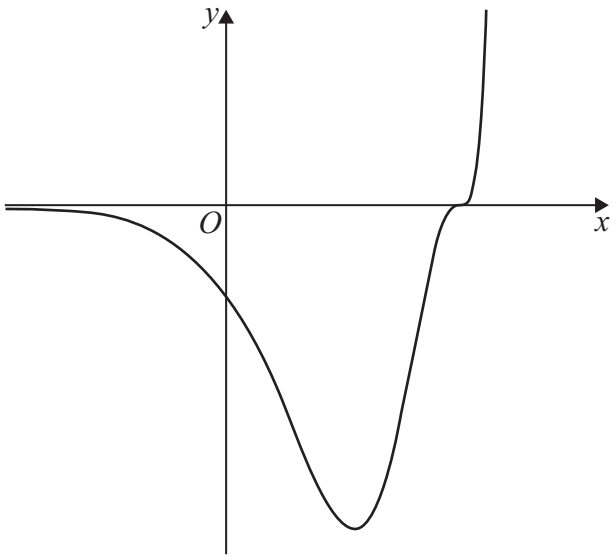


Figure 2

Figure 2 shows a sketch of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = (2x - 3)^3 \mathrm{e}^{4x-2}$$

(a) Show that

$$\mathrm{f}'(x) = 2(Px + Q)(2x - 3)^n \mathrm{e}^{4x-2}$$

where P, Q and n are constants to be found.

(4)

(b) Hence find

(i) the x coordinates of the stationary points on the curve with equation $y = \mathrm{f}(x)$,

(ii) the range of the function g defined by

$$\mathrm{g}(x) = 8(2x - 3)^3 \mathrm{e}^{4x-2} \qquad x \leqslant \frac{3}{2}$$

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

A curve C has equation

$$y = 2x\mathrm{e}^{x^2+(3k-2)x}$$

where k is a constant.

Given that C has two distinct turning points, find the range of possible values of k.

(7)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

2. (i) Given $y = x^2 \ln 2x$, find in simplest form, $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(2)

(ii) Given

$$\mathrm{f}(x) = \frac{\mathrm{e}^{2x}}{(4x+3)^2} \qquad x > -\frac{3}{4}$$

show that

$$\mathrm{f}'(x) = \frac{2\mathrm{e}^{2x}(ax+b)}{(4x+3)^3}$$

where a and b are constants to be found.

(4)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

8. A curve has equation

$$x = \mathrm{e}^{2\tan y} \qquad -\frac{\pi}{2} < y < \frac{\pi}{2}$$

Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{a}{x\left(b + (\ln x)^n\right)}$$

where a, b and n are integers to be found.

(4)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

3. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{4x^2 - 14x + 15}{x - 3} \qquad x < 3$$

(a) Find $\mathrm{f}'(x)$, giving the answer in the form

$$\frac{ax^2 + bx + c}{(x-3)^2}$$

where a, b and c are constants.

(4)

(b) Hence find the coordinates of the turning point of C.

(2)

The point $P\left(\frac{5}{2}, -10\right)$ lies on C.

(c) Find the coordinates of the point to which P is mapped, when the curve with equation $y = \mathrm{f}(x)$ is transformed to the curve with equation $y = \mathrm{f}(2x) + 3$

(2)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

6. The curve C has equation

$$x = \left(3\sin 2y - \frac{6y}{\pi}\right)^2$$

Given that

- the point P with y coordinate $\frac{\pi}{4}$ lies on C
- the tangent to C at P cuts the x-axis at the point Q

use calculus to find the x coordinate of Q

(6)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

3. The curve C has equation

$$x = \frac{2y - 1}{(y - 1)^2} \qquad x > 0 \quad y > 1$$

(a) Find $\frac{dx}{dy}$ in terms of y, giving the answer in simplest form.

(3)

The point P lies on C.

Given that P has y coordinate 2

(b) find the equation of the tangent to C at P, giving the answer in the form $ax + by + c = 0$, where a, b and c are integers.

(3)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362