2. $f(x) = x^3 + 3x^2 + 4x - 12$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\left(\frac{4(3 - x)}{(3 + x)}\right)} \quad x \neq -3$$

(3)

The equation $x^3 + 3x^2 + 4x - 12 = 0$ has a single root which is between 1 and 2

(b) Use the iteration formula

$$x_{n+1} = \sqrt{\left(\frac{4(3 - x_n)}{(3 + x_n)}\right)} \quad n \geqslant 0$$

with $x_0 = 1$ to find, to 2 decimal places, the value of x_1, x_2 and x_3

(3)

The root of $f(x) = 0$ is α.

(c) By choosing a suitable interval, prove that $\alpha = 1.272$ to 3 decimal places.

(2)

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7.

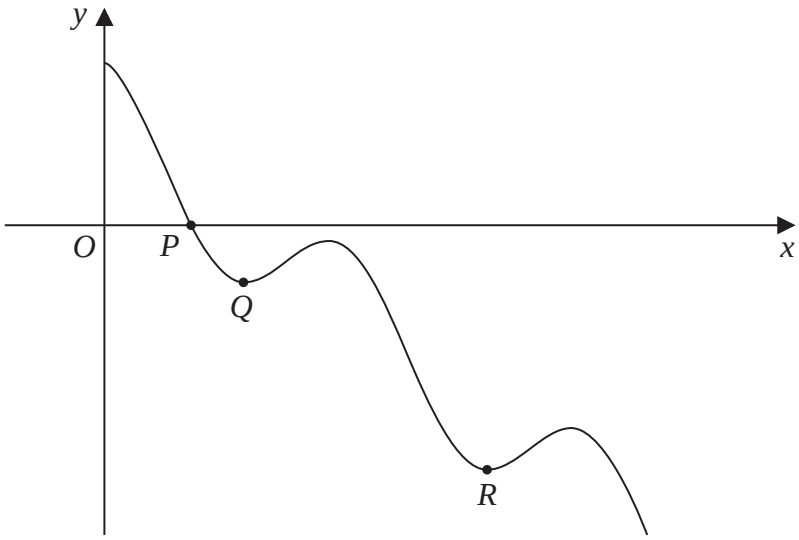


Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 2\cos 3x - 3x + 4 \qquad x > 0$$

where x is measured in radians.

The curve crosses the x-axis at the point P, as shown in Figure 3.

Given that the x coordinate of P is α,

(a) show that α lies between 0.8 and 0.9

(2)

The iteration formula

$$x_{n+1} = \frac{1}{3}\arccos(1.5x_n - 2)$$

can be used to find an approximate value for α.

(b) Using this iteration formula with $x_1 = 0.8$ find, to 4 decimal places, the value of

(i) x_2

(ii) x_5

(3)

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6.

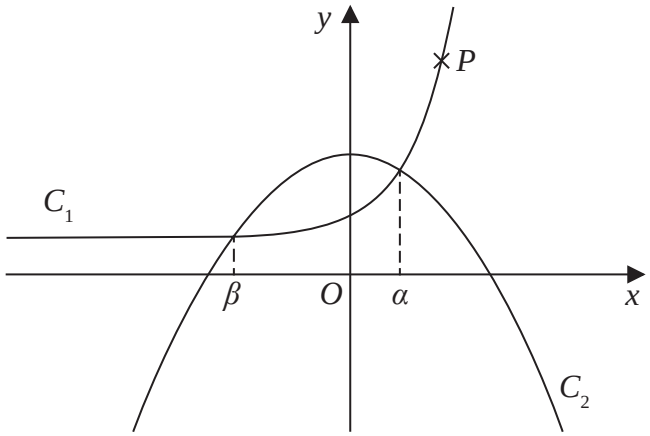


Figure 3

Figure 3 shows a sketch of curve C_1 with equation $y = 5e^{x-1} + 3$

and curve C_2 with equation $y = 10 - x^2$

The point P lies on C_1 and has y coordinate 18

(a) Find the x coordinate of P, writing your answer in the form $\ln k$, where k is a constant to be found.

(3)

The curve C_1 meets the curve C_2 at $x = \alpha$ and at $x = \beta$, as shown in Figure 3.

(b) Using a suitable interval and a suitable function that should be stated, show that to 3 decimal places $\alpha = 1.134$

(3)

The iterative equation

$$x_{n+1} = -\sqrt{7 - 5e^{x_n - 1}}$$

is used to find an approximation to β.

Using this iterative formula with $x_1 = -3$

(c) find the value of x_2 and the value of β, giving each answer to 6 decimal places.

(3)

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6.

$$f(x) = x\cos\left(\frac{x}{3}\right) \qquad x > 0$$

(a) Find $f'(x)$

(2)

(b) Show that the equation $f'(x) = 0$ can be written as

$$x = k\arctan\left(\frac{k}{x}\right)$$

where k is an integer to be found.

(2)

(c) Starting with $x_1 = 2.5$ use the iteration formula

$$x_{n+1} = k\arctan\left(\frac{k}{x_n}\right)$$

with the value of k found in part (b), to calculate the values of x_2 and x_6 giving your answers to 3 decimal places.

(2)

(d) Using a suitable interval and a suitable function that should be stated, show that a root of $f'(x) = 0$ is 2.581 correct to 3 decimal places.

(2)

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June 2021

1.

Using the iteration formula

$$x_{n+1} = 2\arctan\left(\frac{4}{x_n}\right) \qquad x_1 = 2$$

(b) find the value of x_2 and the value of x_6, giving your answers to 3 decimal places.

(3)

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9.

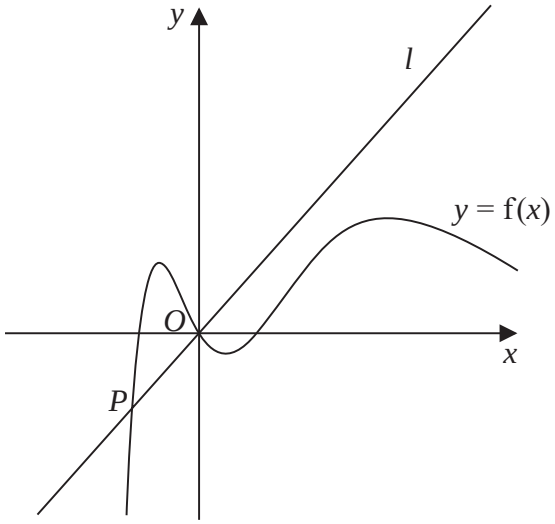


Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = x(x^2 - 4)e^{-\frac{1}{2}x}$$

The line l is the normal to the curve at O and meets the curve again at the point P.

The point P lies in the 3rd quadrant, as shown in Figure 3.

x coordinate of P is a solution of the equation

$$x = -\frac{1}{2}\sqrt{16 + e^{\frac{1}{2}x}}$$

(c) Using the iterative formula

$$x_{n+1} = -\frac{1}{2}\sqrt{16 + e^{\frac{1}{2}x_n}} \quad \text{with } x_1 = -2$$

find, to 4 decimal places,

(i) the value of x_2

(ii) the x coordinate of P.

(3)

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5.

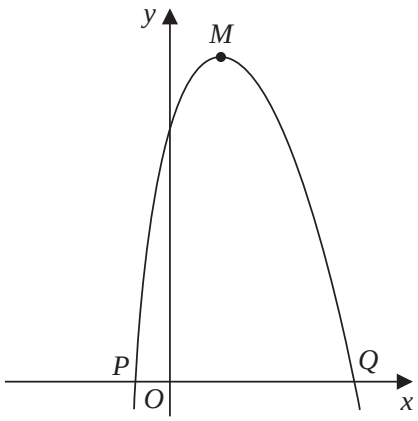


Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = 6\ln(2x + 3) - \frac{1}{2}x^2 + 4 \qquad x > -\frac{3}{2}$$

The curve cuts the negative x-axis at the point P, as shown in Figure 1.

(a) Show that the x coordinate of P lies in the interval $[-1.25, -1.2]$

(2)

The curve cuts the positive x-axis at the point Q, also shown in Figure 1.

Using the iterative formula

$$x_{n+1} = \sqrt{12\ln(2x_n + 3) + 8} \quad \text{with} \quad x_1 = 6$$

(b) (i) find, to 4 decimal places, the value of x_2

(ii) find, by continued iteration, the x coordinate of Q. Give your answer to 4 decimal places.

(3)

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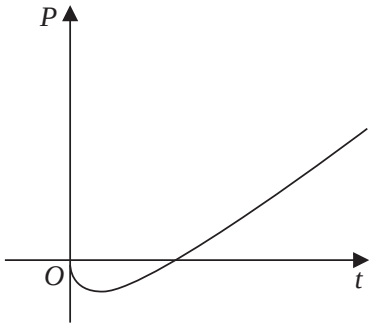


Figure 2

The profit made by a company, £P million, t years after the company started trading, is modelled by the equation

$$P = \frac{4t-1}{10} + \frac{3}{4}\ln\left[\frac{t+1}{(2t+1)^2}\right]$$

The graph of P against t is shown in Figure 2.

(c) Show that the equation $P = 0$ can be expressed in the form

$$t = \frac{1}{4} + \frac{15}{8}\ln\left[\frac{(2t+1)^2}{t+1}\right]$$

(2)

(d) Using the iteration formula

$$t_{n+1} = \frac{1}{4} + \frac{15}{8}\ln\left[\frac{(2t_n+1)^2}{t_n+1}\right] \quad \text{with} \quad t_1 = 6$$

find the value of t_2 and the value of t_6, giving your answers to 3 decimal places.

(3)

(e) Hence find, according to the model, how many months it takes in total, from when the company started trading for it to make a profit.

(2)

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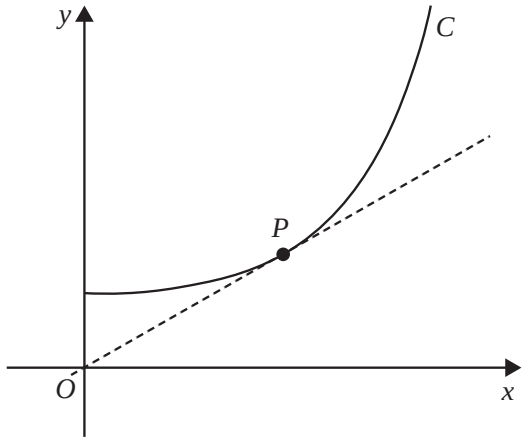


Figure 3

Figure 3 shows a sketch of part of the curve C with equation

$$y = \sqrt{3 + 4e^{x^2}} \qquad x \geqslant 0$$

The point P with x coordinate α lies on C.

$x = \alpha$ is a solution of the equation

$$4x^2e^{x^2} - 4e^{x^2} - 3 = 0$$

(c) Hence show that α lies between 1 and 2

(2)

(d) Show that the equation in part (b) can be written in the form

$$x = \frac{1}{2}\sqrt{4 + 3e^{-x^2}}$$

(1)

The iteration formula

$$x_{n+1} = \frac{1}{2}\sqrt{4 + 3e^{-x_n^2}}$$

with $x_1 = 1$ is used to find an approximation for α.

(e) Use the iteration formula to find, to 4 decimal places, the value of

(i) x_3

(ii) α

(3)

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May 2023

1. $g(x) = x^6 + 2x - 1000$

(a) Show that $g(x) = 0$ has a root α in the interval $[3, 4]$

(2)

Using the iteration formula

$$x_{n+1} = \sqrt[6]{1000 - 2x_n} \qquad \text{with } x_1 = 3$$

(b) (i) find, to 4 decimal places, the value of x_2

(ii) find, by repeated iteration, the value of α.
Give your answer to 4 decimal places.

(3)

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October 2023

1. A curve has equation $y = f(x)$ where

$$f(x) = x^2 - 5x + e^x \qquad x \in \mathbb{R}$$

(a) Show that the equation $f(x) = 0$ has a root, α, in the interval $[1, 2]$

(2)

The iterative formula

$$x_{n+1} = \sqrt{5x_n - e^{x_n}}$$

with $x_1 = 1$ is used to find an approximate value for the root α.

(b) (i) Find the value of x_2 to 4 decimal places.

(ii) Find, by repeated iteration, the value of α, giving your answer to 4 decimal places.

(3)

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2. A curve has equation $y = f(x)$ where

$$f(x) = x^4 - 5x^2 + 4x - 7 \qquad x \in \mathbb{R}$$

(a) Show that the equation $f(x) = 0$ has a root, α, in the interval $[2, 3]$

(2)

(b) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt[3]{\frac{5x^2 - 4x + 7}{x}}$$

(1)

The iterative formula

$$x_{n+1} = \sqrt[3]{\frac{5x_n^{\,2} - 4x_n + 7}{x_n}}$$

is used to find α

(c) Starting with $x_1 = 2$ and using the iterative formula,

(i) find, to 4 decimal places, the value of x_2

(ii) find, to 4 decimal places, the value of α

(3)

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8.

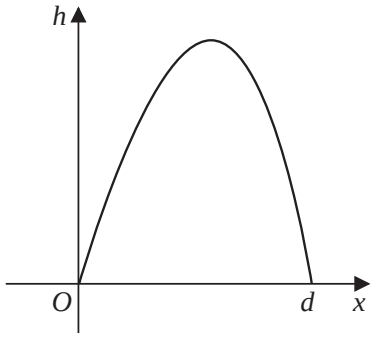


Figure 4

Figure 4 is a graph showing the path of a golf ball after the ball has been hit until it first hits the ground.

The vertical height, h metres, of the ball above the ground has been plotted against the horizontal distance travelled, x metres, measured from where the ball was hit.

The ball travels a horizontal distance of d metres before it first hits the ground.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

The path of the ball is modelled by the equation

$$h = 1.5x - 0.5x\,e^{0.02x} \qquad 0 \leqslant x \leqslant d$$

Use the model to answer parts (a), (b) and (c).

(3)

the maximum value of h occurs when

$$x = 50\ln\left(\frac{150}{x + 50}\right)$$

(4)

Using the iteration formula

$$x_{n+1} = 50\ln\left(\frac{150}{x_n + 50}\right) \qquad \text{with } x_1 = 30$$

(c) (i) find the value of x_2 to 2 decimal places,

(ii) find, by repeated iteration, the horizontal distance travelled by the golf ball before it reaches its maximum height. Give your answer to 2 decimal places.

(3)

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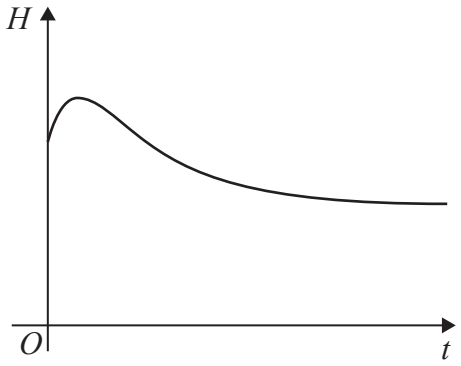


Figure 4

The heart rate of a horse is being monitored.
The heart rate H, measured in beats per minute (bpm), is modelled by the equation

$$H = 32 + 40e^{-0.2t} - 20\,e^{-0.9t}$$

where t minutes is the time after monitoring began.

Figure 4 is a sketch of H against t.

Use the equation of the model to answer parts (a) to (e).

(a) State the initial heart rate of the horse.

(1)

In the long term, the heart rate of the horse approaches L bpm.

(b) State the value of L.

(1)

The heart rate of the horse reaches its maximum value after T minutes.

(c) Find the value of T, giving your answer to 3 decimal places.

(*Solutions based entirely on calculator technology are not acceptable.*)

(5)

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The heart rate of the horse is 37 bpm after M minutes.

(d) Show that M is a solution of the equation

$$t = 5\ln\left(\frac{8}{1+4e^{-0.9t}}\right)$$

(2)

Using the iteration formula

$$t_{n+1} = 5\ln\left(\frac{8}{1+4e^{-0.9t_n}}\right) \quad \text{with} \quad t_1 = 10$$

(e) (i) find, to 4 decimal places, the value of t_2

(ii) find, to 4 decimal places, the value of M

(3)

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January 2025

1. $f(x) = 2\sec x + 6x - 3 \qquad 0 < x < \frac{\pi}{2}$

The equation $f(x) = 0$ has a single root α

(a) Show that $0.1 < \alpha < 0.2$

(2)

(b) Show that α is a solution of

$$x = \frac{1}{2} - \frac{1}{3\cos x}$$

(1)

The iterative formula

$$x_{n+1} = \frac{1}{2} - \frac{1}{3\cos x_n}$$

is used to find α

(c) Starting with $x_1 = 0.15$ and using the iterative formula,

(i) find, to 4 decimal places, the value of x_2

(ii) find, to 4 decimal places, the value of α

(3)

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7: A continuous curve has equation

$$y = e^{-x^2} \sin 3x \qquad 0 \leqslant x \leqslant \frac{\pi}{3}$$

The curve has a stationary point at the point P.

(a) Show, using calculus, that the x coordinate of P is a solution of the equation

$$x = \frac{1}{3} \arctan\left(\frac{3}{2x}\right)$$

(4)

Using the iteration formula

$$x_{n+1} = \frac{1}{3} \arctan\left(\frac{3}{2x_n}\right) \qquad x_1 = 0.4$$

(b) find the value of

(i) x_2

(ii) x_4

giving your answers to 4 decimal places.

(3)

(c) Using a suitable interval and a suitable function which should be stated, show that the x coordinate of P is 0.430 correct to 3 decimal places.

(2)

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2: $f(x) = 4x^3 + 2x^2 - 12$

The equation $f(x) = 0$ has a single root α.

(a) Show that α lies in the interval [1, 2]

(2)

(b) Show that the equation $f(x) = 0$ can be written as

$$x = \sqrt{\frac{k}{2x+1}}$$

where k is a constant to be found.

(2)

(c) Using the iteration formula

$$x_{n+1} = \sqrt{\frac{k}{2x_n+1}}$$

with $x_1 = 1$ and the value of k found in part (b), find, to 4 decimal places,

(i) the value of x_3

(ii) the value of α.

(3)

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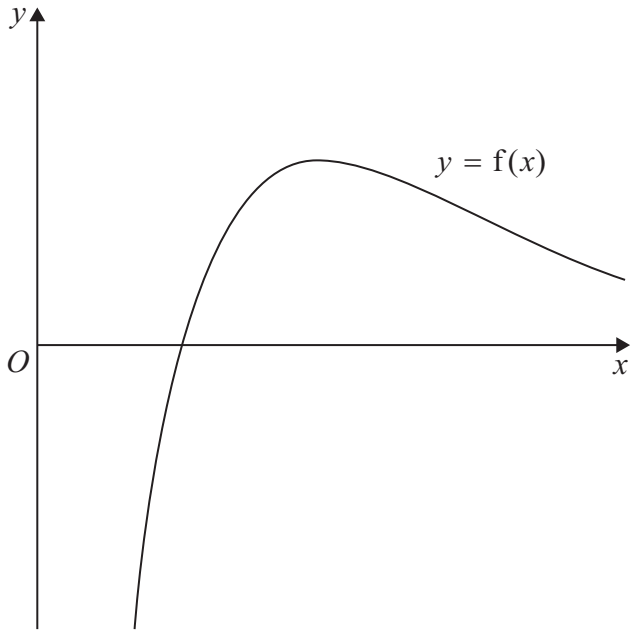


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{2x^2 + 3x - 4}{\mathrm{e}^x} - \frac{1}{x^2} \qquad x \in \mathbb{R} \quad x \neq 0$$

(a) Show that $\mathrm{f}(x) = 0$ has a root α in the interval $[1, 2]$

(2)

(b) Show that the equation $\mathrm{f}(x) = 0$ can be written in the form

$$x = \sqrt[3]{\frac{\mathrm{e}^x + 4x^2}{2x + 3}}$$

(2)

Using the iteration formula

$$x_{n+1} = \sqrt[3]{\frac{\mathrm{e}^{x_n} + 4x_n^{\,2}}{2x_n + 3}} \qquad \text{with } x_1 = 1$$

find, to 4 decimal places,

(c) (i) the value of x_3

(ii) the value of α

(3)

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January 2026

1. $$g(x) = 3x - 5 + 5\sin\left(x^2\right)$$

where x is measured in radians.

(a) Show that $g(x) = 0$ has a root α in the interval $[0.7, 0.8]$

(2)

(b) Show that the equation $g(x) = 0$ can be written in the form

$$x = \left[\arcsin(1 - px)\right]^{\frac{1}{2}}$$

where p is a constant to be found.

(1)

Using the iteration formula

$$x_{n+1} = \left[\arcsin(1 - px_n)\right]^{\frac{1}{2}}$$

with the value of p found in part (b) and starting with $x_1 = 0.7$

(c) (i) find, to 4 decimal places, the value of x_2

(ii) find, by repeated iteration, the value of α, giving your answer to 4 decimal places.

(3)

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June 2026

1. A curve has equation $y = f(x)$ where

$$f(x) = \frac{2}{3}x^3 - 3x^2 + 5x + e^{2x}(1 - 2x) \qquad x \in \mathbb{R}$$

(a) Find $f'(x)$ giving the answer in simplest form.

(3)

The equation $f'(x) = 0$ has a single root, α

(b) Show that α lies in the interval $[0, 1]$

(2)

Given that the equation $f'(x) = 0$ can be written as

$$x = \frac{2x^2 + 5}{4e^{2x} + 6}$$

(c) use the iteration formula

$$x_{n+1} = \frac{2x_n^{\,2} + 5}{4e^{2x_n} + 6} \qquad \text{with } x_1 = 0$$

to find, to 4 decimal places,

(i) the value of x_3

(ii) the value of α

(3)

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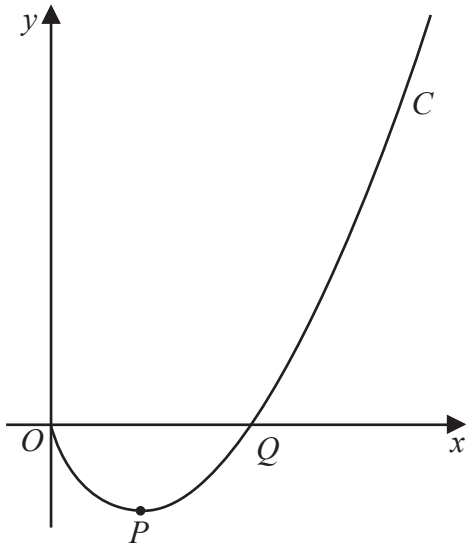


Figure 1

Figure 1 is a sketch of part of the curve C with equation

$$y = x(3 + x)\ln(2x) \qquad x > 0$$

The curve cuts the x-axis at the point Q.

(a) Find the x coordinate of Q.

(1)

The point P with x coordinate α is the stationary point on C.

(b) Show that α is a solution of

$$x = \frac{1}{2}\mathrm{e}^{-\left(\frac{x+3}{2x+3}\right)}$$

(4)

The iteration formula

$$x_{n+1} = \frac{1}{2}\mathrm{e}^{-\left(\frac{x_n+3}{2x_n+3}\right)}$$

is used to find an approximate value of α

Starting with $x_1 = 0.5$

(c) (i) find, to 4 decimal places, the value of x_2

(ii) find, by repeated iteration, the value of α, giving the answer to 4 decimal places.

(3)

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