4. (i) Find

$$\int_5^{13} \frac{1}{(2x-1)}\,\mathrm{d}x$$

writing your answer in its simplest form.

(4)

(ii) Use integration to find the exact value of

$$\int_0^{\frac{\pi}{2}} \sin 2x + \sec\frac{1}{3}x\tan\frac{1}{3}x\,\mathrm{d}x$$

(3)

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8. (i) Find, using algebraic integration, the exact value of

$$\int_3^{42} \frac{2}{3x-1}\,\mathrm{d}x$$

giving your answer in simplest form.

(4)

(ii) $$\mathrm{h}(x) = \frac{2x^3 - 7x^2 + 8x + 1}{(x-1)^2} \qquad x > 1$$

Given $\mathrm{h}(x) = Ax + B + \frac{C}{(x-1)^2}$ where A, B and C are constants to be found, find

$$\int \mathrm{h}(x)\,\mathrm{d}x$$

(6)

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5. (a) Show that

$$\sin 3x \equiv 3\sin x - 4\sin^3 x$$

(4)

(b) Hence find, using algebraic integration,

$$\int_0^{\frac{\pi}{3}} \sin^3 x \, \mathrm{d}x$$

(4)

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9. (a) Given that

$$\frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} \equiv x^2 + P + \frac{Q}{x - 4} \qquad x > -3$$

find the value of the constant P and show that $Q = 5$

(4)

The curve C has equation $y = \mathrm{g}(x)$, where

$$\mathrm{g}(x) = \frac{x^4 - x^3 - 10x^2 + 3x - 9}{x^2 - x - 12} \qquad -3 < x < 3.5 \qquad x \in \mathbb{R}$$

(b) Find the equation of the tangent to C at the point where $x = 2$
Give your answer in the form $y = mx + c$, where m and c are constants to be found.

(5)

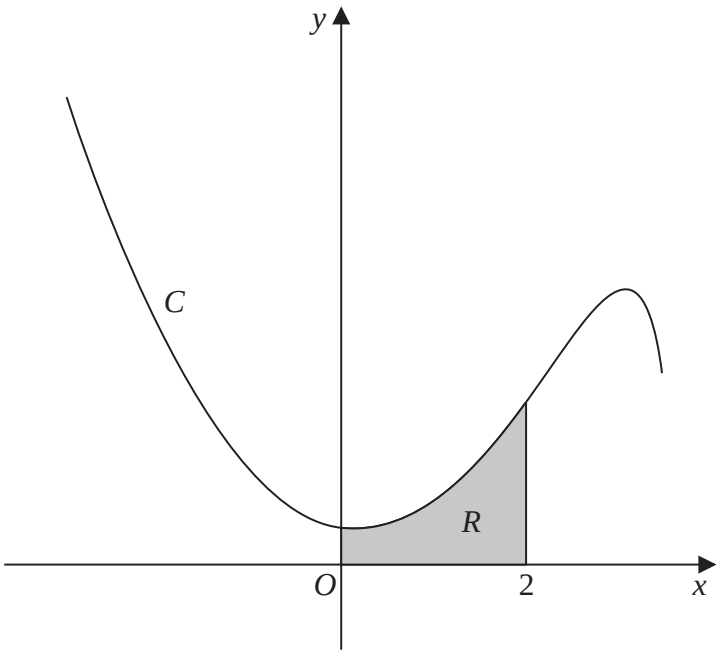


Figure 4

Figure 4 shows a sketch of the curve C.
The region R, shown shaded in Figure 4, is bounded by C, the y-axis, the x-axis and the line with equation $x = 2$

(c) Find the exact area of R, writing your answer in the form $a + b\ln 2$, where a and b are constants to be found.

(5)

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January 2021

1. Find

$$\int \frac{x^2 - 5}{2x^3} \mathrm{d}x \qquad x > 0$$

giving your answer in simplest form.

(3)

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9. Find

(i) $\int \frac{3x - 2}{3x^2 - 4x + 5} \, dx$

(2)

(ii) $\int \frac{e^{2x}}{(e^{2x} - 1)^3} \, dx \qquad x \neq 0$

(2)

3. (i) Find

$$\int \frac{12}{(2x-1)^2}\,dx$$

giving your answer in simplest form.

(2)

(ii) (a) Write $\frac{4x+3}{x+2}$ in the form

$$A + \frac{B}{x+2}$$ where A and B are constants to be found

(b) Hence find, using algebraic integration, the exact value of

$$\int_{-8}^{-5} \frac{4x+3}{x+2}\,dx$$

giving your answer in simplest form.

(6)

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5. (i) Find, by algebraic integration, the exact value of

$$\int_2^4 \frac{8}{(2x-3)^3}\,\mathrm{d}x$$

(4)

(ii) Find, in simplest form,

$$\int x\left(x^2+3\right)^7\,\mathrm{d}x$$

(2)

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3. (i) Find, in simplest form,

$$\int (2x - 5)^7 \, dx$$

(2)

(ii) Show, by algebraic integration, that

$$\int_0^{\frac{\pi}{3}} \frac{4\sin x}{1 + 2\cos x} dx = \ln a$$

where a is a rational constant to be found.

(4)

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3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

Given that k is a positive constant,

(a) find

$$\int \frac{9x}{3x^2 + k} \, \mathrm{d}x$$

(2)

Given also that

$$\int_2^5 \frac{9x}{3x^2 + k} \, \mathrm{d}x = \ln 8$$

(b) find the value of k

(4)

8.

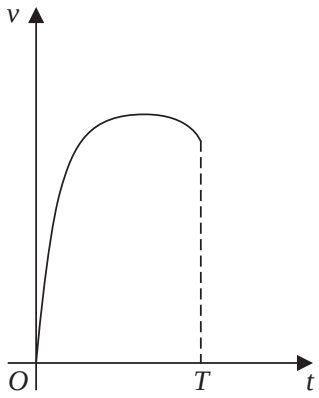


Figure 4

Figure 4 is a graph showing the velocity of a sprinter during a 100 m race.

The sprinter's velocity during the race, $v\,\text{m s}^{-1}$, is modelled by the equation

$$v = 12 - e^{t-10} - 12e^{-0.75t} \qquad t \geqslant 0$$

where t seconds is the time after the sprinter begins to run.

According to the model,

(a) find, using calculus, the sprinter's maximum velocity during the race.

(5)

Given that the sprinter runs 100 m in T seconds, such that

$$\int_0^T v\,dt = 100$$

(b) show that T is a solution of the equation

$$T = \frac{1}{12}\left(116 - 16e^{-0.75T} + e^{T-10} - e^{-10}\right)$$

(4)

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October 2022

1. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$\mathrm{f}(x) = \frac{2x^3 - 4x - 15}{x^2 + 3x + 4}$$

(a) Show that

$$\mathrm{f}(x) \equiv Ax + B + \frac{C(2x + 3)}{x^2 + 3x + 4}$$

where A, B and C are integers to be found.

(4)

(b) Hence, find

$$\int_3^5 \mathrm{f}(x)\,\mathrm{d}x$$

giving your answer in the form $p + \ln q$, where p and q are integers.

(5)

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4. $$f(x) = \frac{2x^4 + 15x^3 + 35x^2 + 21x - 4}{(x+3)^2} \qquad x \in \mathbb{R} \quad x > -3$$

(a) Find the values of the constants A, B, C and D such that

$$f(x) = Ax^2 + Bx + C + \frac{D}{(x+3)^2}$$

(4)

(b) Hence find,

$$\int f(x)\,dx$$

(3)

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8. Find, in simplest form,

$$\int (2\cos x - \sin x)^2 \, dx$$

(5)

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3. (i) Find $\frac{\mathrm{d}}{\mathrm{d}x}\ln\left(\sin^2 3x\right)$ writing your answer in simplest form.

(2)

(b) Hence show that

$$\int_0^{\sqrt{2}} x\left(3x^2 - 4\right)^5 \mathrm{d}x = R$$

where R is an integer to be found.

(*Solutions relying on calculator technology are not acceptable.*)

(3)

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3.

(b) Hence, using algebraic integration, find the exact value of

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{8}} (5 - 4\cos^2 3x)\,\mathrm{d}x$$

(4)

7. (d) show, using algebraic integration that,

$$\int_1^3 \frac{16}{9(3x-k)}\,\mathrm{d}x = \lambda \ln 10 \quad , \quad K = \frac{7}{3}$$

(4)

where λ is a constant to be found.

2. $$g(x) = \frac{2x^2 - 5x + 8}{x - 2}$$

(a) Write $g(x)$ in the form

$$Ax + B + \frac{C}{x - 2}$$

where A, B and C are integers to be found.

(3)

(b) Hence use algebraic integration to show that

$$\int_4^8 g(x)\,dx = \alpha + \beta \ln 3$$

where α and β are integers to be found.

(4)

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6.

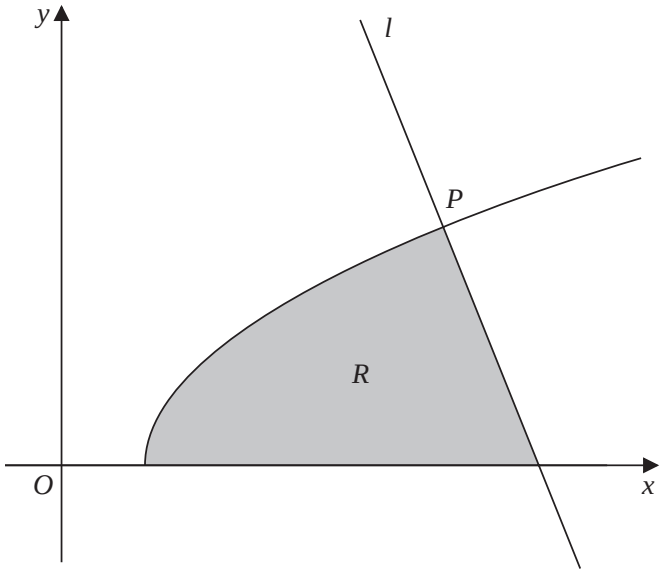


Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = \sqrt{4x - 7}$$

The line l, shown in Figure 3, is the normal to the curve at the point $P(8, 5)$

Equation of l is

$$5x + 2y - 50 = 0$$

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and l.

(b) Use algebraic integration to find the exact area of R.

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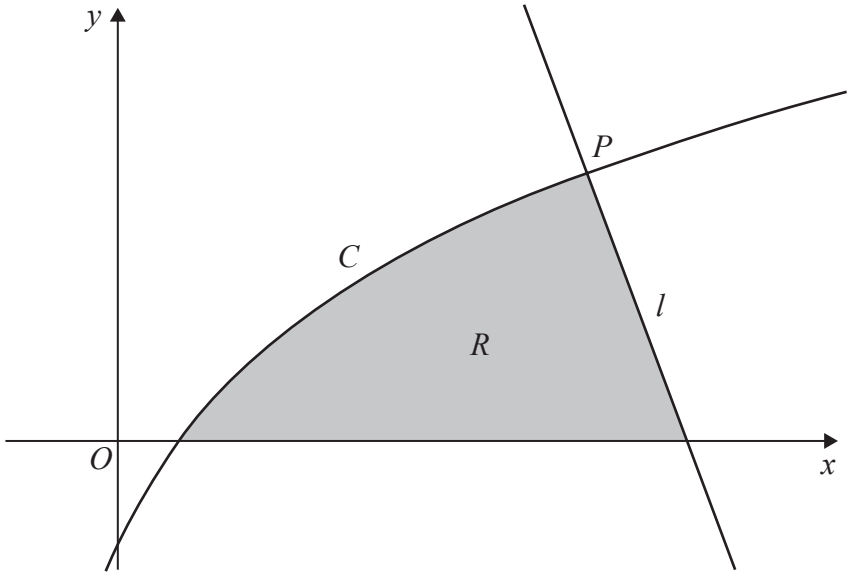


Figure 5

Figure 5 shows a sketch of part of the curve C with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{6x^2 + 4x - 2}{2x + 1} \qquad x > -\frac{1}{2}$$

(a) Find $\mathrm{f}'(x)$, giving the answer in simplest form.

(3)

The line l is the normal to C at the point $P(2, 6)$

(b) Show that an equation for l is

$$16y + 5x = 106$$

(3)

(c) Write $\mathrm{f}(x)$ in the form $Ax + B + \dfrac{D}{2x + 1}$ where A, B and D are constants.

(3)

The region R, shown shaded in Figure 5, is bounded by C, l and the x-axis.

(d) Use algebraic integration to find the exact area of R, giving your answer in the form $P + Q\ln 3$, where P and Q are rational constants.

(*Solutions based entirely on calculator technology are not acceptable.*)

(5)

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4. Given that

$$\frac{4x^3 + 2x^2 + 3x + 8}{x^2 + 4} \equiv Ax + B + \frac{Cx + D}{x^2 + 4}$$

(a) (i) find the values of the constants A, B and C

(ii) show that $D = 0$

(4)

(b) Hence, using algebraic integration, find

$$\int_1^4 \frac{4x^3 + 2x^2 + 3x + 8}{x^2 + 4}\,\mathrm{d}x$$

giving your answer in the form $p + q \ln 2$, where p and q are integers.

(5)

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5: Find

(i) $\int \sin^2 3x \, dx$

(2)

(ii) $\int x(x^2 + 4)^{\frac{3}{2}} dx$

(2)

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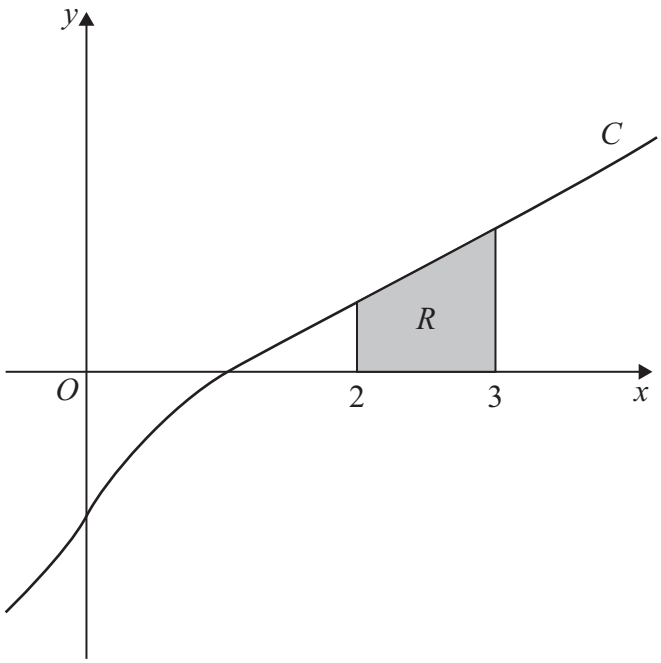


Figure 1

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Figure 1 shows a sketch of part of the curve C with equation $y = \mathrm{g}(x)$ where

$$\mathrm{g}(x) = \frac{3x^3 - 5x^2 + 7x - 5}{x^2 + 1}$$

Given that

$$\frac{3x^3 - 5x^2 + 7x - 5}{x^2 + 1} \equiv Ax + B + \frac{Cx + D}{x^2 + 1}$$

(a) (i) find the values of the constants A, B and C

(ii) show that $D = 0$

(4)

The finite region R, shown shaded in Figure 1, is bounded by C, the x-axis and the lines with equations $x = 2$ and $x = 3$

(b) Find the exact area of region R using algebraic integration.
Give the answer in the form $\alpha + \ln\beta$ where α and β are constants to be found.

(4)

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6. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) Given $y = \ln\left(2x^2 + 5\right)$ find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$

(2)

(ii) A curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{21x}{3x^2 + k} \qquad x \in \mathbb{R}$$

where k is a positive integer, $k > 1$

Given that

$$\int_1^k \mathrm{f}(x)\,\mathrm{d}x < 7\ln 8$$

find the greatest possible value of k

(5)

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3. (i) Find, in simplest form,

$$\int 2(3x+5)^7 \, dx$$

(2)

(ii) $$f(x) = \frac{4x^3 + x^2 - 7x + 14}{(x-1)(x+2)} \qquad x > 1$$

(a) Write $f(x)$ in the form

$$ax + b + \frac{c}{x-1}$$

where a, b and c are constants to be found.

(4)

(b) Hence, using algebraic integration, find

$$\int_3^9 f(x) \, dx$$

writing the answer in simplest form.

(4)

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4. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Using algebraic integration, find the exact value of

$$\int_{\frac{\pi}{18}}^{\frac{\pi}{12}} (\sin 3x + \cos 3x)^2 \, \mathrm{d}x$$

(5)

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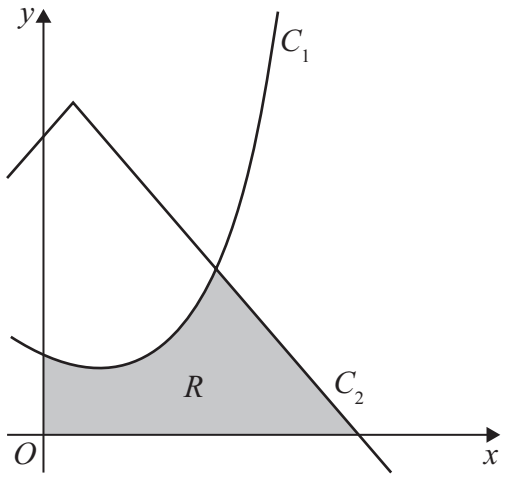


Figure 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

(a) Find $\int \left(\frac{11}{2x+5} + 4e^{x-3} \right) dx$

(2)

Figure 2 shows a sketch of part of the curve C_1 and part of the graph C_2
The curve C_1 has equation

$$y = \frac{11}{2x+5} + 4e^{x-3}$$

The graph C_2 has equation

$$y = a - |bx - 1|$$

where a and b are positive integers.

(b) State, in terms of a and/or b, the coordinates of the maximum point of C_2

(2)

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Given that

- C_1 intersects C_2 at $x = 3$
- C_2 crosses the y-axis at $(0, 9)$

(c) find the value of a and the value of b

(3)

The region R, shown shaded in Figure 2, is bounded by C_1, C_2, the x-axis and the y-axis.

(d) Find the area of R, giving the answer to 3 significant figures.

(4)

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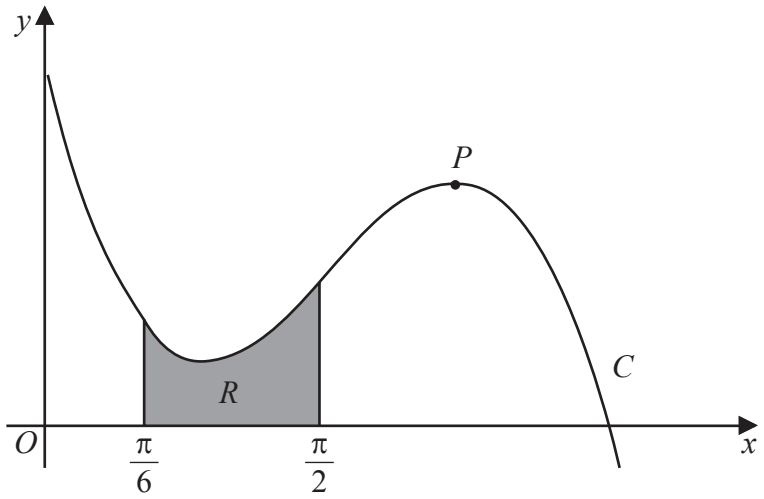


Figure 3

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve C with equation

$$y = 1 - 2x + 5\sin^2\left(\frac{1}{2}x\right) \qquad x > 0$$

where x is measured in radians.

The point P lies on C and is a local maximum, as shown in Figure 3.

(a) Use calculus and a suitable trigonometric identity to find the x coordinate of P.
Give the answer to 3 significant figures.

(4)

The region R, shown shaded in Figure 3, is bounded by C, the x-axis and the lines with equations $x = \frac{\pi}{6}$ and $x = \frac{\pi}{2}$

(b) Use algebraic integration to find the exact area of R.

(4)

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