7.

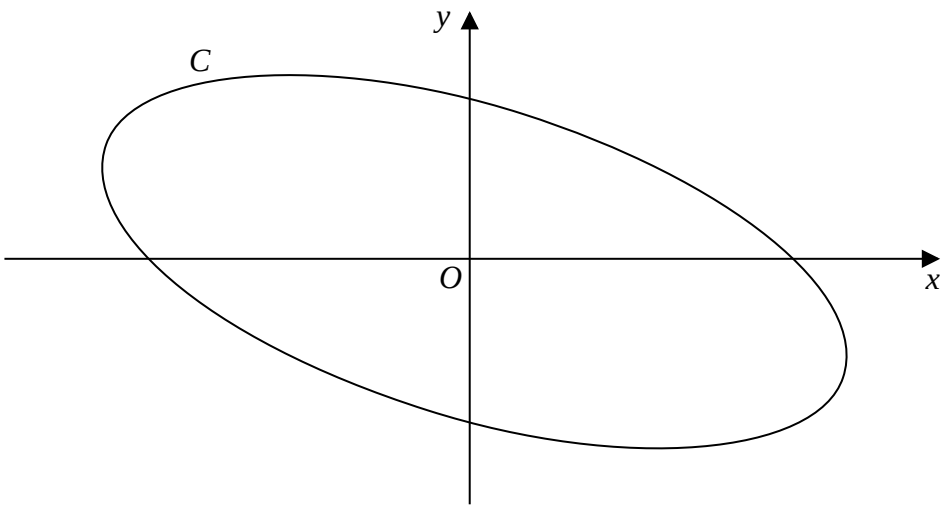


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 4\cos\left(t + \frac{\pi}{6}\right) \qquad y = 2\sin t \qquad 0 \leqslant t \leqslant 2\pi$$

(a) Show that

$$x + y = 2\sqrt{3}\cos t$$

(3)

(b) Show that a cartesian equation of C is

$$(x + y)^2 + ay^2 = b$$

where a and b are integers to be found.

(2)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

4.

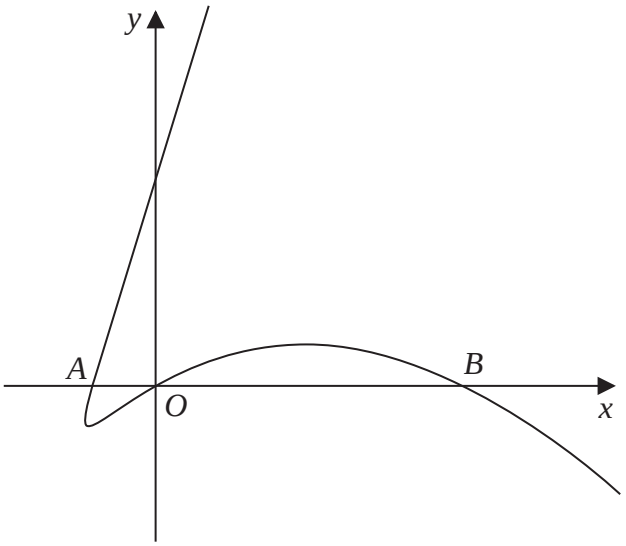


Figure 2

Figure 2 shows a sketch of part of the curve with parametric equations

$$x = 2t^2 - 6t, \qquad y = t^3 - 4t, \qquad t \in \mathbb{R}$$

The curve cuts the x-axis at the origin and at the points A and B, as shown in Figure 2.

(a) Find the coordinates of A and show that B has coordinates (20, 0).

(3)

(b) Show that the equation of the tangent to the curve at B is

$$7y + 4x - 80 = 0$$

(5)

The tangent to the curve at B cuts the curve again at the point P.

(c) Find, using algebra, the x coordinate of P.

(4)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

4. The curve C is defined by the parametric equations

$$x = \frac{1}{t} + 2 \qquad y = \frac{1 - 2t}{3 + t} \qquad t > 0$$

(a) Show that the equation of C can be written in the form $y = \mathrm{g}(x)$ where g is the function

$$\mathrm{g}(x) = \frac{ax + b}{cx + d} \qquad x > k$$

where a, b, c, d and k are integers to be found.

(5)

(b) Hence, or otherwise, state the range of g.

(2)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

3. The curve C has parametric equations

$$x = 3 + 2\sin t \qquad y = \frac{6}{7 + \cos 2t} \qquad -\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}$$

(a) Show that C has Cartesian equation

$$y = \frac{12}{(7 - x)(1 + x)} \qquad p \leqslant x \leqslant q$$

where p and q are constants to be found.

(6)

(b) Hence, find a Cartesian equation for C in the form

$$y = \frac{a}{x + b} + \frac{c}{x + d} \qquad p \leqslant x \leqslant q$$

where a, b, c and d are constants.

(3)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

October 2022

1. A curve C has parametric equations

$$x = \frac{t}{t-3} \qquad y = \frac{1}{t} + 2 \qquad t \in \mathbb{R} \quad t > 3$$

Show that all points on C lie on the curve with Cartesian equation

$$y = \frac{ax - 1}{bx}$$

where a and b are constants to be found.

(3)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

2. A set of points $P(x, y)$ is defined by the parametric equations

$$x = \frac{t-1}{2t+1} \qquad y = \frac{6}{2t+1} \qquad t \neq -\frac{1}{2}$$

(a) Show that all points $P(x, y)$ lie on a straight line.

(4)

(b) Hence or otherwise, find the x coordinate of the point of intersection of this line and the line with equation $y = x + 12$

(2)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

5.

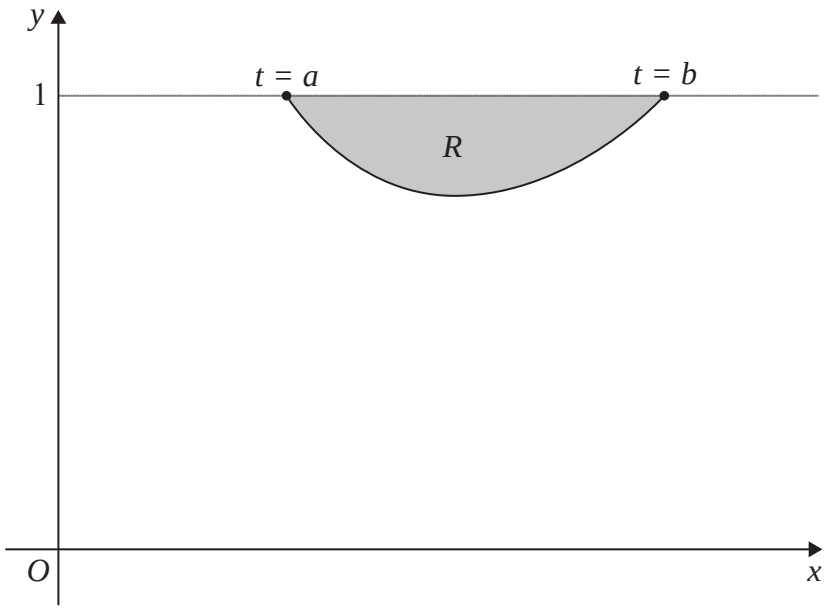


Figure 2

Figure 2 shows a sketch of the curve defined by the parametric equations

$$x = t^2 + 2t \qquad y = \frac{2}{t(3-t)} \qquad a \leqslant t \leqslant b$$

where a and b are constants.

The ends of the curve lie on the line with equation $y = 1$

(a) Find the value of a and the value of b

(2)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

3.

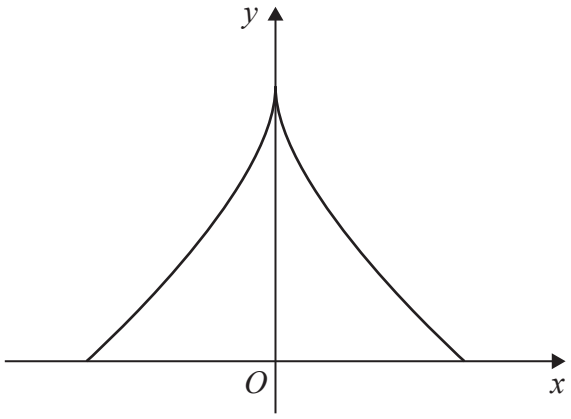


Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = 3\sin^3\theta \qquad y = 1 + \cos 2\theta \qquad -\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$$

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = k\,\mathrm{cosec}\,\theta \qquad \theta \neq 0$$

where k is a constant to be found.

(3)

The point P lies on C where $\theta = \frac{\pi}{6}$

(b) Find the equation of the tangent to C at P, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(3)

(c) Show that C has Cartesian equation

$$8x^2 = 9(2 - y)^3 \qquad -q \leqslant x \leqslant q$$

where q is a constant to be found.

(3)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

9.

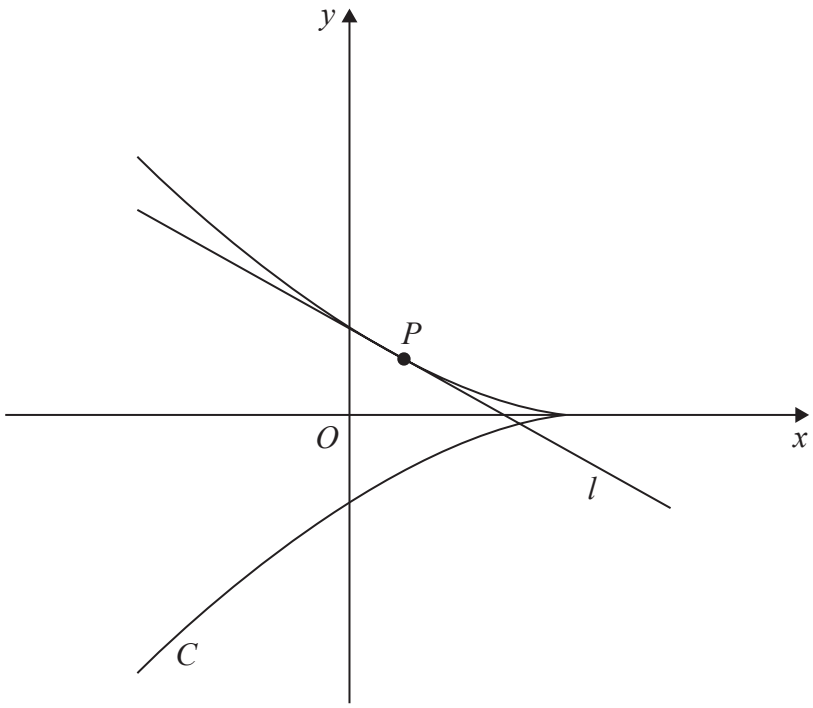


Figure 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 2\cos 2t \qquad y = \sin^3 t \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

where t is a parameter.

The point P lies on C where $t = \frac{\pi}{6}$

The line l, shown in Figure 2, is the tangent to C at P.

(a) Use parametric differentiation to show that

(i) $\frac{dy}{dx} = k\sin t$ where k is a constant to be found

(ii) an equation for l is $3x + 16y - 5 = 0$

(6)

The line l intersects the curve C again at the point Q.

(b) Using algebra and showing detailed reasoning, find the exact coordinates of Q.

(6)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

5. The curve C has parametric equations

$$x = \frac{3 + 2t}{1 - t} \qquad y = 1 - t^2 \qquad t \neq 1$$

The point P, where $t = 2$, lies on C.

(a) Use parametric differentiation to find the equation of the normal to C at P. Give your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(5)

(b) Show that a Cartesian equation for C can be expressed in the form

$$y = \frac{px + q}{(x + r)^2} \qquad x \neq k$$

where p, q, r and k are integers to be found.

(4)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

6: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

A curve C has parametric equations

$$x = 8\cos t + 2 \qquad y = 6\sin t - 3 \qquad 0 < t \leqslant 2\pi$$

The point P lies on C where $t = \frac{\pi}{3}$

(a) Use parametric differentiation to find the exact gradient of the **tangent** to C at P.

(3)

The **normal** to C at P intersects the x-axis at the point R.

(b) Find, using algebra, the exact coordinates of R.

(6)

(c) Find a Cartesian equation of C. The equation must not involve trigonometric expressions.

(2)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

October 2025

3. A curve has parametric equations

$$x = \frac{t + 15}{t + 4} \qquad y = \frac{5}{t + 2} \qquad t \geqslant 0$$

(a) Show that a Cartesian equation of the curve is $y = g(x)$ where

$$g(x) = \frac{ax + b}{cx + d} \qquad e < x \leqslant f$$

and a, b, c, d, e and f are constants to be found.

(5)

(b) State the range of g.

(2)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

5.

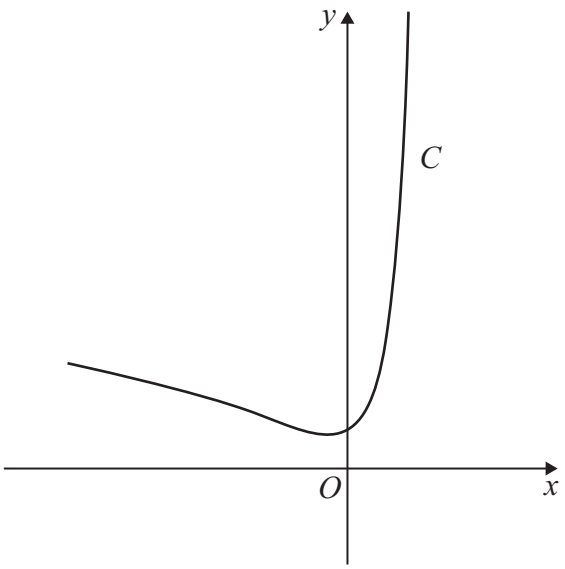


Figure 1

The curve C, shown in Figure 1, has parametric equations

$$x = \frac{4t - 5}{2t + 1} \qquad y = 2t^2 - 4t + 4 \qquad t \geqslant 0$$

The point P lies on C and has parameter $t = 3$

(a) Find the coordinates of P.

(2)

(b) Find $\frac{dy}{dx}$ in terms of t, writing the answer in simplest form.

(4)

(c) Hence find the equation of the tangent to C at P. Write the answer in the form $y = mx + c$, where m and c are constants.

(2)

Given that

- the Cartesian equation of C is $y = \mathrm{f}(x)$
- the function f is defined for all values of x on C

(d) (i) find the domain of f

(ii) find the range of f

(3)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

7.

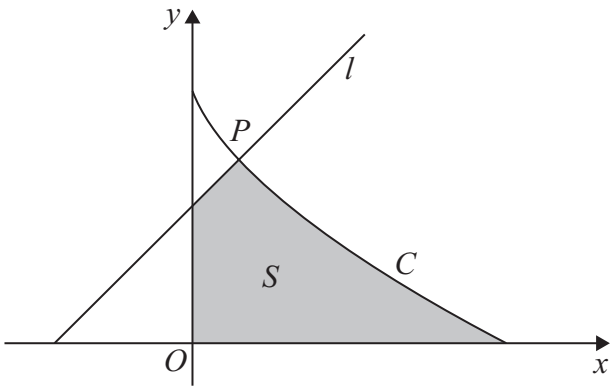


Figure 1

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve C with parametric equations

$$x = 8\cos^3\theta \qquad y = 6\sin^2\theta \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

Given that the point P lies on C and has parameter $\theta = \frac{\pi}{3}$

(a) find the coordinates of P.

(2)

The line l is the normal to C at P.

(b) Find an equation for l in the form $y = mx + c$, where m and c are constants.

(5)

The finite region S, shown shaded in Figure 1, is bounded by the curve C, the line l, the y-axis and the x-axis.

(c) Show that the area of S is given by

$$4 + 144\int_0^{\frac{\pi}{3}} (\sin\theta\cos^2\theta - \sin\theta\cos^4\theta)\,\mathrm{d}\theta$$

(6)

(d) Hence, using algebraic integration, find the exact area of S.

(3)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

3. The curve C is defined by the parametric equations

$$x = \frac{4}{t+2} \qquad y = \frac{6t}{t+4} \qquad t > -1$$

The point P with parameter $t = 2$ lies on C.

(a) Use parametric differentiation to find the equation of the tangent to C at P. Give the answer in the form $y = mx + c$, where m and c are constants.

(5)

(b) Show that the equation of C can be written in the form $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{ax+b}{x+d} \qquad 0 < x < k$$

and where a, b, d and k are integers to be found.

(4)

(c) Hence, or otherwise, find the range of f.

(2)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

4. The curve C has parametric equations

$$x = \frac{3}{5t-2} \qquad y = \frac{t}{4-t} \qquad t \in \mathbb{R} \qquad t \neq \frac{2}{5} \qquad t \neq 4$$

(a) Show that the Cartesian equation of C is

$$y = \frac{ax+b}{cx+d} \qquad x \neq -\frac{d}{c}$$

where a, b, c and d are integers to be found.

(3)

The line l has equation $y = -x + 1$

(b) Use algebra to prove by contradiction that l and C do not intersect.

(4)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362

10.

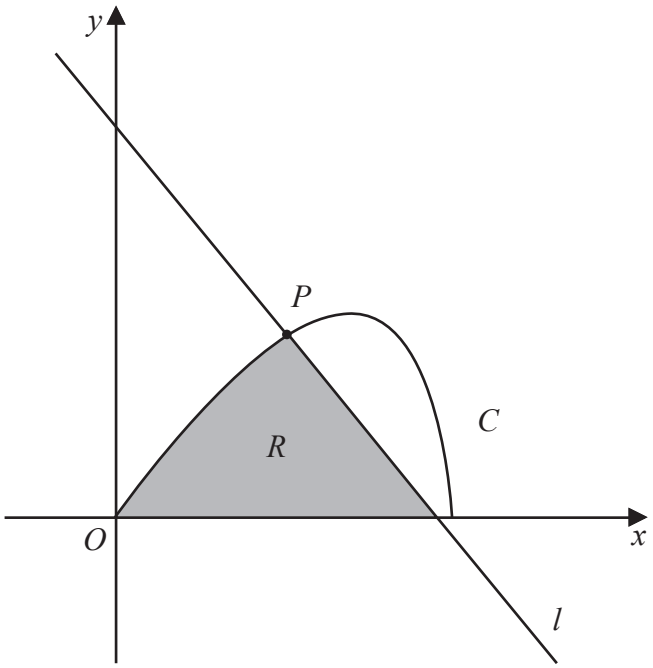


Figure 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 8\cos\theta \qquad y = 5\sin 2\theta \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

The point P with parameter $\theta = \frac{\pi}{3}$ lies on C.

The line l is the normal to C at P, as shown in Figure 2.

(a) Use calculus to show that l has equation

$$y = p\sqrt{3}\,x + q\sqrt{3}$$

where p and q are rational constants to be found.

(4)

The finite region R, shown shaded in Figure 2, is bounded by C, l and the x-axis.

(b) Using algebraic integration, find the exact area of R, giving the answer in the form $A + B\sqrt{3}$ where A and B are rational constants.

(6)

(Total for Question 10 is 10 marks)

SirAyazRao aayazrao ayazrao.com AyazRao +92 346 2656 362