2. A curve C has the equation

$$x^3 + 2xy - x - y^3 - 20 = 0$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

(b) Find an equation of the tangent to C at the point $(3, -2)$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(2)

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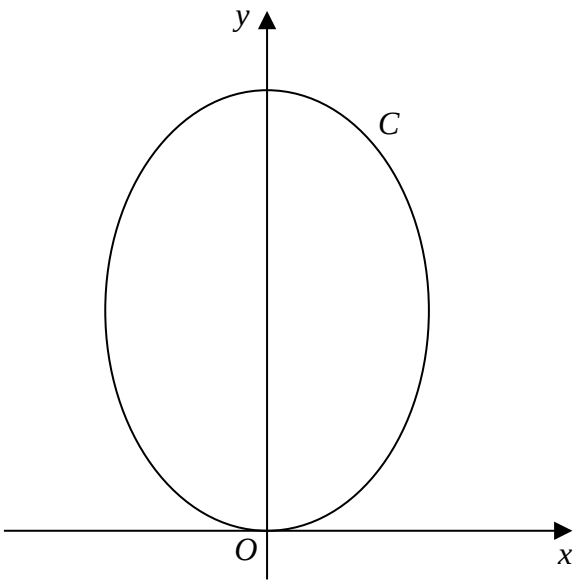


Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = \sqrt{3}\sin 2t \qquad y = 4\cos^2 t \qquad 0 \leqslant t \leqslant \pi$$

(a) Show that $\frac{dy}{dx} = k\sqrt{3}\tan 2t$, where k is a constant to be found.

(5)

(b) Find an equation of the tangent to C at the point where $t = \frac{\pi}{3}$

Give your answer in the form $y = ax + b$, where a and b are constants.

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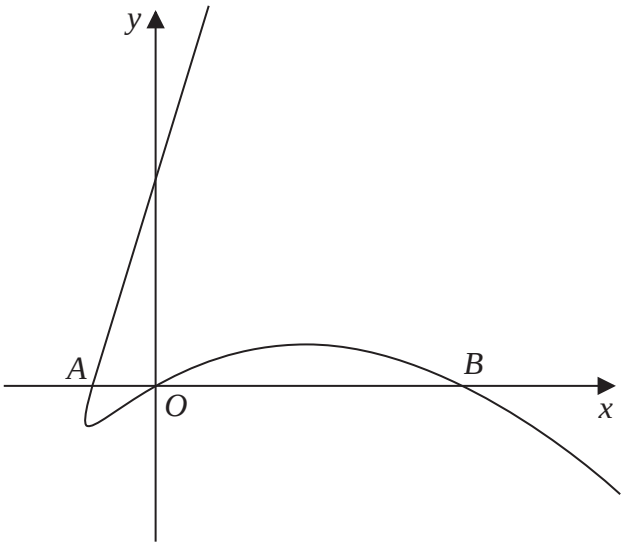


Figure 2

Figure 2 shows a sketch of part of the curve with parametric equations

$$x = 2t^2 - 6t, \qquad y = t^3 - 4t, \qquad t \in \mathbb{R}$$

The curve cuts the x-axis at the origin and at the points A and B, as shown in Figure 2.

(a) Find the coordinates of A and show that B has coordinates (20, 0).

(3)

(b) Show that the equation of the tangent to the curve at B is

$$7y + 4x - 80 = 0$$

(5)

The tangent to the curve at B cuts the curve again at the point P.

(c) Find, using algebra, the x coordinate of P.

(4)

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6. A curve C has equation

$$y = x^{\sin x} \qquad x > 0 \qquad y > 0$$

(a) Find, by firstly taking natural logarithms, an expression for $\frac{dy}{dx}$ in terms of x and y.

(5)

(b) Hence show that the x coordinates of the stationary points of C are solutions of the equation

$$\tan x + x \ln x = 0$$

(2)

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6. A curve has equation

$$4y^2 + 3x = 6ye^{-2x}$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

The curve crosses the y-axis at the origin and at the point P.

(b) Find the equation of the normal to the curve at P, writing your answer in the form $y = mx + c$ where m and c are constants to be found.

(4)

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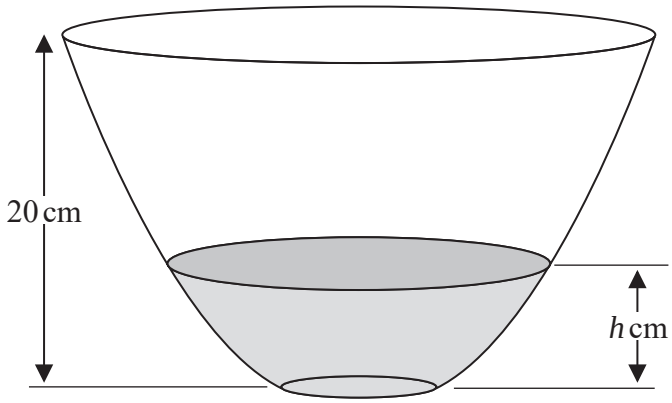


Figure 2

A bowl with circular cross section and height 20 cm is shown in Figure 2.

The bowl is initially empty and water starts flowing into the bowl.

When the depth of water is h cm, the volume of water in the bowl, V cm^3, is modelled by the equation

$$V = \frac{1}{3}h^2(h + 4) \qquad 0 \leqslant h \leqslant 20$$

Given that the water flows into the bowl at a constant rate of 160 cm^3 s^{-1}, find, according to the model,

(a) the time taken to fill the bowl,

(2)

(b) the rate of change of the depth of the water, in cm s^{-1}, when $h = 5$

(5)

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5. A curve has equation

$$y^2 = y\mathrm{e}^{-2x} - 3x$$

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2y\mathrm{e}^{-2x} + 3}{\mathrm{e}^{-2x} - 2y}$$

(4)

The curve crosses the y-axis at the origin and at the point P.

The tangent to the curve at the origin and the tangent to the curve at P meet at the point R.

(b) Find the coordinates of R.

(5)

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October 2021

1. The curve C has equation

$$2x - 4y^2 + 3x^2y = 4x^2 + 8$$

The point $P(3, 2)$ lies on C.

Find the equation of the normal to C at the point P, writing your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(7)

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3. $$g(x) = \frac{3x^3 + 8x^2 - 3x - 6}{x(x+3)} \equiv Ax + B + \frac{C}{x} + \frac{D}{x+3}$$

(a) Find the values of the constants A, B, C and D.

(5)

A curve has equation

$$y = g(x) \qquad x > 0$$

Using the answer to part (a),

(b) find $g'(x)$.

(2)

(c) Hence, explain why $g'(x) > 3$ for all values of x in the domain of g.

(1)

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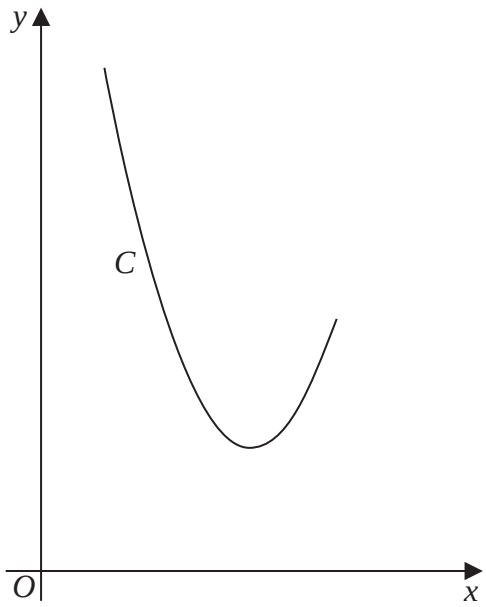


Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = 5 + 2\tan t \qquad y = 8\sec^2 t \qquad -\frac{\pi}{3} \leqslant t \leqslant \frac{\pi}{4}$$

(a) Use parametric differentiation to find the gradient of C at $x = 3$

(4)

The curve C has equation $y = \mathrm{f}(x)$, where f is a quadratic function.

(b) Find $\mathrm{f}(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants to be found.

(3)

(c) Find the range of f.

(2)

January 2022

1. The curve C has equation

$$xy^2 = x^2y + 6 \qquad x \neq 0 \quad y \neq 0$$

Find an equation for the tangent to C at the point $P(2, 3)$, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(6)

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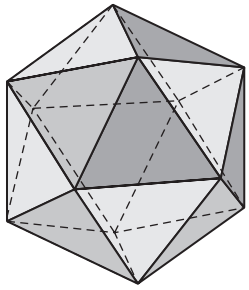


Figure 1

A regular icosahedron of side length x cm, shown in Figure 1, is expanding uniformly.

The icosahedron consists of 20 congruent equilateral triangular faces of side length x cm.

(a) Show that the surface area, A cm^2, of the icosahedron is given by

$$A = 5\sqrt{3}x^2$$

(2)

Given that the volume, V cm^3, of the icosahedron is given by

$$V = \frac{5}{12}\left(3+\sqrt{5}\right)x^3$$

(b) show that $\dfrac{\mathrm{d}V}{\mathrm{d}A} = \dfrac{\left(3+\sqrt{5}\right)x}{8\sqrt{3}}$

(3)

The surface area of the icosahedron is increasing at a constant rate of $0.025\,\text{cm}^2\,\text{s}^{-1}$

(c) Find the rate of change of the volume of the icosahedron when $x = 2$, giving your answer to 2 significant figures.

(3)

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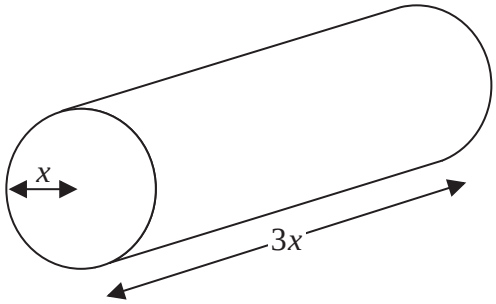


Figure 1

A tablet is dissolving in water.

The tablet is modelled as a cylinder, shown in Figure 1.

At t seconds after the tablet is dropped into the water, the radius of the tablet is x mm and the length of the tablet is $3x$ mm.

The cross-sectional area of the tablet is decreasing at a constant rate of $0.5\,\text{mm}^2\,\text{s}^{-1}$

(a) Find $\frac{\mathrm{d}x}{\mathrm{d}t}$ when $x = 7$

(4)

(b) Find, according to the model, the rate of decrease of the volume of the tablet when $x = 4$

(4)

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4. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

A curve has equation

$$16x^3 - 9kx^2y + 8y^3 = 875$$

where k is a constant.

(a) Show that

$$\frac{dy}{dx} = \frac{6kxy - 16x^2}{8y^2 - 3kx^2}$$

(4)

Given that the curve has a turning point at $x = \frac{5}{2}$

(b) find the value of k

(4)

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7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The curve C has parametric equations

$$x = \sin t - 3\cos^2 t \qquad y = 3\sin t + 2\cos t \qquad 0 \leqslant t \leqslant 5$$

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 3$ where $t = \pi$

(4)

The point P lies on C where $t = \pi$

(b) Find the equation of the tangent to the curve at P in the form $y = mx + c$ where m and c are constants to be found.

(3)

Given that the tangent to the curve at P cuts C at the point Q

(c) show that the value of t at point Q satisfies the equation

$$9\cos^2 t + 2\cos t - 7 = 0$$

(2)

(d) Hence find the exact value of the y coordinate of Q

(3)

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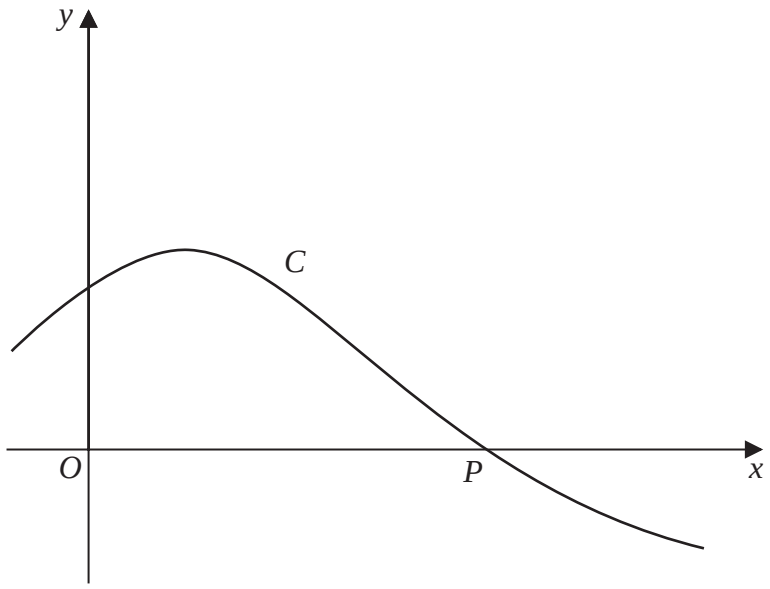


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 1 + 3\tan t \qquad y = 2\cos 2t \qquad -\frac{\pi}{6} \leqslant t \leqslant \frac{\pi}{3}$$

The curve crosses the x-axis at point P, as shown in Figure 3.

(a) Find the equation of the tangent to C at P, writing your answer in the form $y = mx + c$, where m and c are constants to be found.

(5)

The curve C has equation $y = \mathrm{f}(x)$, where f is a function with domain $\left[k, 1 + 3\sqrt{3}\right]$

(b) Find the exact value of the constant k.

(1)

(c) Find the range of f.

(2)

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10. A spherical ball of ice of radius 12 cm is placed in a bucket of water.

In a model of the situation,

- the ball remains spherical as it melts
- t minutes after the ball of ice is placed in the bucket, its radius is r cm
- the rate of decrease of the radius of the ball of ice is inversely proportional to the square of the radius
- the radius of the ball of ice is 6 cm after 15 minutes

Using the model and the information given,

(a) find an equation linking r and t,

(5)

(b) find the time taken for the ball of ice to melt completely.

(2)

(c) On Diagram 1 on page 27, sketch a graph of r against t.

(1)

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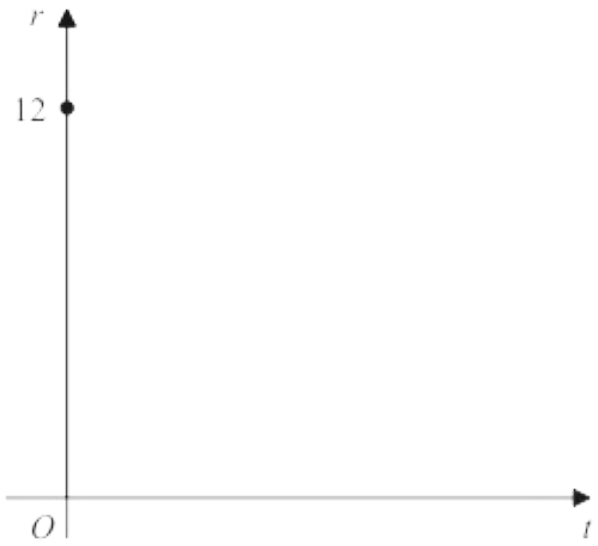


Diagram 1

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11.

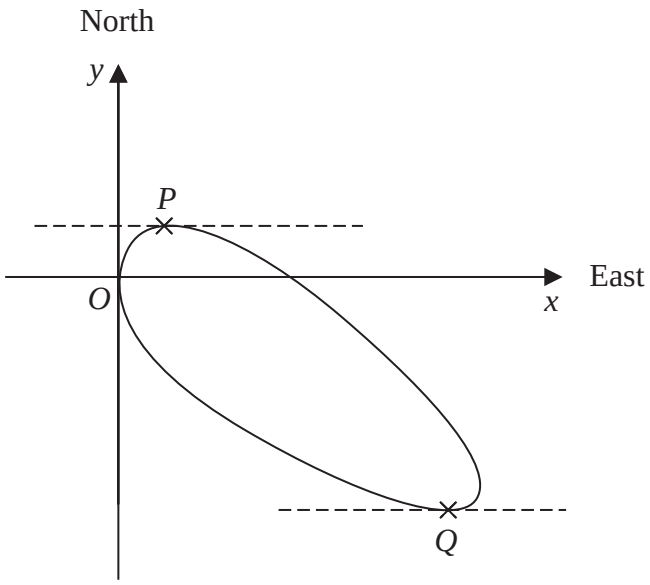


Figure 4

Figure 4 shows a sketch of the closed curve with equation

$$(x + y)^3 + 10y^2 = 108x$$

(a) Show that

$$\frac{dy}{dx} = \frac{108 - 3(x + y)^2}{20y + 3(x + y)^2}$$

(5)

The curve is used to model the shape of a cycle track with both x and y measured in km.

The points P and Q represent points that are furthest north and furthest south of the origin O, as shown in Figure 4.

Using the result given in part (a),

(b) find how far the point Q is south of O. Give your answer to the nearest 100 m.

(4)

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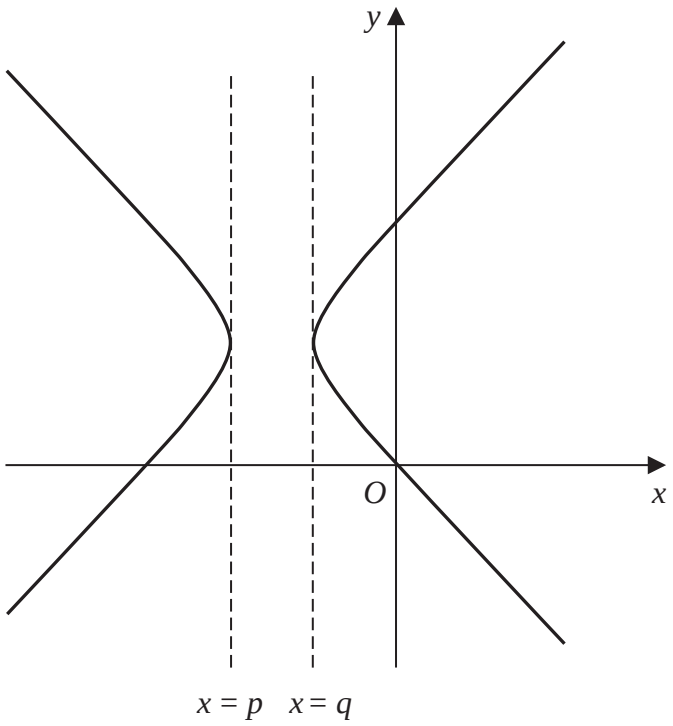


Figure 2

Figure 2 shows a sketch of the curve with equation

$$y^2 = 2x^2 + 15x + 10y$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(4)

The curve is not defined for values of x in the interval (p, q), as shown in Figure 2.

(b) Using your answer to part (a) or otherwise, find the value of p and the value of q.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(3)

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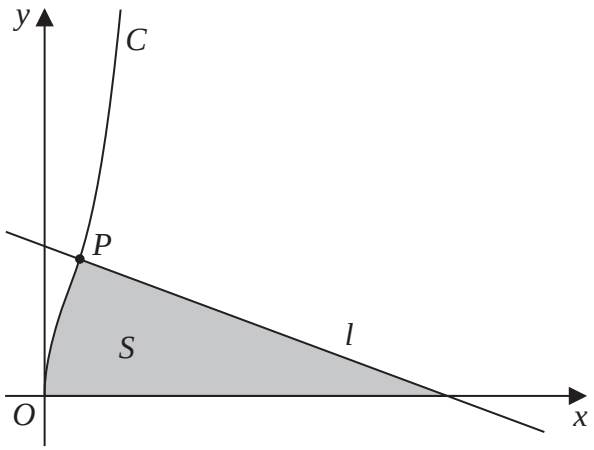


Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A curve C has parametric equations

$$x = \sin^2 t \qquad y = 2\tan t \qquad 0 \leqslant t < \frac{\pi}{2}$$

The point P with parameter $t = \frac{\pi}{4}$ lies on C.

The line l is the normal to C at P, as shown in Figure 3.

(a) Show, using calculus, that an equation for l is

$$8y + 2x = 17$$

(5)

The region S, shown shaded in Figure 3, is bounded by C, l and the x-axis.

(b) Find, using calculus, the exact area of S.

(6)

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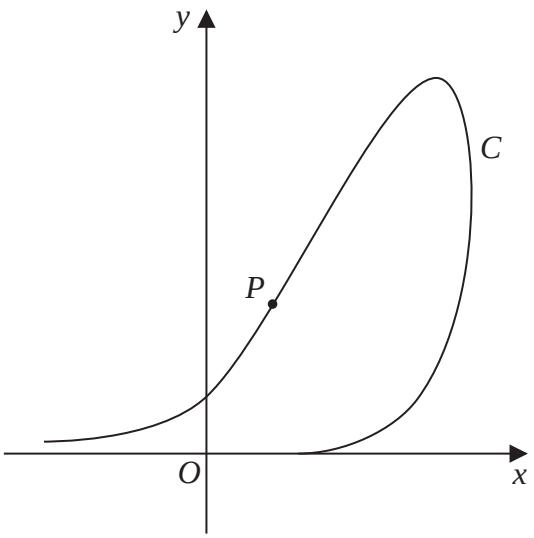


Figure 1

Figure 1 shows a sketch of part of the curve C with equation

$$2^x - 4xy + y^2 = 13 \qquad y \geqslant 0$$

The point P lies on C and has x coordinate 2

(a) Find the y coordinate of P.

(2)

(b) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.

(5)

The tangent to C at P crosses the x-axis at the point Q.

(c) Find the x coordinate of Q, giving your answer in the form $\dfrac{a\ln 2 + b}{c\ln 2 + d}$ where a, b, c and d are integers to be found.

(3)

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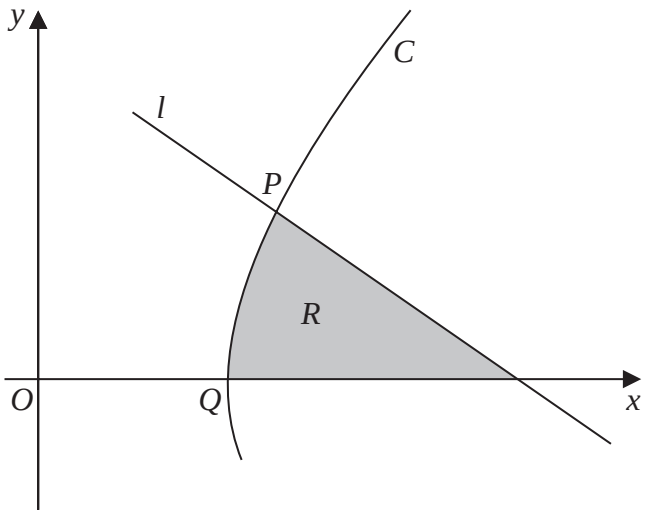


Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = t + \frac{1}{t} \qquad y = t - \frac{1}{t} \qquad t > 0.7$$

The curve C intersects the x-axis at the point Q.

(a) Find the x coordinate of Q.

(1)

The line l is the normal to C at the point P as shown in Figure 2.

Given that $t = 2$ at P

(b) write down the coordinates of P

(1)

(c) Using calculus, show that an equation of l is

$$3x + 5y = 15$$

(3)

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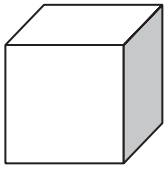


Figure 1

Figure 1 shows a cube which is increasing in size.

At time t seconds,

- the length of each edge of the cube is x cm
- the surface area of the cube is S cm^2
- the volume of the cube is V cm^3

Given that the surface area of the cube is increasing at a constant rate of $4\,\text{cm}^2\,\text{s}^{-1}$

(a) show that $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{k}{x}$ where k is a constant to be found,

(4)

(b) show that $\dfrac{\mathrm{d}V}{\mathrm{d}t} = V^p$ where p is a constant to be found.

(3)

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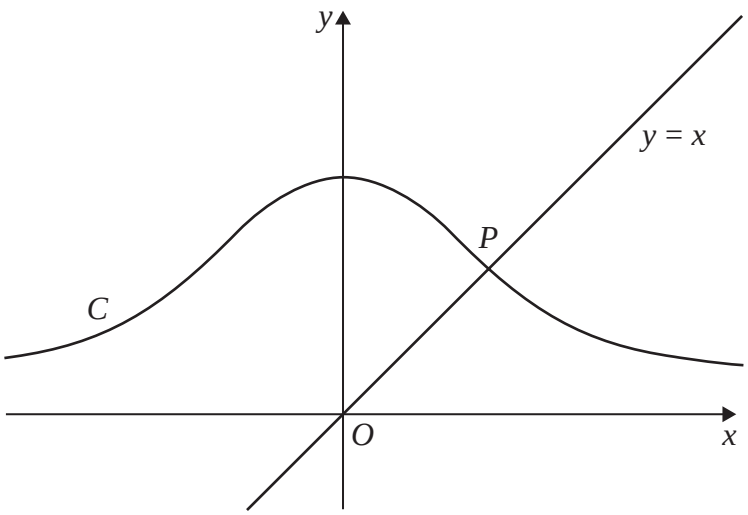


Figure 2

Figure 2 shows a sketch of the curve C with equation

$$y^3 - x^2 + 4x^2y = k$$

where k is a positive constant greater than 1

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(5)

The point P lies on C.

Given that the normal to C at P has equation $y = x$, as shown in Figure 2,

(b) find the value of k.

(5)

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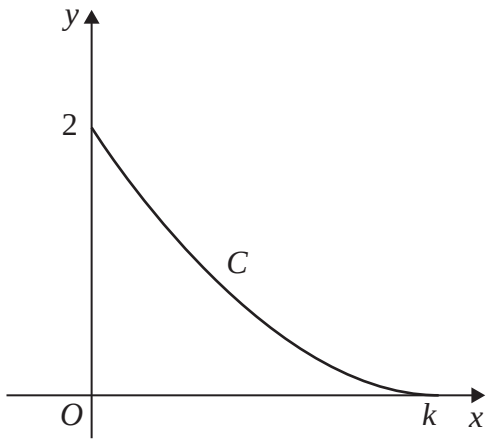


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 6t - 3\sin 2t \qquad y = 2\cos t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The curve meets the y-axis at 2 and the x-axis at k, where k is a constant.

(a) State the value of k.

(1)

(b) Use parametric differentiation to show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \lambda \operatorname{cosec} t$$

where λ is a constant to be found.

(4)

The point P with parameter $t = \frac{\pi}{4}$ lies on C.

The tangent to C at the point P cuts the y-axis at the point N.

(c) Find the exact y coordinate of N, giving your answer in simplest form.

(3)

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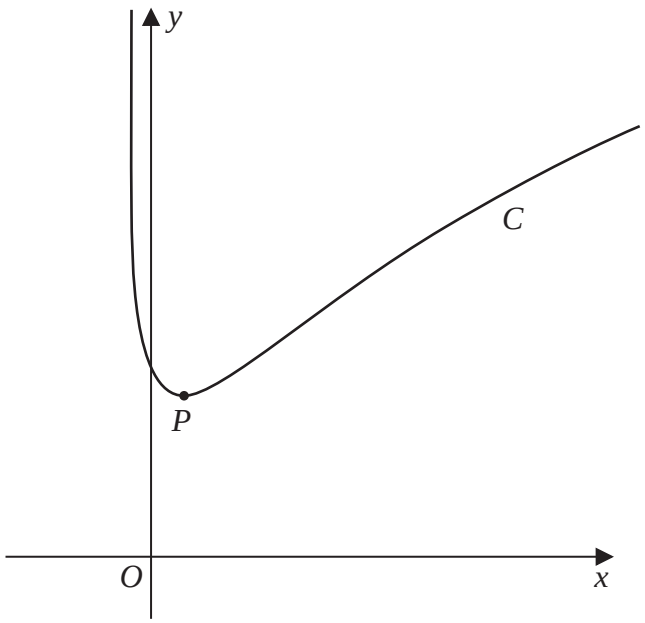


Figure 1

The curve C, shown in Figure 1, has equation

$$y^2x + 3y = 4x^2 + k \qquad y > 0$$

where k is a constant.

(a) Find $\frac{dy}{dx}$ in terms of x and y

(5)

The point $P(p, 2)$, where p is a constant, lies on C.

Given that P is the minimum turning point on C,

(b) find

(i) the value of p

(ii) the value of k

(4)

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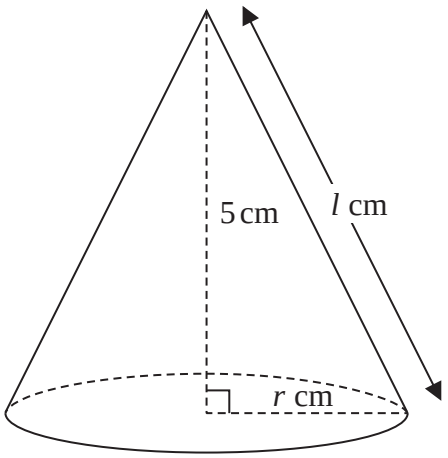


Figure 2

A cone, shown in Figure 2, has

- fixed height 5 cm
- base radius r cm
- slant height l cm

(a) Find an expression for l in terms of r

(1)

Given that the base radius is increasing at a constant rate of 3 cm per minute,

(b) find the rate at which the total surface area of the cone is changing when the radius of the cone is 1.5 cm. Give your answer in cm^2 per minute to one decimal place.

[*The total surface area, S, of a cone is given by the formula* $S = \pi r^2 + \pi rl$]

(4)

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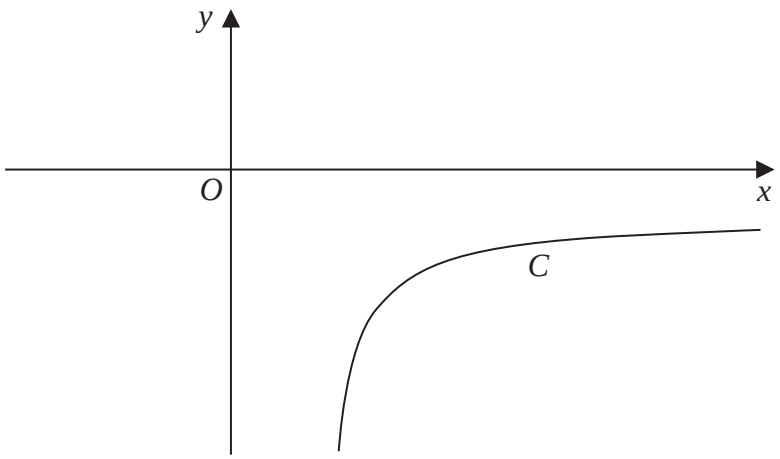


Figure 4

Figure 4 shows a sketch of the curve C with parametric equations

$$x = \sec t \qquad y = \sqrt{3}\tan\left(t + \frac{\pi}{3}\right) \qquad \frac{\pi}{6} < t < \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of t

(3)

(b) Find an equation for the tangent to C at the point where $t = \frac{\pi}{3}$

Give your answer in the form $y = mx + c$, where m and c are constants.

(4)

(c) Show that all points on C satisfy the equation

$$y = \frac{Ax^2 + B\sqrt{3x^2 - 3}}{4 - 3x^2}$$

where A and B are constants to be found.

(5)

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3. The curve C is defined by the equation

$$8x^3 - 3y^2 + 2xy = 9$$

Find an equation of the normal to C at the point (2, 5), giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(7)

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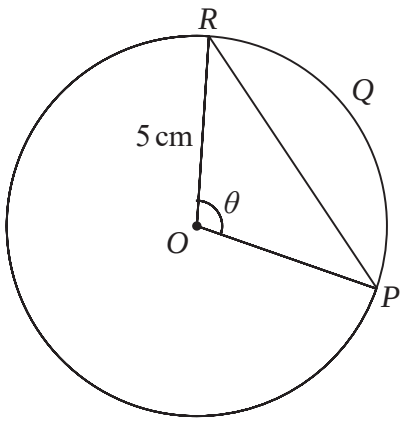


Figure 1

Figure 1 shows a sketch of a segment $PQRP$ of a circle with centre O and radius 5 cm.

Given that

- angle POR is θ radians
- θ is increasing, from 0 to π, at a constant rate of 0.1 radians per second
- the area of the segment $PQRP$ is $A\,\text{cm}^2$

(a) show that

$$\frac{\mathrm{d}A}{\mathrm{d}\theta} = K\left(1 - \cos\theta\right)$$

where K is a constant to be found.

(2)

(b) Find, in $\text{cm}^2\,\text{s}^{-1}$, the rate of increase of the area of the segment when $\theta = \frac{\pi}{3}$

(4)

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October 2024

3.

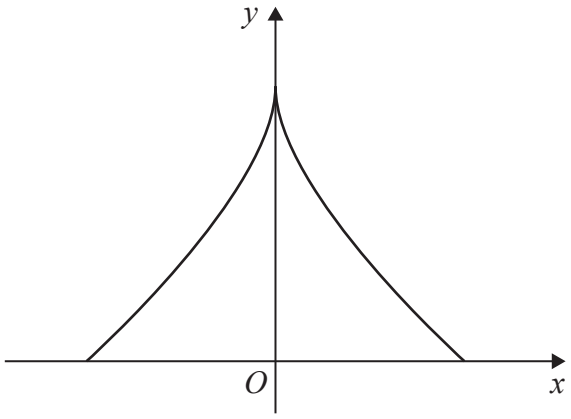


Figure 1

Figure 1 shows a sketch of the curve C with parametric equations

$$x = 3\sin^3\theta \qquad y = 1 + \cos 2\theta \qquad -\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$$

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = k\,\mathrm{cosec}\,\theta \qquad \theta \neq 0$$

where k is a constant to be found.

(3)

The point P lies on C where $\theta = \frac{\pi}{6}$

(b) Find the equation of the tangent to C at P, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(3)

(c) Show that C has Cartesian equation

$$8x^2 = 9(2 - y)^3 \qquad -q \leqslant x \leqslant q$$

where q is a constant to be found.

(3)

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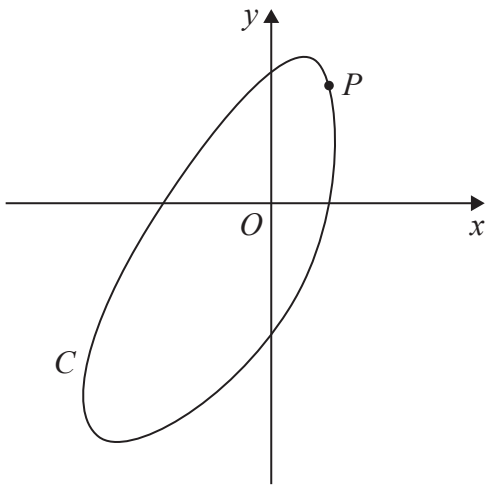


Figure 2

Figure 2 shows a sketch of the curve C with equation

$$3x^2 + 2y^2 - 4xy + 8^x - 11 = 0$$

The point P has coordinates $(1, 2)$.

(a) Verify that P lies on C.

(1)

(b) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y.

(5)

The normal to C at P crosses the x-axis at a point Q.

(c) Find the x coordinate of Q, giving your answer in the form $a + b\ln 2$ where a and b are integers.

(3)

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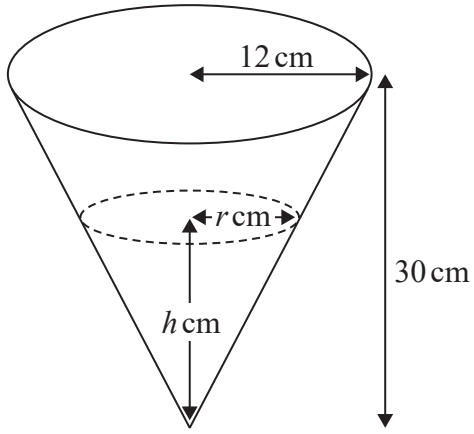


Figure 3

Figure 3 shows a container in the shape of a hollow, inverted, right circular cone.

The height of the container is 30 cm and the radius is 12 cm, as shown in Figure 3.

The container is initially empty when water starts flowing into it.

When the height of water is h cm, the surface of the water has radius r cm and the volume of water is V cm^3

(a) Show that

$$V = \frac{4\pi h^3}{75}$$

[*The volume V of a right circular cone with vertical height h and base radius r is given by the formula* $V = \frac{1}{3}\pi r^2 h$]

(2)

Given that water flows into the container at a constant rate of 2π cm^3 s^{-1}

(b) find, in cm s^{-1}, the rate at which h is changing, exactly 1.5 **minutes** after water starts flowing into the container.

(4)

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January 2025

2. The curve C has equation

$$3x + 5y^2 + 4x^2y = 10(2^x) + 35 \qquad y > 0$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of x and y

(6)

Curve C cuts the y-axis at the point P

(b) Find the exact value of the gradient of the tangent to C at P

(2)

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4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) The volume, V, of a spherical balloon is increasing at a constant rate of $70\pi\,\text{cm}^3\,\text{s}^{-1}$

Find the rate of increase of the radius of the balloon, in $\text{cm}\,\text{s}^{-1}$, at the instant when the radius of the balloon is 5 cm.

[The volume V of a sphere of radius r is given by the formula $V = \frac{4}{3}\pi r^3$]

(4)

(ii) The depth of water in a cave is being monitored.

The rate of increase in the depth of water, h cm, at a particular point in the cave is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{k}{h^3}$$

where k is a constant and t hours is the time after monitoring began.

Given that

- initially the depth of water was 4 cm
- 5 hours after monitoring began, the depth of water was 6 cm
- T hours after monitoring began, the depth of water was 10 cm

solve the differential equation to find the value of T.

Give your answer to one decimal place.

(6)

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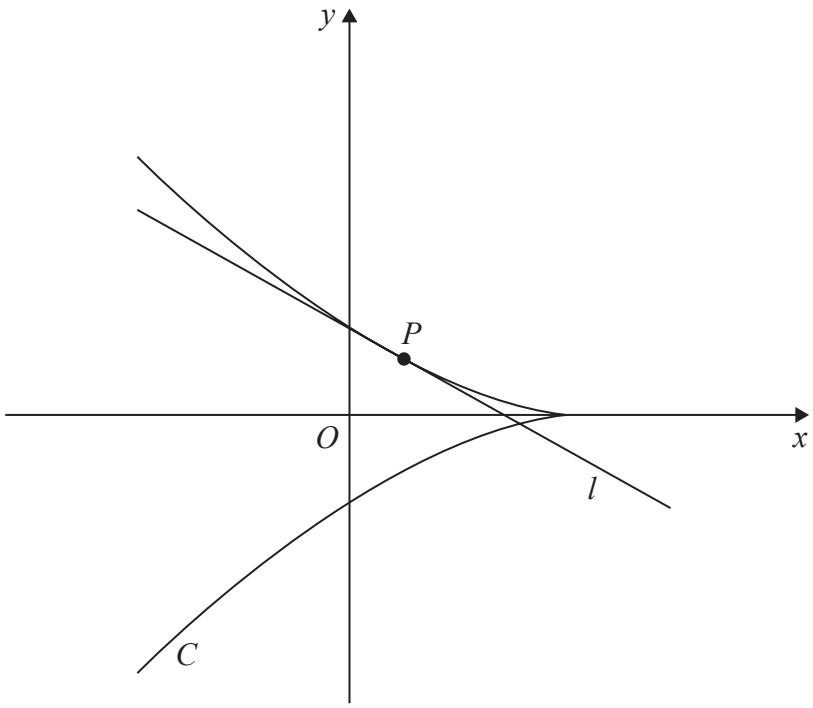


Figure 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 2\cos 2t \qquad y = \sin^3 t \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}$$

where t is a parameter.

The point P lies on C where $t = \frac{\pi}{6}$

The line l, shown in Figure 2, is the tangent to C at P.

(a) Use parametric differentiation to show that

(i) $\frac{\mathrm{d}y}{\mathrm{d}x} = k\sin t$ where k is a constant to be found

(ii) an equation for l is $3x + 16y - 5 = 0$

(6)

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The line l intersects the curve C again at the point Q.

(b) Using algebra and showing detailed reasoning, find the exact coordinates of Q.

(6)

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June 2025

1. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

The curve C has equation

$$2y^2 - 6xy = 7e^{2x-1} + 13$$

The point P with x coordinate $\frac{1}{2}$ lies on C.

(a) Find the two possible y coordinates of P.

(2)

Given that P lies above the x-axis,

(b) find an equation for the tangent to C at P, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(6)

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June 2025

2. A spherical ball of ice with radius r cm is melting.

The volume of the ball of ice, V cm^3, is decreasing at a constant rate, k cm^3 per second, where k is a constant.

Given that $V = \frac{4}{3}\pi r^3$, show that the rate of decrease of the radius of the ball of ice with respect to time is inversely proportional to the square of the radius.

(4)

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5. The curve C has parametric equations

$$x = \frac{3 + 2t}{1 - t} \qquad y = 1 - t^2 \qquad t \neq 1$$

The point P, where $t = 2$, lies on C.

(a) Use parametric differentiation to find the equation of the normal to C at P. Give your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(5)

(b) Show that a Cartesian equation for C can be expressed in the form

$$y = \frac{px + q}{(x + r)^2} \qquad x \neq k$$

where p, q, r and k are integers to be found.

(4)

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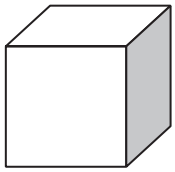


Figure 1

Figure 1 shows a cube which is increasing in size.

At time t seconds,

- the length of each edge of the cube is x cm
- the volume of the cube is V cm^3

Given that the volume of the cube is increasing at a constant rate of $5\,\text{cm}^3\text{s}^{-1}$

(a) find $\frac{\mathrm{d}x}{\mathrm{d}t}$ giving your answer in terms of x.

(3)

(b) Hence find the value of $\frac{\mathrm{d}x}{\mathrm{d}t}$ when $V = 64$

(2)

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5: **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

A curve C has equation

$$2x + 4y^2 = 6 + 3e^{xy}$$

The point $P\left(\frac{9}{2}, 0\right)$ lies on C.

Find an equation of the tangent to C at P, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers.

(6)

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6: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

A curve C has parametric equations

$$x = 8\cos t + 2 \qquad y = 6\sin t - 3 \qquad 0 < t \leqslant 2\pi$$

The point P lies on C where $t = \frac{\pi}{3}$

(a) Use parametric differentiation to find the exact gradient of the **tangent** to C at P.

(3)

The **normal** to C at P intersects the x-axis at the point R.

(b) Find, using algebra, the exact coordinates of R.

(6)

(c) Find a Cartesian equation of C. The equation must not involve trigonometric expressions.

(2)

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4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$4x^2 + y^2 - 2xy = 24x$$

(a) Find $\frac{dy}{dx}$ giving your answer in simplest form in terms of x and y.

(5)

The point P lies on C.

Given that

- the gradient of C at P is 2
- P has coordinates (a, b) where $a > 0$ and $b > 0$

(b) find the value of a and the value of b.

(5)

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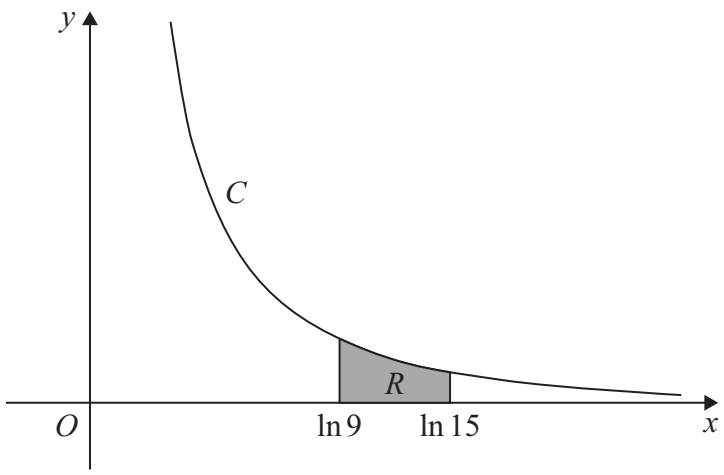


Figure 3

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Figure 3 shows a sketch of the curve C with parametric equations

$$x = \ln(2t + 5) \qquad y = \frac{1}{t + 1} \qquad t > -1$$

A point P lies on C.

Given that the gradient of C at P is -4

(a) use calculus to find the exact y coordinate of P.

(6)

The region R, shown shaded in Figure 3, is bounded by C, the line with equation $x = \ln 9$, the x-axis and the line with equation $x = \ln 15$

(b) (i) Show that the area of R is given by

$$\int_a^b \frac{k}{(t + 1)(2t + 5)}\,\mathrm{d}t$$

where a, b and k are constants to be found.

(ii) Hence, using algebraic integration, find the exact area of R.

Write your answer in the form $\frac{1}{3}\ln\alpha$, where α is a rational number.

(9)

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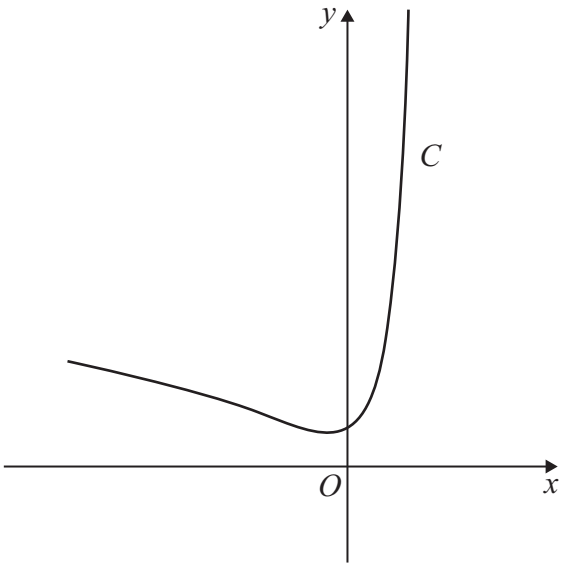


Figure 1

The curve C, shown in Figure 1, has parametric equations

$$x = \frac{4t - 5}{2t + 1} \qquad y = 2t^2 - 4t + 4 \qquad t \geqslant 0$$

The point P lies on C and has parameter $t = 3$

(a) Find the coordinates of P.

(2)

(b) Find $\frac{dy}{dx}$ in terms of t, writing the answer in simplest form.

(4)

(c) Hence find the equation of the tangent to C at P. Write the answer in the form $y = mx + c$, where m and c are constants.

(2)

Given that

- the Cartesian equation of C is $y = f(x)$
- the function f is defined for all values of x on C

(d) (i) find the domain of f

(ii) find the range of f

(3)

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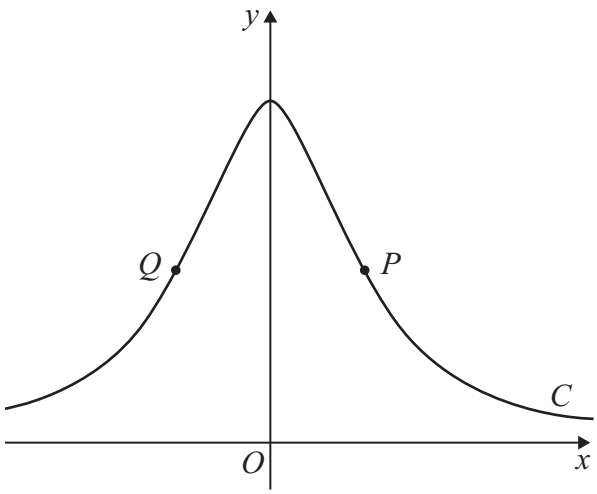


Figure 2

The curve C, shown in Figure 2 has equation

$$x^2 \tan y + \frac{32y^2}{\pi^2} = 11 \qquad 0 < y \leqslant \frac{\pi}{2}$$

The points P and Q, shown in Figure 2, lie on C and have y coordinate $\frac{\pi}{4}$

(a) Find the x coordinate of P and the x coordinate of Q.

(2)

(b) Use algebraic differentiation to find the exact value of $\frac{\mathrm{d}y}{\mathrm{d}x}$ at P.

(5)

(c) Hence, showing your working, find the value of

(the gradient of the tangent to C at P) $\times$ (the gradient of the normal to C at Q)

(2)

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8. A large spherical balloon starts to deflate.

The balloon remains spherical whilst deflating.

At time t seconds after the balloon starts to deflate

- the balloon has radius r cm
- the balloon has volume V cm^3

Given that the volume of the balloon is modelled to decrease at a constant rate,

(a) use the model to show that

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{k}{r^2}$$

where k is a positive constant.

(3)

Given that

- the initial radius of the balloon is 30 cm
- after 24 seconds the radius of the balloon is 12 cm

(b) solve the differential equation in part (a) to find an equation for r in terms of t.

(5)

Given that $V = 0$ when $t = T$

(c) use the model to find the value of T to one decimal place.

(2)

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2. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

A curve has equation

$$4x^2 - y^2 + 2xy + 5 = 0$$

Given that

- points P and Q lie on the curve
- $\frac{dy}{dx} = 2$ at P and at Q

find the coordinates of P and Q.

(7)

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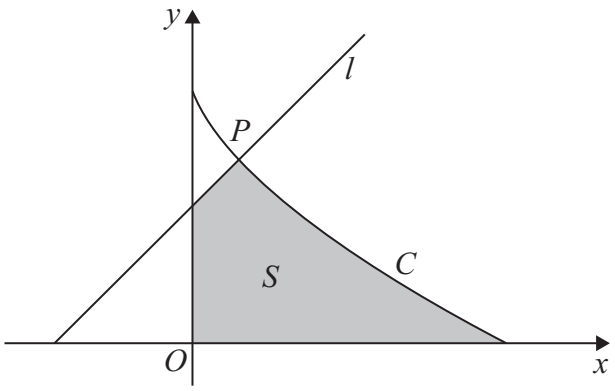


Figure 1

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve C with parametric equations

$$x = 8\cos^3\theta \qquad y = 6\sin^2\theta \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

Given that the point P lies on C and has parameter $\theta = \frac{\pi}{3}$

(a) find the coordinates of P.

(2)

The line l is the normal to C at P.

(b) Find an equation for l in the form $y = mx + c$, where m and c are constants.

(5)

The finite region S, shown shaded in Figure 1, is bounded by the curve C, the line l, the y-axis and the x-axis.

(c) Show that the area of S is given by

$$4 + 144\int_0^{\frac{\pi}{3}} (\sin\theta\cos^2\theta - \sin\theta\cos^4\theta)\,\mathrm{d}\theta$$

(6)

(d) Hence, using algebraic integration, find the exact area of S.

(3)

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June 2026

3. The curve C is defined by the parametric equations

$$x = \frac{4}{t+2} \qquad y = \frac{6t}{t+4} \qquad t > -1$$

The point P with parameter $t = 2$ lies on C.

(a) Use parametric differentiation to find the equation of the tangent to C at P. Give the answer in the form $y = mx + c$, where m and c are constants.

(5)

(b) Show that the equation of C can be written in the form $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = \frac{ax+b}{x+d} \qquad 0 < x < k$$

and where a, b, d and k are integers to be found.

(4)

(c) Hence, or otherwise, find the range of f.

(2)

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4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Water flows at a constant rate into a container.

When the depth of water in the container is h cm, the volume of water in the container, V cm^3, is modelled by the equation

$$V = \frac{1}{3}h^2\sqrt{(h+4)} \qquad 0 \leqslant h \leqslant 21$$

Given that

- the container is initially empty
- water flows into the container at a rate of $30\ \text{cm}^3\text{s}^{-1}$
- the container is full when the depth of the water is 21 cm

find, according to the model,

(a) the time taken to fill the container,

(2)

(b) the exact rate of change of the depth of the water, in cm s^{-1}, when $h = 5$

(5)

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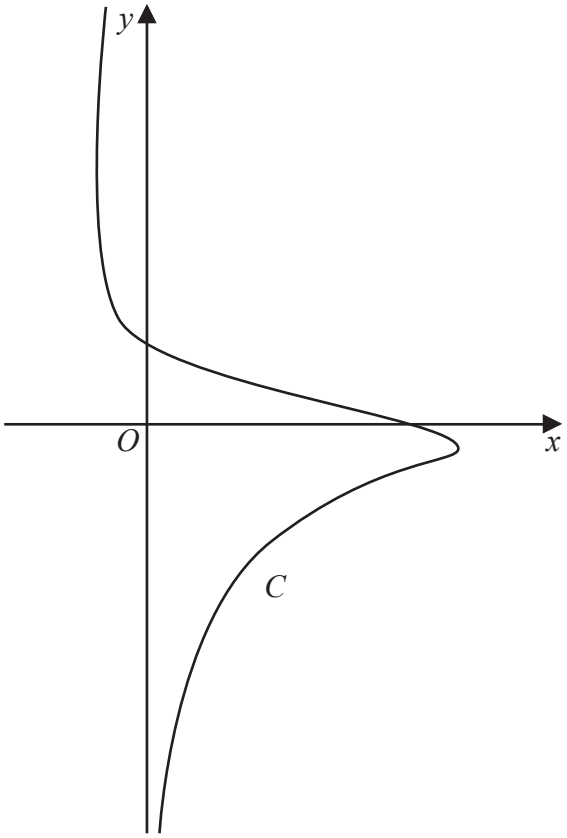


Figure 2

Figure 2 shows a sketch of the curve C with equation

$$4xy^2 + 12y + 6x = 53 \qquad y \in \mathbb{R}$$

(a) Use implicit differentiation to find $\frac{dy}{dx}$ in simplest form in terms of x and y.

(5)

The point $P(a, b)$ lies on C.

Given that a is the maximum possible value of x

(b) find the value of a and the value of b.

(*Solutions relying on calculator technology are not acceptable.*)

(5)

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8. The growth of algae on the surface of a pond is being monitored.

The surface area of the pond covered by algae, $S\ \text{m}^2$, is modelled by the equation

$$S = 140 - \frac{k}{0.3t + 8}$$

where t days is the time after monitoring began and k is a constant.

Given that $18\ \text{m}^2$ of the surface area of the pond was covered by algae when monitoring began,

(a) find, according to this model,

(i) the value of k,

(ii) the value of $\frac{\text{d}S}{\text{d}t}$ when $t = 10$

(*Solutions relying entirely on calculator technology are not acceptable.*)

(3)

In an improved model, S increases with respect to t according to the differential equation

$$\frac{\text{d}S}{\text{d}t} = \frac{128 - S}{50}$$

(b) Solve the differential equation, with $S = 18$ when $t = 0$, to find an equation for S in terms of t.

(6)

Given that the algae grows until it covers the entire surface of the pond,

(c) find, according to the **improved** model, the maximum possible area of the pond.

(1)

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3. A solid metal cube is being heated and is expanding.

The volume of the cube is increasing at a constant rate of $0.085\ \text{cm}^3\,\text{s}^{-1}$

At time t seconds,

- the side length of the cube is x cm
- the volume of the cube is $V\ \text{cm}^3$

At the instant the volume of the cube is $100\ \text{cm}^3$, the side length is increasing at a rate of $k\ \text{cm}\,\text{s}^{-1}$

Find the value of k, giving your answer to 3 significant figures.

(4)

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6. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

The curve C has equation

$$x^2 - xy + y^2 + 4x = 12$$

(a) Find $\frac{dy}{dx}$ in terms of x and y.

(4)

Given that the point $P(-4, -6)$ lies on C,

(b) find the equation of the normal to C at P, giving the answer in the form $y = mx + c$ where m and c are constants.

(3)

The normal to C at P intersects C again at the point Q.

(c) Find the exact coordinates of Q.

(3)

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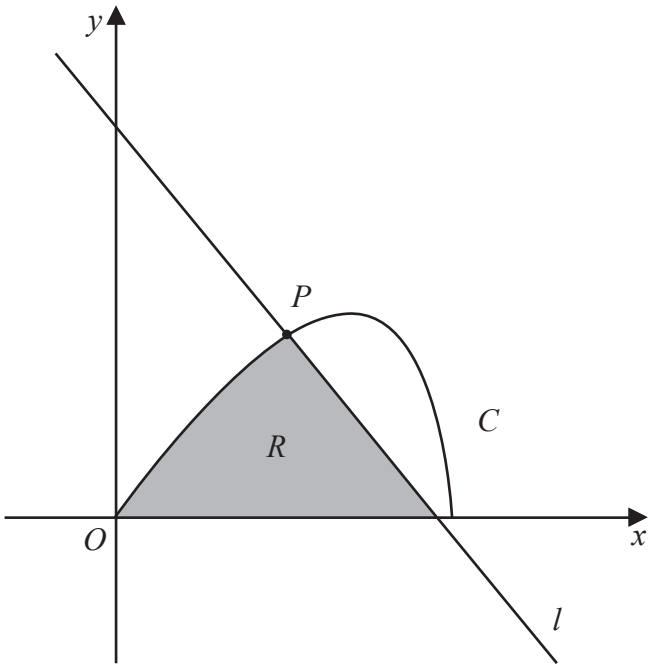


Figure 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 8\cos\theta \qquad y = 5\sin 2\theta \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

The point P with parameter $\theta = \frac{\pi}{3}$ lies on C.

The line l is the normal to C at P, as shown in Figure 2.

(a) Use calculus to show that l has equation

$$y = p\sqrt{3}\,x + q\sqrt{3}$$

where p and q are rational constants to be found.

(4)

The finite region R, shown shaded in Figure 2, is bounded by C, l and the x-axis.

(b) Using algebraic integration, find the exact area of R, giving the answer in the form $A + B\sqrt{3}$ where A and B are rational constants.

(6)

(Total for Question 10 is 10 marks)

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