6. Prove by contradiction that, if a, b are positive real numbers, then $a + b \geqslant 2\sqrt{ab}$

(4)

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1. Given that n is an integer, use algebra, to prove by contradiction, that if n^3 is even then n is even.

(4)

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3. Prove by contradiction that there is no greatest odd integer.

(2)

9.

(ii) Given that $n \in \mathbb{N}$, prove by contradiction that if n^2 is a multiple of 3 then n is a multiple of 3

(5)

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10. (a) A student's attempt to answer the question

"Prove by contradiction that if n^3 is even, then n is even"

is shown below. Line 5 of the proof is missing.

Assume that there exists a number n such that n^3 is even, but n is odd.
If n is odd then $n = 2p + 1$ where $p \in \mathbb{Z}$
So $n^3 = (2p + 1)^3$
$= 8p^3 + 12p^2 + 6p + 1$
$=$
This contradicts our initial assumption, so if n^3 is even, then n is even.

Complete this proof by filling in line 5.

(1)

(b) Hence, prove by contradiction that $\sqrt[3]{2}$ is irrational.

(5)

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6. Three **consecutive** terms in a sequence of real numbers are given by

$$k,\ 1 + 2k \text{ and } 3 + 3k$$

where k is a constant.

Use proof by contradiction to show that this sequence is not a geometric sequence.

(5)

9. Use proof by contradiction to show that, when n is an integer,

$$n^2 - 2$$

is **never** divisible by 4

(4)

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8. A student was asked to prove by contradiction that

"there are no positive integers x and y such that $3x^2 + 2xy - y^2 = 25$"

The start of the student's proof is shown in the box below.

Assume that integers x and y exist such that $3x^2 + 2xy - y^2 = 25$

$$\Rightarrow (3x - y)(x + y) = 25$$

If $(3x - y) = 1$ and $(x + y) = 25$

$$\left.\begin{aligned} 3x - y &= 1 \\ x + y &= 25 \end{aligned}\right\} \Rightarrow 4x = 26 \Rightarrow x = 6.5, y = 18.5$$ Not integers

Show the calculations and statements that are needed to complete the proof.

(4)

9. A student was asked to prove, for $p \in \mathbb{N}$, that

"if p^3 is a multiple of 3, then p must be a multiple of 3"

The start of the student's proof by contradiction is shown in the box below.

Assumption:

There exists a number p, $p \in \mathbb{N}$, such that p^3 is a multiple of 3, and p is NOT a multiple of 3

Let $p = 3k + 1$, $k \in \mathbb{N}$.

Consider $p^3 = (3k + 1)^3 = 27k^3 + 27k^2 + 9k + 1$

$= 3(9k^3 + 9k^2 + 3k) + 1$ which is not a multiple of 3

(a) Show the calculations and statements that are required to complete the proof.

(3)

(b) Hence prove, by contradiction, that $\sqrt[3]{3}$ is an irrational number.

(5)

7. Use proof by contradiction to prove that $\sqrt{7}$ is irrational.

(You may assume that if k is an integer and k^2 is a multiple of 7 then k is a multiple of 7)

(4)

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4. (a) Prove by contradiction that for all positive numbers k

$$k + \frac{9}{k} \geqslant 6$$

(4)

(b) Show that the result in part (a) is not true for all real numbers.

(1)

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8. Use proof by contradiction to prove that the curve with equation

$$y = 2x + x^3 + \cos x$$

has no stationary points.

(4)

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2. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

The curve C_1 has equation

$$y = x^4 + 10x^2 + 8 \qquad x \in \mathbb{R}$$

The curve C_2 has equation

$$y = 2x^2 - 7 \qquad x \in \mathbb{R}$$

Use algebra to prove by contradiction that C_1 and C_2 do **not** intersect.

(4)

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6. Given that $n \in \mathbb{N}$, use algebra to prove by contradiction that

" if $n^2 - 4n + 5$ is even then n is odd"

(4)

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June 2025

10. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Use proof by contradiction to show that for all angles x, where $90° < x < 180°$

$$\left| \frac{\cos 2x}{\cos x - \sin x} \right| < 1$$

(4)

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10: [*In this question you may assume that if n is an integer and n^2 is even, then n is even.*]

Given that p and q are integers, use proof by contradiction to show that

$$p^2 - 4q + 2 \neq 0$$

(5)

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10. (a) Statement X: "If x^2 is an odd square number, then x is an odd number."

A student was asked to prove Statement X using proof by contradiction.

The student begins by writing the following.

> Assume that x^2 is an odd square number and x is an even number.
>
> If x is an even number it can be expressed in the form $x = 2m \quad m \in \mathbb{N}$

Complete the proof.

(2)

(b) Use proof by contradiction to show that

"There are no integers a and b such that $a^2 - 4b = 27$"

You may assume Statement X without further justification.

(4)

3. (a) Factorise $x^2 - 4y^2$

(1)

(b) Use proof by contradiction to prove that there are no positive integers x and y such that

$$x^2 - 4y^2 = 27$$

(4)

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8. (a) Factorise $4p^2 - q^2$

(1)

(b) Hence, prove by contradiction, that there are no positive integers p and q such that

$$4p^2 - q^2 = 46$$

(4)

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