3. $$f(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}$$

(a) Find the values of the constants A, B and C

(4)

(b) (i) Hence find $\int f(x)\,dx$

(ii) Find $\int_1^2 f(x)\,dx$, giving your answer in the form $a + \ln b$, where a and b are constants.

(6)

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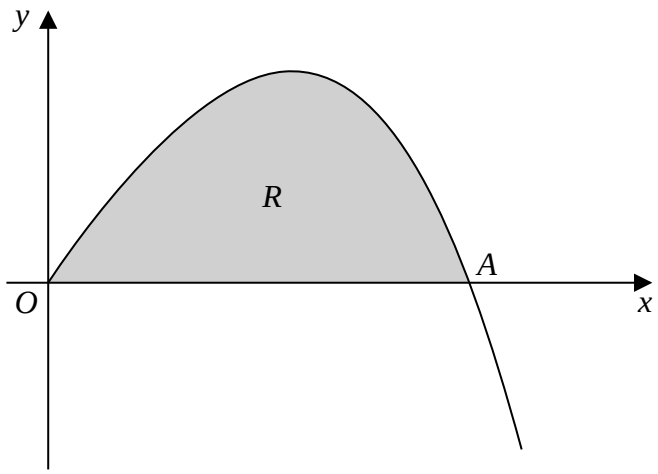


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \geqslant 0$

The curve meets the x-axis at the origin O and cuts the x-axis at the point A.

(a) Find, in terms of $\ln 2$, the x coordinate of the point A.

(2)

(b) Find $\int xe^{\frac{1}{2}x}\,dx$

(3)

The finite region R, shown shaded in Figure 2, is bounded by the x-axis and the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \geqslant 0$

(c) Find, by integration, the exact value for the area of R.

Give your answer in terms of $\ln 2$

(3)

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8. Water is being heated in a kettle. At time t seconds, the temperature of the water is θ °C.

The rate of increase of the temperature of the water at time t is modelled by the differential equation

$$\frac{d\theta}{dt} = \lambda(120 - \theta) \qquad \theta \leqslant 100$$

where λ is a positive constant.

Given that $\theta = 20$ when $t = 0$

(a) solve this differential equation to show that

$$\theta = 120 - 100e^{-\lambda t}$$

(8)

When the temperature of the water reaches 100 °C, the kettle switches off.

(b) Given that $\lambda = 0.01$, find the time, to the nearest second, when the kettle switches off.

(3)

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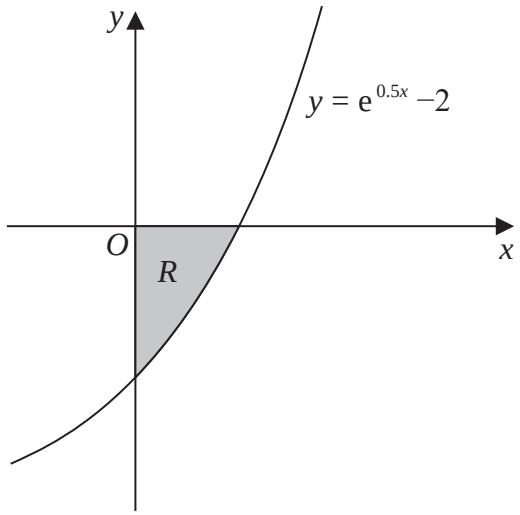


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = e^{0.5x} - 2$

The region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the y-axis.

The region R is rotated 360° about the x-axis to form a solid of revolution.

Show that the volume of this solid can be written in the form $a \ln 2 + b$, where a and b are constants to be found.

(6)

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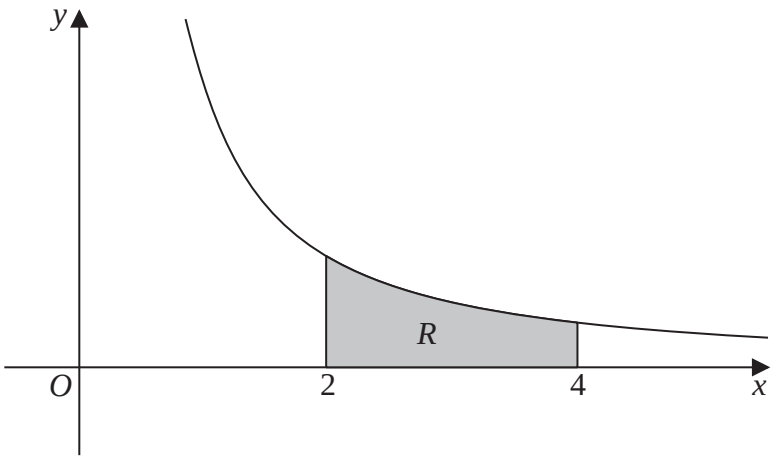


Figure 3

(a) Find $\int \frac{\ln x}{x^2} \mathrm{d}x$

(3)

Figure 3 shows a sketch of part of the curve with equation

$$y = \frac{3 + 2x + \ln x}{x^2} \qquad x > 0.5$$

The finite region R, shown shaded in Figure 3, is bounded by the curve, the line with equation $x = 2$, the x-axis and the line with equation $x = 4$

(b) Use the answer to part (a) to find the exact area of R, writing your answer in simplest form.

(4)

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7. (i) Using a suitable substitution, find, using calculus, the value of

$$\int_1^5 \frac{3x}{\sqrt{2x-1}}\,\mathrm{d}x$$

(*Solutions relying entirely on calculator technology are not acceptable*.)

(6)

(ii) Find

$$\int \frac{6x^2-16}{(x+1)(2x-3)}\,\mathrm{d}x$$

(6)

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9. Bacteria are growing on the surface of a dish in a laboratory.

The area of the dish, $A\,\text{cm}^2$, covered by the bacteria, t days after the bacteria start to grow, is modelled by the differential equation

$$\frac{\mathrm{d}A}{\mathrm{d}t} = \frac{A^{\frac{3}{2}}}{5t^2} \qquad t > 0$$

Given that $A = 2.25$ when $t = 3$

(a) show that

$$A = \left(\frac{pt}{qt + r} \right)^2$$

where p, q and r are integers to be found.

(7)

According to the model, there is a limit to the area that will be covered by the bacteria.

(b) Find the value of this limit.

(2)

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5. **In this question you should show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Using the substitution $u = 3 + \sqrt{2x - 1}$ find the exact value of

$$\int_{1}^{13} \frac{4}{3 + \sqrt{2x - 1}} \, \mathrm{d}x$$

giving your answer in the form $p + q \ln 2$, where p and q are integers to be found.

(8)

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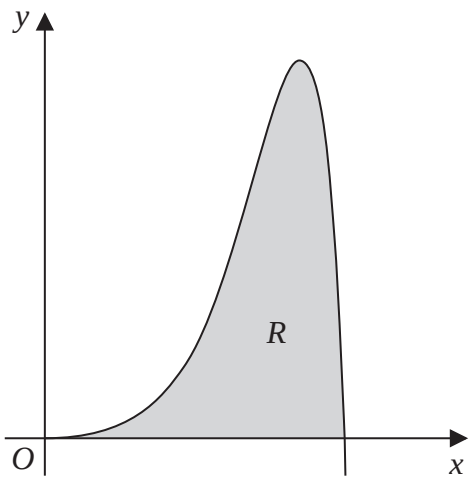


Figure 2

(a) Find $\int e^{2x} \sin x \, dx$

(5)

Figure 2 shows a sketch of part of the curve with equation

$$y = e^{2x} \sin x \qquad x \geqslant 0$$

The finite region R is bounded by the curve and the x-axis and is shown shaded in Figure 2.

(b) Show that the exact area of R is $\dfrac{e^{2\pi} + 1}{5}$

(*Solutions relying on calculator technology are not acceptable.*)

(2)

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9.

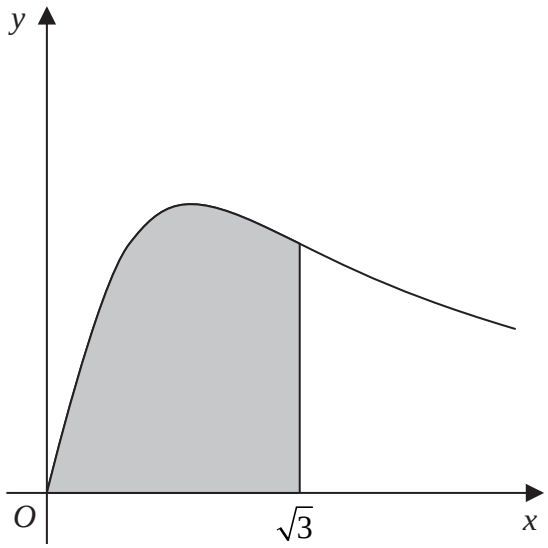


Figure 3

Figure 3 shows a sketch of part of the curve with parametric equations

$$x = \tan\theta \qquad y = 2\sin 2\theta \qquad \theta \geqslant 0$$

The finite region, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line with equation $x = \sqrt{3}$

The region is rotated through 2π radians about the x-axis to form a solid of revolution.

(a) Show that the exact volume of this solid of revolution is given by

$$\int_0^k p(1 - \cos 2\theta)\, \mathrm{d}\theta$$

where p and k are constants to be found.

(7)

(b) Hence find, by algebraic integration, the exact volume of this solid of revolution.

(3)

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10. (a) Write $\dfrac{1}{(H-5)(H+3)}$ in partial fraction form.

(3)

The depth of water in a storage tank is being monitored.

The depth of water in the tank, H metres, is modelled by the differential equation

$$\frac{\mathrm{d}H}{\mathrm{d}t} = -\frac{(H-5)(H+3)}{40}$$

where t is the time, in days, from when monitoring began.

Given that the initial depth of water in the tank was 13 m,

(b) solve the differential equation to show that

$$H = \frac{10 + 3\mathrm{e}^{-0.2t}}{2 - \mathrm{e}^{-0.2t}}$$

(7)

(c) Hence find the time taken for the depth of water in the tank to fall to 8 m.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(3)

According to the model, the depth of water in the tank will eventually fall to k metres.

(d) State the value of the constant k.

(1)

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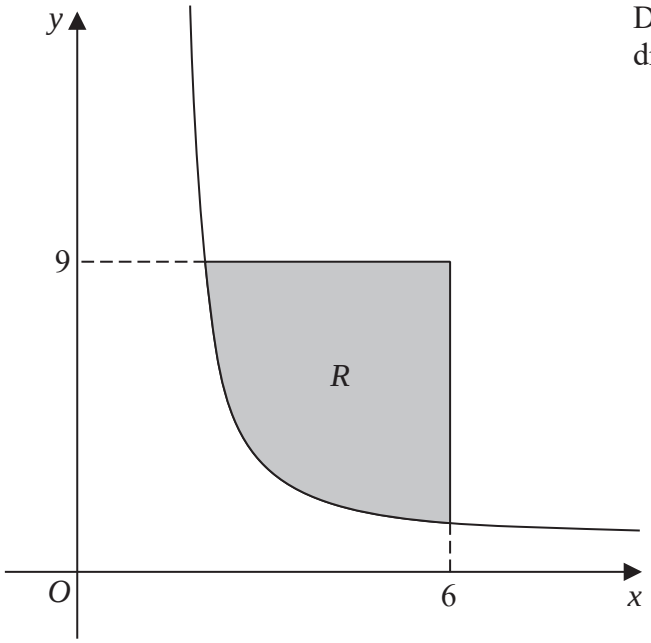


Diagram **NOT** drawn to scale

Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = \frac{9}{(2x - 3)^{1.25}} \qquad x > \frac{3}{2}$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the line with equation $y = 9$ and the line with equation $x = 6$

This region is rotated through 2π radians about the x-axis to form a solid of revolution.

Find, by algebraic integration, the exact volume of the solid generated.

(7)

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4. Use algebraic integration and the substitution $u = \sqrt{x}$ to find the exact value of

$$\int_1^4 \frac{10}{5x + 2x\sqrt{x}} \mathrm{d}x$$

Write your answer in the form $4\ln\left(\frac{a}{b}\right)$, where a and b are integers to be found.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(8)

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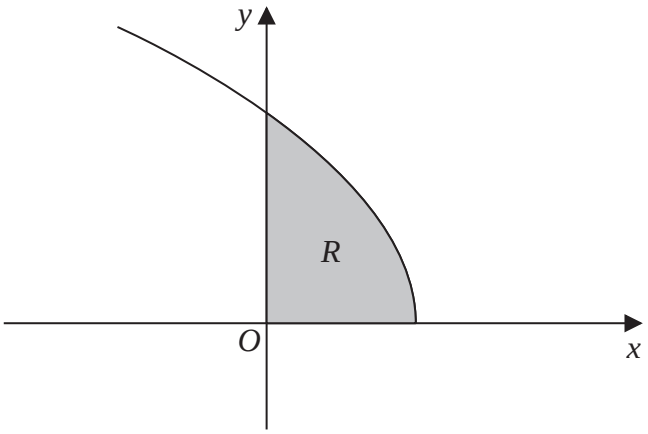


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 2\cos 2t \qquad y = 4\sin t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the y-axis.

(a) (i) Show, making your working clear, that the area of $R = \int_0^{\frac{\pi}{4}} 32\sin^2 t\cos t \, \mathrm{d}t$

(ii) Hence find, by algebraic integration, the exact value of the area of R.

(6)

(b) Show that all points on C satisfy $y = \sqrt{ax + b}$, where a and b are constants to be found.

(3)

The curve C has equation $y = \mathrm{f}(x)$ where f is the function

$$\mathrm{f}(x) = \sqrt{ax + b} \qquad -2 \leqslant x \leqslant 2$$

and a and b are the constants found in part (b).

(c) State the range of f.

(1)

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8. (a) Given that $y = 1$ at $x = 0$, solve the differential equation

$$\frac{dy}{dx} = \frac{6xy^{\frac{1}{3}}}{e^{2x}} \qquad y \geqslant 0$$

giving your answer in the form $y^2 = g(x)$.

(7)

(b) Hence find the equation of the horizontal asymptote to the curve with equation $y^2 = g(x)$.

(2)

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2. Find the particular solution of the differential equation

$$\frac{dy}{dx} = \frac{4y^2}{\sqrt{4x+5}} \qquad x > -\frac{5}{4}$$

for which $y = \frac{1}{3}$ at $x = -\frac{1}{4}$ giving your answer in the form $y = f(x)$

(6)

6. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

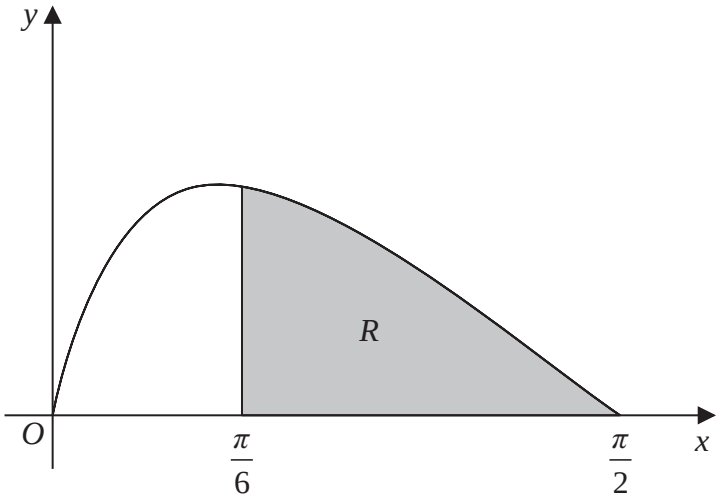


Figure 2

Figure 2 shows a sketch of the curve with equation

$$y = \frac{16\sin 2x}{(3 + 4\sin x)^2} \qquad 0 \leqslant x \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line with equation $x = \frac{\pi}{6}$

Using the substitution $u = 3 + 4\sin x$, show that the area of R can be written in the form $a + \ln b$, where a and b are rational constants to be found.

(7)

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8. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

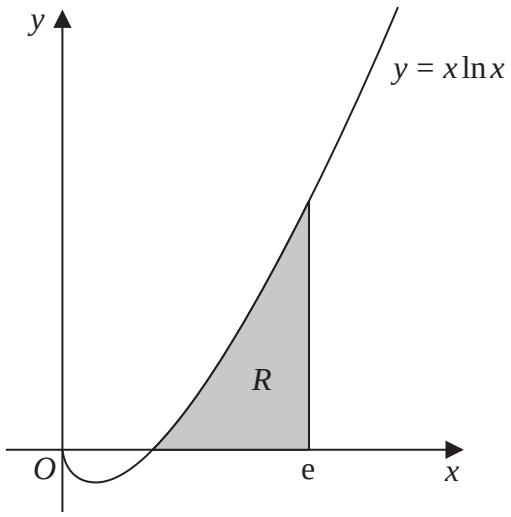


Figure 3

(a) Find $\int x^2 \ln x \, dx$

(3)

Figure 3 shows a sketch of part of the curve with equation

$$y = x \ln x \qquad x > 0$$

The region R, shown shaded in Figure 3, lies entirely above the x-axis and is bounded by the curve, the x-axis and the line with equation $x = e$.

This region is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Find the exact volume of the solid formed, giving your answer in simplest form.

(4)

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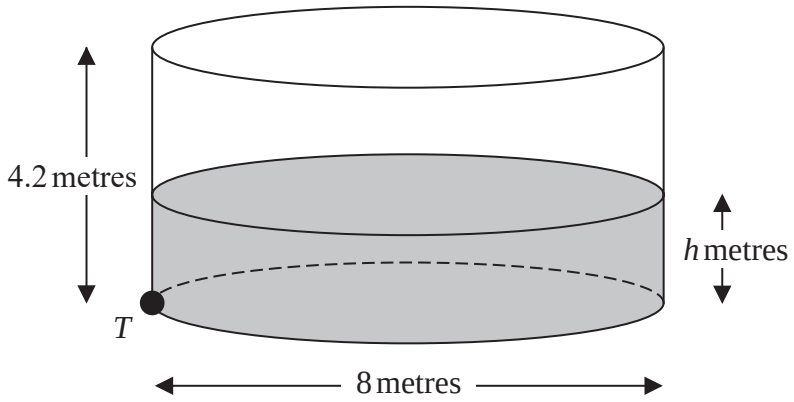


Figure 4

Figure 4 shows a cylindrical tank that contains some water.

The tank has an internal diameter of 8 m and an internal height of 4.2 m.

Water is flowing into the tank at a constant rate of $(0.6\pi)\,\text{m}^3$ per minute.

There is a tap at point T at the bottom of the tank.

At time t minutes after the tap has been opened,

- the depth of the water is h metres
- the water is leaving the tank at a rate of $(0.15\pi h)\,\text{m}^3$ per minute

(a) Show that

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{12 - 3h}{320}$$

(4)

Given that the depth of the water in the tank is 0.5 m when the tap is opened,

(b) find the time taken for the depth of water in the tank to reach 3.5 m.

(6)

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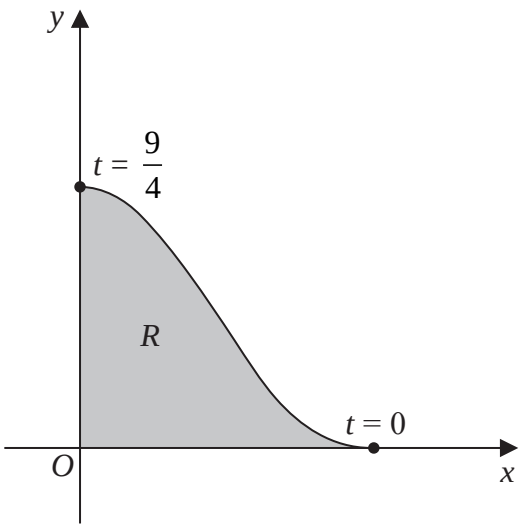


Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = \sqrt{9 - 4t} \qquad y = \frac{t^3}{\sqrt{9 + 4t}} \qquad 0 \leqslant t \leqslant \frac{9}{4}$$

The curve touches the x-axis when $t = 0$ and meets the y-axis when $t = \frac{9}{4}$

The region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the y-axis.

(a) Show that the area of R is given by

$$K\int_0^{\frac{9}{4}} \frac{t^3}{\sqrt{81 - 16t^2}}\,\mathrm{d}t$$

where K is a constant to be found.

(4)

(b) Using the substitution $u = 81 - 16t^2$, or otherwise, find the exact area of R.

(*Solutions relying on calculator technology are not acceptable.*)

(6)

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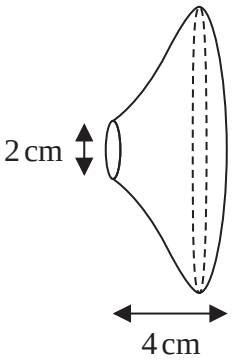


Figure 3

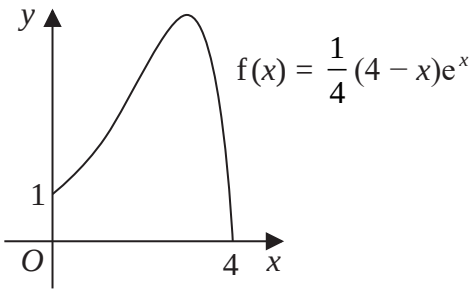


Figure 4

Figure 3 shows the design of a doorknob.

The shape of the doorknob is formed by rotating the curve shown in Figure 4 through 360° about the x-axis, where the units are centimetres.

The equation of the curve is given by

$$\mathrm{f}(x) = \frac{1}{4}(4 - x)\mathrm{e}^{x} \qquad 0 \leqslant x \leqslant 4$$

(a) Show that the volume, $V\,\mathrm{cm}^3$, of the doorknob is given by

$$V = K\int_0^4 \left(x^2 - 8x + 16\right)\mathrm{e}^{2x}\,\mathrm{d}x$$

where K is a constant to be found.

(3)

(b) Hence, find the exact value of the volume of the doorknob.

Give your answer in the form $p\pi(\mathrm{e}^q + r)\,\mathrm{cm}^3$ where p, q and r are simplified rational numbers to be found.

(5)

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9. (a) Find the derivative with respect to y of

$$\frac{1}{(1+2\ln y)^2}$$

(2)

(b) Hence find a general solution to the differential equation

$$3\text{cosec}(2x)\frac{\mathrm{d}y}{\mathrm{d}x} = y(1+2\ln y)^3 \qquad y>0 \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$$

(4)

(c) Show that the particular solution of this differential equation for which $y = 1$ at $x = \frac{\pi}{6}$ is given by

$$y = \mathrm{e}^{A\sec x - \frac{1}{2}}$$

where A is an irrational number to be found.

(5)

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2. (a) Express $\dfrac{1}{(1+3x)(1-x)}$ in partial fractions.

(3)

(b) Hence find the solution of the differential equation

$$(1+3x)(1-x)\frac{dy}{dx} = \tan y \qquad -\frac{1}{3} < x \leqslant \frac{1}{2}$$

for which $x = \dfrac{1}{2}$ when $y = \dfrac{\pi}{2}$

Give your answer in the form $\sin^n y = f(x)$ where n is an integer to be found.

(6)

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5. **In this question you must show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Use the substitution $x = 2\sin u$ to show that

$$\int_0^1 \frac{3x+2}{(4-x^2)^{\frac{3}{2}}}\,\mathrm{d}x = \int_0^p \left(\frac{3}{2}\sec u \tan u + \frac{1}{2}\sec^2 u\right)\mathrm{d}u$$

where p is a constant to be found.

(4)

(b) Hence find the exact value of

$$\int_0^1 \frac{3x+2}{(4-x^2)^{\frac{3}{2}}}\,\mathrm{d}x$$

(4)

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3. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

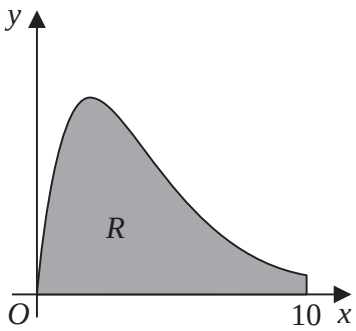


Figure 2

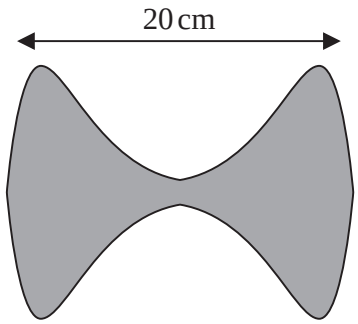


Figure 3

Figure 2 shows the curve with equation

$$y = 10xe^{-\frac{1}{2}x} \qquad 0 \leqslant x \leqslant 10$$

The finite region R, shown shaded in Figure 2, is bounded by the curve, the x-axis and the line with equation $x = 10$

The region R is rotated through 2π radians about the x-axis to form a solid of revolution.

(a) Show that the volume, V, of this solid is given by

$$V = k\int_0^{10} x^2 e^{-x}\,dx$$

where k is a constant to be found.

(2)

(b) Find $\int x^2 e^{-x}\,dx$

(3)

Figure 3 represents an exercise weight formed by joining two of these solids together.

The exercise weight has mass 5 kg and is 20 cm long.

Given that

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

and using your answers to part (a) and part (b),

(c) find the density of this exercise weight. Give your answer in grams per cm^3 to 3 significant figures.

(5)

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2. (a) Express $\dfrac{3x}{(2x-1)(x-2)}$ in partial fraction form.

(3)

(b) Hence show that

$$\int_{5}^{25} \frac{3x}{(2x-1)(x-2)}\,\mathrm{d}x = \ln k$$

where k is a fully simplified fraction to be found.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(4)

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5. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

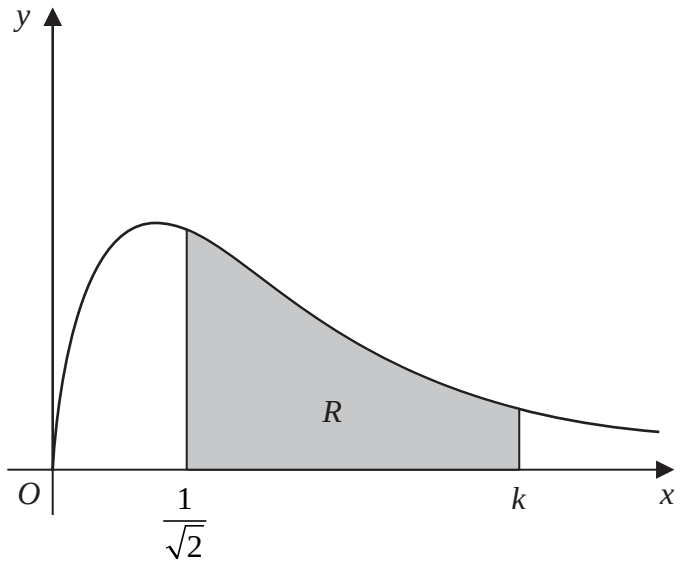


Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$y = \frac{12\sqrt{x}}{(2x^2 + 3)^{1.5}}$$

The region R, shown shaded in Figure 2, is bounded by the curve, the line with equation $x = \frac{1}{\sqrt{2}}$, the x-axis and the line with equation $x = k$.

This region is rotated through 360° about the x-axis to form a solid of revolution.

Given that the volume of this solid is $\frac{713}{648}\pi$, use algebraic integration to find the exact value of the constant k.

(6)

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7. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Use the substitution $u = e^x - 3$ to show that

$$\int_{\ln 5}^{\ln 7} \frac{4e^{3x}}{e^x - 3}\,dx = a + b\ln 2$$

where a and b are constants to be found.

(7)

(ii) Show, by integration, that

$$\int 3e^x \cos 2x\,dx = pe^x \sin 2x + qe^x \cos 2x + c$$

where p and q are constants to be found and c is an arbitrary constant.

(5)

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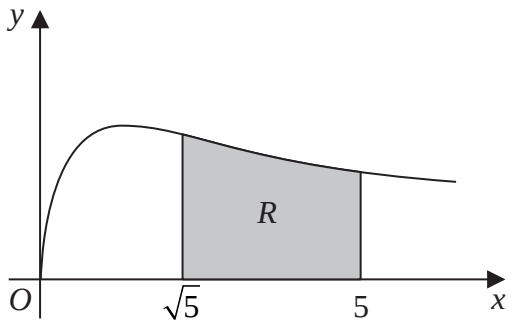


Figure 1

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve with equation

$$y = \sqrt{\frac{3x}{3x^2 + 5}} \qquad x \geqslant 0$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis and the lines with equations $x = \sqrt{5}$ and $x = 5$

The region R is rotated through 360° about the x-axis.

Use integration to find the exact volume of the solid generated. Give your answer in the form $a \ln b$, where a is an irrational number and b is a prime number.

(5)

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4. (a) Using the substitution $u = \sqrt{2x+1}$, show that

$$\int_4^{12} \sqrt{8x+4}\, e^{\sqrt{2x+1}}\, dx$$

may be expressed in the form

$$\int_a^b ku^2 e^u\, du$$

where a, b and k are constants to be found.

(4)

(b) Hence find, by algebraic integration, the exact value of

$$\int_4^{12} \sqrt{8x+4}\, e^{\sqrt{2x+1}}\, dx$$

giving your answer in simplest form.

(5)

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7. The volume $V\text{cm}^3$ of a spherical balloon with radius r cm is given by the formula

$$V = \frac{4}{3}\pi r^3$$

(a) Find $\frac{dV}{dr}$ giving your answer in simplest form.

(1)

At time t seconds, the volume of the balloon is increasing according to the differential equation

$$\frac{dV}{dt} = \frac{900}{(2t+3)^2} \qquad t \geqslant 0$$

Given that $V = 0$ when $t = 0$

(b) (i) solve this differential equation to show that

$$V = \frac{300t}{2t+3}$$

(ii) Hence find the upper limit to the volume of the balloon.

(5)

(c) Find the radius of the balloon at $t = 3$, giving your answer in cm to 3 significant figures.

(3)

(d) Find the rate of increase of the radius of the balloon at $t = 3$, giving your answer to 2 significant figures. Show your working and state the units of your answer.

(3)

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3. $$f(x) = \frac{8x - 5}{(2x - 1)(4x - 3)} \qquad x > 1$$

(a) Express $f(x)$ in partial fractions.

(3)

(b) Hence find $\int f(x)\,dx$

(3)

(c) Use the answer to part (b) to find the value of k for which

$$\int_k^{3k} f(x)\,dx = \frac{1}{2}\ln 20$$

(5)

5. (i) Find

$$\int x^2 e^x dx$$

(4)

(ii) Use the substitution $u = \sqrt{1-3x}$ to show that

$$\int \frac{27x}{\sqrt{1-3x}} dx = -2(1-3x)^{\frac{1}{2}}(Ax + B) + k$$

where A and B are integers to be found and k is an arbitrary constant.

(6)

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6. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

The temperature, θ°C, of a car engine, t minutes after the engine is turned off, is modelled by the differential equation

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -k(\theta - 15)^2$$

where k is a constant.

Given that the temperature of the car engine
- is 85 °C at the instant the engine is turned off
- is 40 °C exactly 10 minutes after the engine is turned off

(a) solve the differential equation to show that, according to the model

$$\theta = \frac{at + b}{ct + d}$$

where a, b, c and d are integers to be found.

(7)

(b) Hence find, according to the model, the time taken for the temperature of the car engine to reach 20 °C. Give your answer to the nearest minute.

(2)

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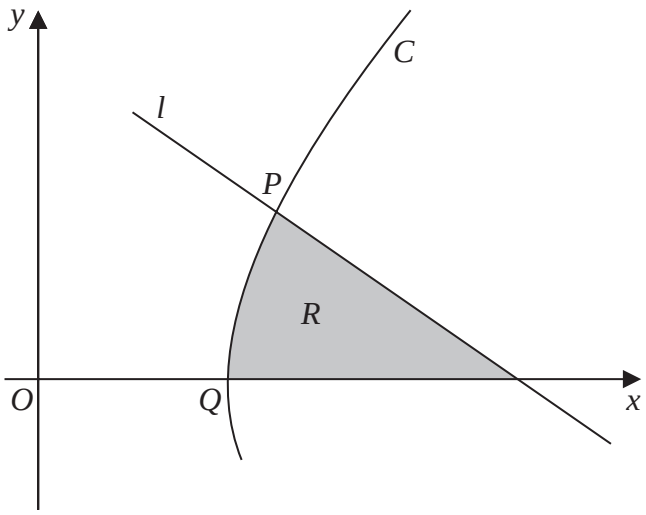


Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = t + \frac{1}{t} \qquad y = t - \frac{1}{t} \qquad t > 0.7$$

The curve C intersects the x-axis at the point Q.

(a) Find the x coordinate of Q.

(1)

The line l is the normal to C at the point P as shown in Figure 2.

Given that $t = 2$ at P

(b) write down the coordinates of P

(1)

(c) Using calculus, show that an equation of l is

$$3x + 5y = 15$$

(3)

The region, R, shown shaded in Figure 2 is bounded by the curve C, the line l and the x-axis.

(d) Using algebraic integration, find the exact volume of the solid of revolution formed when the region R is rotated through 2π radians about the x-axis.

(7)

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3. **In this question you must show all stages of your working.**

Solutions based on calculator technology are not acceptable.

(i) Use integration by parts to find the exact value of

$$\int_0^4 x^2 e^{2x}\,dx$$

giving your answer in simplest form.

(5)

(ii) Use integration by substitution to show that

$$\int_3^{\frac{21}{2}} \frac{4x}{(2x-1)^2}\,dx = a + \ln b$$

where a and b are constants to be found.

(7)

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7. The number of goats on an island is being monitored.

When monitoring began there were 3000 goats on the island.

In a simple model, the number of goats, x, in thousands, is modelled by the equation

$$x = \frac{k(9t + 5)}{4t + 3}$$

where k is a constant and t is the number of years after monitoring began.

(a) Show that $k = 1.8$

(2)

(b) Hence calculate the long-term population of goats predicted by this model.

(1)

In a **second** model, the number of goats, x, in thousands, is modelled by the differential equation

$$3\frac{dx}{dt} = x(9 - 2x)$$

(c) Write $\frac{3}{x(9 - 2x)}$ in partial fraction form.

(3)

(d) Solve the differential equation with the initial condition to show that

$$x = \frac{9}{2 + e^{-3t}}$$

(5)

(e) Find the long-term population of goats predicted by this **second** model.

(1)

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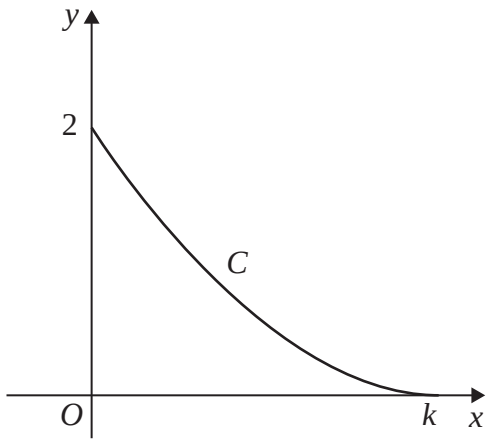


Figure 3

Figure 3 shows a sketch of the curve C with parametric equations

$$x = 6t - 3\sin 2t \qquad y = 2\cos t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The curve meets the y-axis at 2 and the x-axis at k, where k is a constant.

(a) State the value of k.

(1)

The region bounded by the curve, the x-axis and the y-axis is rotated through 2π radians about the x-axis to form a solid of revolution.

(d) (i) Show that the volume of this solid is given by

$$\int_0^{\alpha} \beta\left(1 - \cos 4t\right) \mathrm{d}t$$

where α and β are constants to be found.

(ii) Hence, using algebraic integration, find the exact volume of this solid.

(6)

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2. Given that

$$\frac{3x+4}{(x-2)(2x+1)^2} \equiv \frac{A}{x-2} + \frac{B}{2x+1} + \frac{C}{(2x+1)^2}$$

(a) find the values of the constants A, B and C.

(4)

(b) Hence find the exact value of

$$\int_7^{12} \frac{3x+4}{(x-2)(2x+1)^2}\,\mathrm{d}x$$

giving your answer in the form $p\ln q + r$ where p, q and r are rational numbers.

(6)

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5. (a) Find $\int x^2 \cos 2x \, dx$

(4)

(b) Hence solve the differential equation

$$\frac{dy}{dt} = \left(\frac{t \cos t}{y} \right)^2$$

giving your answer in the form $y^n = f(t)$ where n is an integer.

(5)

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7. (a) Using the substitution $u = 4x + 2\sin 2x$, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} e^{4x+2\sin 2x}\cos^2 x\,dx = \frac{1}{8}(e^{2\pi} - 1)$$

(5)

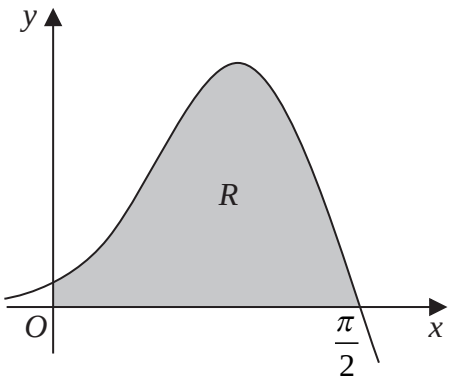


Figure 3

The curve shown in Figure 3, has equation

$$y = 6e^{2x+\sin 2x}\cos x$$

The region R, shown shaded in Figure 3, is bounded by the positive x-axis, the positive y-axis and the curve.

The region R is rotated through 2π radians about the x-axis to form a solid.

(b) Use the answer to part (a) to find the volume of the solid formed, giving the answer in simplest form.

(3)

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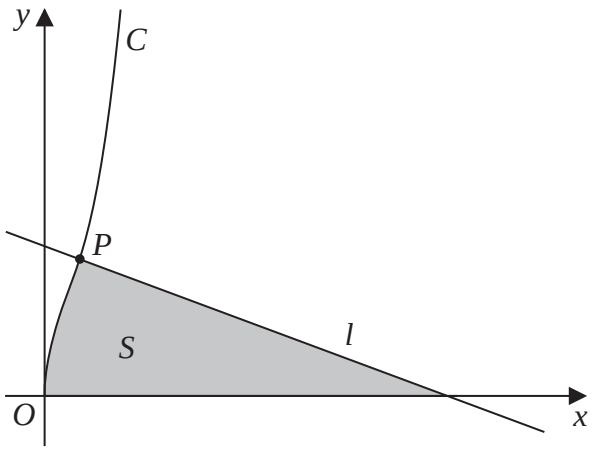


Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

A curve C has parametric equations

$$x = \sin^2 t \qquad y = 2\tan t \qquad 0 \leqslant t < \frac{\pi}{2}$$

The point P with parameter $t = \frac{\pi}{4}$ lies on C.

The line l is the normal to C at P, as shown in Figure 3.

The region S, shown shaded in Figure 3, is bounded by C, l and the x-axis.

(b) Find, using calculus, the exact area of S.

equation for l is

$$8y + 2x = 17$$

(6)

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June 2024

1. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Find

$$\int_0^{\frac{\pi}{6}} x\cos 3x \,\mathrm{d}x$$

giving your answer in simplest form.

(5)

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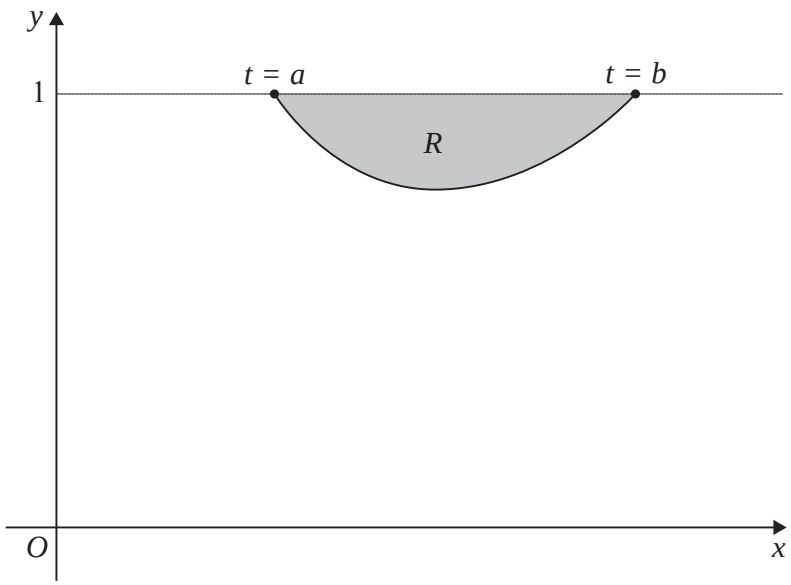


Figure 2

Figure 2 shows a sketch of the curve defined by the parametric equations

$$x = t^2 + 2t \qquad y = \frac{2}{t(3-t)} \qquad a \leqslant t \leqslant b$$

where a and b are constants.

The ends of the curve lie on the line with equation $y = 1$

(a) Find the value of a and the value of b

(2)

The region R, shown shaded in Figure 2, is bounded by the curve and the line with equation $y = 1$

(b) Show that the area of region R is given by

$$M - k\int_a^b \frac{t+1}{t(3-t)}\,\mathrm{d}t$$

where M and k are constants to be found.

(5)

(c) (i) Write $\dfrac{t+1}{t(3-t)}$ in partial fractions.

(ii) Use algebraic integration to find the exact area of R, giving your answer in simplest form.

(6)

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7. The current, x amps, at time t seconds after a switch is closed in a particular electric circuit is modelled by the equation

$$\frac{dx}{dt} = k - 3x$$

where k is a constant.

Initially there is zero current in the circuit.

(a) Solve the differential equation to find an equation, in terms of k, for the current in the circuit at time t seconds.
Give your answer in the form $x = f(t)$.

(6)

Given that in the long term the current in the circuit approaches 7 amps,

(b) find the value of k.

(2)

(c) Hence find the time in seconds it takes for the current to reach 5 amps, giving your answer to 2 significant figures.

(3)

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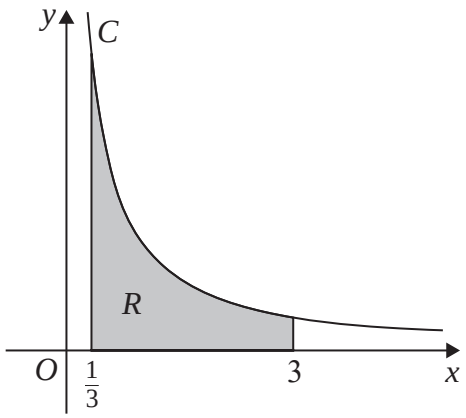


Figure 3

The curve C, shown in Figure 3, has equation

$$y = \frac{x^{-\frac{1}{4}}}{\sqrt{1+x}\left(\arctan\sqrt{x}\right)}$$

The region R, shown shaded in Figure 3, is bounded by C, the line with equation $x = 3$, the x-axis and the line with equation $x = \frac{1}{3}$

The region R is rotated through 360° about the x-axis to form a solid.

Using the substitution $\tan u = \sqrt{x}$

(a) show that the volume V of the solid formed is given by

$$k\int_a^b \frac{1}{u^2}\,\mathrm{d}u$$

where k, a and b are constants to be found.

(6)

(b) Hence, using algebraic integration, find the value of V in simplest form.

(3)

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6. Use the substitution $u = \sqrt{x^3 + 1}$ to show that

$$\int \frac{9x^5}{\sqrt{x^3 + 1}} \mathrm{d}x = 2\left(x^3 + 1\right)^k \left(x^3 - A\right) + c$$

where k and A are constants to be found and c is an arbitrary constant.

(5)

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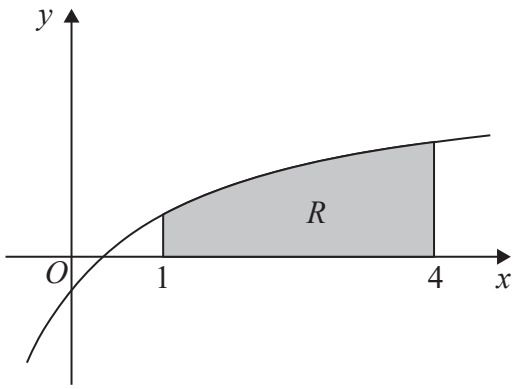


Figure 4

Figure 4 shows a sketch of part of the curve with equation

$$y = \frac{3x - 1}{x + 2} \qquad x > -2$$

(a) Show that

$$\frac{3x - 1}{x + 2} \equiv A + \frac{B}{x + 2}$$

where A and B are constants to be found.

(2)

The finite region R, shown shaded in Figure 4, is bounded by the curve, the line with equation $x = 4$, the x-axis and the line with equation $x = 1$

This region is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Use the answer to part (a) and algebraic integration to find the exact volume of the solid generated, giving your answer in the form

$$\pi(p + q \ln 2)$$

where p and q are rational constants.

(6)

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9. (a) Express $\dfrac{1}{x(2x-1)}$ in partial fractions.

(2)

The height above ground, h metres, of a carriage on a fairground ride is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{1}{50}h(2h-1)\cos\left(\frac{t}{10}\right)$$

where t seconds is the time after the start of the ride.

Given that, at the start of the ride, the carriage is 2.5 m above ground,

(b) solve the differential equation to show that, according to the model,

$$h = \frac{5}{10 - 8\mathrm{e}^{k\sin\left(\frac{t}{10}\right)}}$$

where k is a constant to be found.

(6)

(c) Hence find, according to the model, the time taken for the carriage to reach its maximum height above ground for the 3rd time.
Give your answer to the nearest second.

(*Solutions relying entirely on calculator technology are not acceptable.*)

(2)

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10.

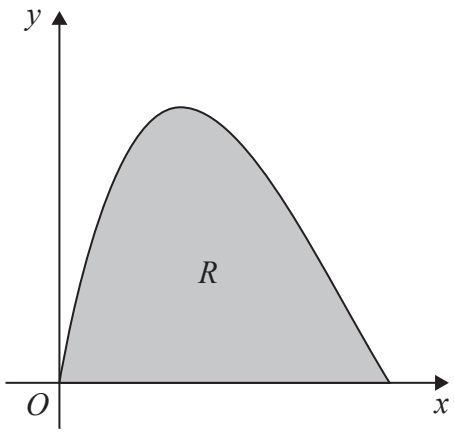


Figure 5

Figure 5 shows a sketch of the curve with parametric equations

$$x = 3t^2 \qquad y = \sin t \sin 2t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region R, shown shaded in Figure 5, is bounded by the curve and the x-axis.

(a) Show that the area of R is

$$k\int_0^{\frac{\pi}{2}} t\sin^2 t\cos t\,\mathrm{d}t$$

where k is a constant to be found.

(3)

(b) Hence, using algebraic integration, find the exact area of R, giving your answer in the form

$$p\pi + q$$

where p and q are constants.

(5)

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January 2025

1.

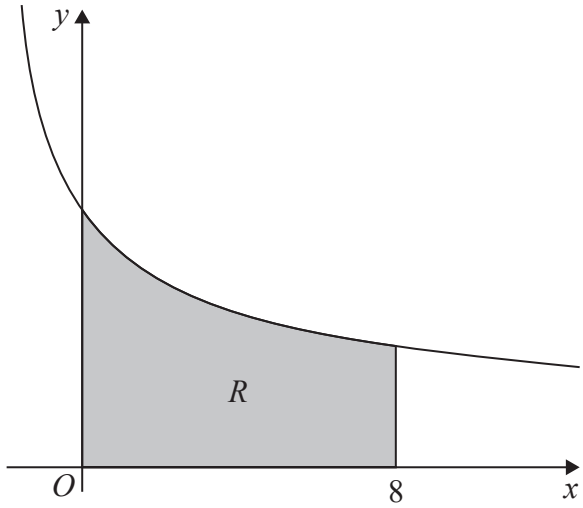


Figure 1

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

The curve shown in Figure 1 has equation

$$y = \frac{4}{x+2} \qquad x > -2$$

The region R, bounded by the curve, the y-axis, the x-axis and the line with equation $x = 8$ is shown shaded in Figure 1

Region R is rotated through 360 degrees about the x-axis.

Use calculus to find the exact value of the volume of the solid generated, writing your answer in simplest form.

(5)

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4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

(i) The volume, V, of a spherical balloon is increasing at a constant rate of $70\pi\,\text{cm}^3\,\text{s}^{-1}$

Find the rate of increase of the radius of the balloon, in $\text{cm}\,\text{s}^{-1}$, at the instant when the radius of the balloon is 5 cm.

[The volume V of a sphere of radius r is given by the formula $V = \frac{4}{3}\pi r^3$]

(4)

(ii) The depth of water in a cave is being monitored.

The rate of increase in the depth of water, h cm, at a particular point in the cave is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{k}{h^3}$$

where k is a constant and t hours is the time after monitoring began.

Given that

- initially the depth of water was 4 cm
- 5 hours after monitoring began, the depth of water was 6 cm
- T hours after monitoring began, the depth of water was 10 cm

solve the differential equation to find the value of T.

Give your answer to one decimal place.

(6)

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5. (i) Find

$$\int x^2 e^{4x} dx$$

writing the answer in simplest form.

(4)

(ii) Use partial fractions and algebraic integration to show that

$$\int_4^7 \frac{2x+11}{(2x+1)(2-x)} dx = \ln k$$

where k is a fully simplified rational constant to be found.

(6)

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7. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

Use the substitution $x = 4\sin\theta$ to find the exact value of

$$\int_{2}^{2\sqrt{3}} \frac{1}{\left(16-x^2\right)^{\frac{3}{2}}}\,\mathrm{d}x$$

(6)

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3. Given that $y = 4$ at $x = \frac{\pi}{6}$, solve the differential equation

$$y\cos^2(2x)\frac{dy}{dx} = 3\sin(2x) \qquad y > 0 \qquad -\frac{\pi}{4} < x < \frac{\pi}{4}$$

giving your answer in the form $y^2 = g(x)$

(6)

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4. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = \frac{5 + 17x - 10x^2}{x(1 - x)(2x + 1)} \qquad x > 1$$

(a) Write $f(x)$ in the form

$$\frac{A}{x} + \frac{B}{1 - x} + \frac{C}{2x + 1}$$

where A, B and C are constants to be found.

(3)

(b) Hence, use algebraic integration to find the exact value of

$$\int_2^4 f(x)\,dx$$

Write your answer in the form $p \ln 2 + q \ln\left(\frac{3}{5}\right)$, where p and q are integers to be found.

(5)

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7. (a) Show that the substitution $u = \tan x$ transforms

$$\int \frac{\tan x + \tan^3 x}{(4 + \sec^2 x)^3} \, \mathrm{d}x$$

to

$$\int \frac{u}{(k+u^2)^n} \, \mathrm{d}u$$

where k and n are integers to be found.

(4)

(b) Hence find

$$\int \frac{\tan x + \tan^3 x}{(4+\sec^2 x)^3} \, \mathrm{d}x$$

(2)

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9. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

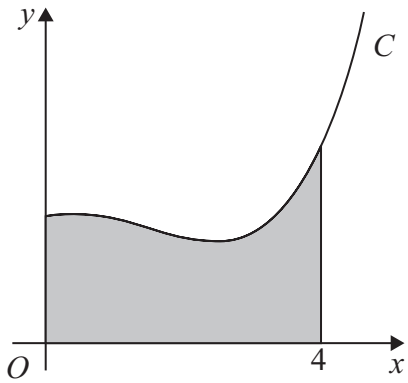


Figure 1

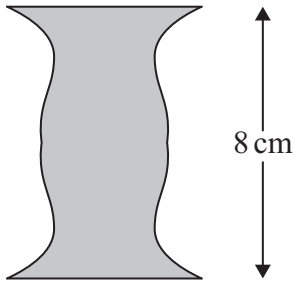


Figure 2

Figure 1 shows a sketch of part of the curve C with equation

$$y = \cos x + \frac{1}{5}e^{x}$$

The finite region, shown shaded in Figure 1, is bounded by C, the y-axis, the x-axis and the line with equation $x = 4$

The region is rotated through 2π radians about the x-axis to form a solid S.

(a) Show that the exact volume of S is given by

$$\pi \int_0^4 \left(A + B\cos 2x + Ce^{x}\cos x + De^{2x} \right) \mathrm{d}x$$

where A, B, C and D are constants to be found.

(4)

(b) Find $\int e^{x}\cos x \, \mathrm{d}x$

(4)

Figure 2 represents a paperweight formed by joining two of these solids together.
The paperweight is 8 cm high.

(c) Using the answers to parts (a) and (b), find, by algebraic integration, the volume of the paperweight, giving your answer to 2 significant figures.

(4)

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1: **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Find, by algebraic integration,

$$\int_1^3 2x \ln(3x)\,\mathrm{d}x$$

giving the answer in the form $A\ln 3 + B$ where A and B are integers.

(5)

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3: Given that $y = 0$ at $x = 5$ solve the differential equation

$$8y\frac{dy}{dx} = xe^{-y^2} \qquad y \geqslant 0 \quad x \geqslant 5$$

giving your answer in the form $y^2 = f(x)$

(5)

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7: **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

(a) Show that the substitution $u = e^x - \frac{1}{2}$ transforms

$$\int_{\ln 3}^{\ln 8} \frac{24e^x - 2}{(2e^x - 1)^2}\,dx$$

to

$$\int_a^b \frac{12u + 5}{u^2(2u + 1)}\,du$$

where a and b are constants to be found.

(4)

(b) Find an expression for $\frac{12u + 5}{u^2(2u + 1)}$ in the form

$$\frac{A}{u^2} + \frac{B}{u} + \frac{C}{(2u + 1)}$$

where A, B and C are integers.

(3)

(c) Hence show that

$$\int_{\ln 3}^{\ln 8} \frac{24e^x - 2}{(2e^x - 1)^2}\,dx = p + \ln q$$

where p and q are constants to be found.

(4)

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9: **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

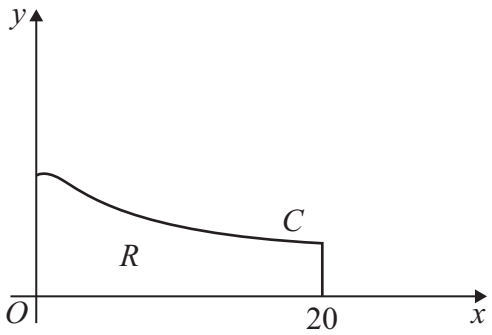


Figure 2

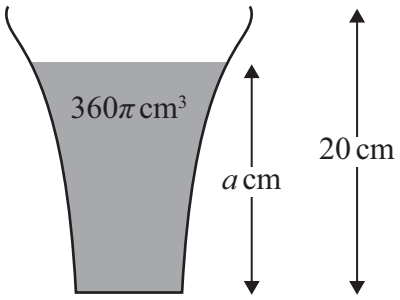


Figure 3

The curve C has equation

$$y = 12\left(\frac{2x+3}{x^2+3x+5}\right)^{\frac{1}{2}}$$

The finite region, R, shown in Figure 2, is bounded by the curve C, the line with equation $x = 20$, the x-axis and the y-axis.

The region R is rotated through 2π radians about the x-axis to form a solid, S.

(a) Use algebraic integration to find the exact volume of S.

Give your answer in the form $k\ln\lambda$ where k and λ are constants.

(4)

Figure 3 shows a hollow vase with identical shape to solid S.

The vase is made of glass, which has negligible thickness.

When the vase is full of water,

volume of water in the vase = volume of S

When the vase contains $360\pi\,\text{cm}^3$ of water, the depth of the water is a cm, as shown in Figure 3.

(b) Use algebra to find the value of a, giving your answer to one decimal place.

(3)

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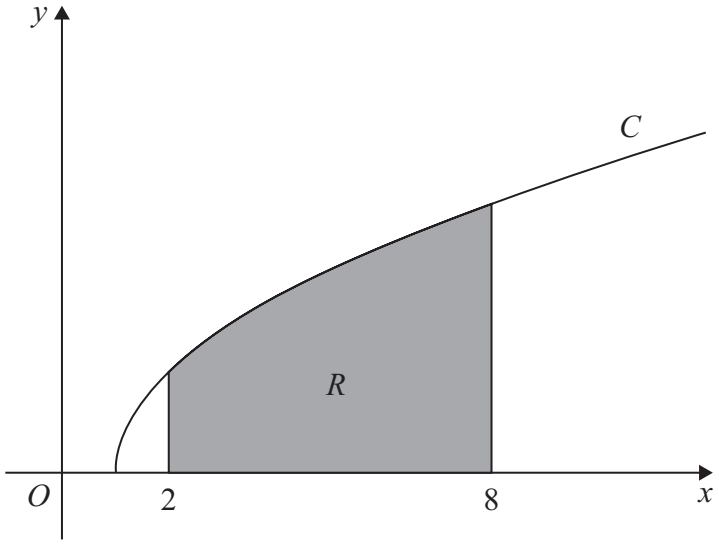


Figure 1

Figure 1 shows a sketch of the curve C with equation

$$y = \frac{5}{3}\sqrt{x-1}$$

The region R, shown shaded in Figure 1, is bounded by C, the line with equation $x = 2$, the x-axis and the line with equation $x = 8$

This region is then rotated through 2π radians about the x-axis to form a solid.

Use algebraic integration to find the exact volume of this solid.

(5)

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6. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Use the substitution $u = 3 + \cos\theta$ to show that

$$\int_0^{\frac{\pi}{2}} \frac{\sin 2\theta}{\sqrt{3 + \cos\theta}} \, \mathrm{d}\theta = a\sqrt{3} + b$$

where a and b are constants to be found.

(7)

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8. A container initially contains 6 litres of water.

Water starts to flow into the container.

The rate at which the water flows into the container is modelled by the differential equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = 12t\mathrm{e}^{-t}$$

where V litres is the volume of water in the container, t minutes after water starts flowing into the container.

(a) Solve the differential equation and use the initial condition to find an equation for V in terms of t

(5)

Given that the capacity of the container is 20 litres,

(b) determine, according to the model, whether or not the container will ever become full, giving a reason for your answer.

(1)

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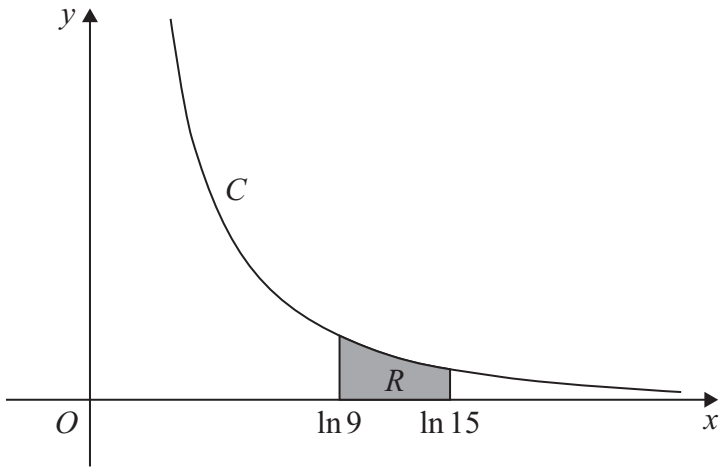


Figure 3

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Figure 3 shows a sketch of the curve C with parametric equations

$$x = \ln(2t + 5) \qquad y = \frac{1}{t + 1} \qquad t > -1$$

A point P lies on C.

Given that the gradient of C at P is -4

(a) use calculus to find the exact y coordinate of P.

(6)

The region R, shown shaded in Figure 3, is bounded by C, the line with equation $x = \ln 9$, the x-axis and the line with equation $x = \ln 15$

(b) (i) Show that the area of R is given by

$$\int_a^b \frac{k}{(t + 1)(2t + 5)}\,\mathrm{d}t$$

where a, b and k are constants to be found.

(ii) Hence, using algebraic integration, find the exact area of R.

Write your answer in the form $\frac{1}{3}\ln\alpha$, where α is a rational number.

(9)

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2. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Using algebraic integration, find the exact value of

$$\int_{2}^{2e^2} x^3 \ln\left(\frac{1}{2}x\right) \mathrm{d}x$$

giving the answer in simplest form.

(5)

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4. (a) Use the substitution $u = e^x + 4$ to show that

$$\int_0^{\ln 6} \frac{2e^x + 3}{e^x + 4}\,dx = \int_a^b \frac{cu + d}{u(u-4)}\,du$$

where a, b, c and d are constants to be found.

(3)

(b) Hence, using algebraic integration, show that

$$\int_0^{\ln 6} \frac{2e^x + 3}{e^x + 4}\,dx = P\ln 2 + Q\ln 3$$

where P and Q are constants to be found.

(6)

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8. A large spherical balloon starts to deflate.

The balloon remains spherical whilst deflating.

At time t seconds after the balloon starts to deflate

- the balloon has radius r cm
- the balloon has volume V cm^3

Given that the volume of the balloon is modelled to decrease at a constant rate,

(a) use the model to show that

$$\frac{\mathrm{d}r}{\mathrm{d}t} = -\frac{k}{r^2}$$

where k is a positive constant.

(3)

Given that

- the initial radius of the balloon is 30 cm
- after 24 seconds the radius of the balloon is 12 cm

(b) solve the differential equation in part (a) to find an equation for r in terms of t.

(5)

Given that $V = 0$ when $t = T$

(c) use the model to find the value of T to one decimal place.

(2)

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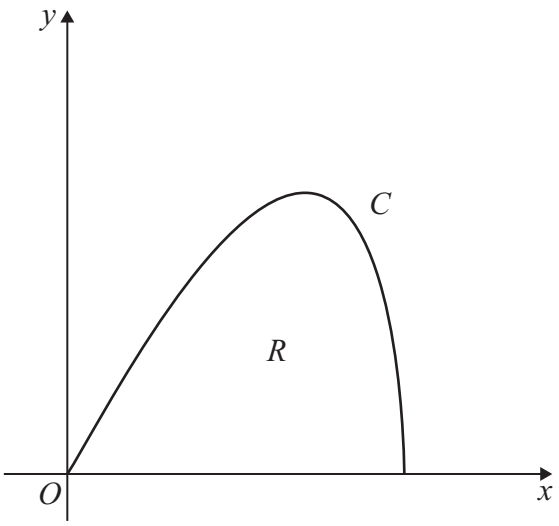


Figure 3

The curve C, shown in Figure 3, has parametric equations

$$x = 6\sin t \qquad y = 5\sin 2t \qquad 0 \leqslant t \leqslant k$$

where k is a constant.

The region R, also shown in Figure 3, is bounded by C and the x-axis.

This region is rotated through 2π radians about the x-axis to form a solid.

(a) Show that the volume, V, of this solid is given by

$$V = P\int_0^k \sin^m t \cos^n t \, \mathrm{d}t$$

where k, P, m and n are constants to be found.

(4)

(b) Hence, using algebraic integration, find the exact value of V.

(4)

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3. The height, h metres, of a shrub, t years after it was planted, is modelled by the differential equation

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{2h^{\frac{3}{2}}}{5t^2} \qquad t > 0$$

Given that $h = 1$ when $t = 1$

(a) solve the differential equation to show that

$$h = \frac{at^2}{(1 + bt)^2}$$

where a and b are constants to be found.

(6)

(b) Hence find, according to the model, the limit of the height of the shrub.

(2)

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4. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

Find

$$\int_1^3 \frac{3x^2 + 8}{x^2 - 4x} \mathrm{d}x$$

giving the answer in the form $a + b \ln 3$ where a and b are constants.

(7)

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6. **In this question you must show all stages of your working.**
Solutions relying on calculator technology are not acceptable.

(a) Use algebraic integration and the substitution $u = 2x - 1$ to show that

$$\int_2^5 \frac{(3x+2)^2}{2x-1}\,dx = 72 + \frac{49}{8}\ln 3$$

(6)

The curve C has equation

$$y = \frac{3x+2}{2\sqrt{2x-1}} \qquad x > 1$$

The finite region R is bounded by C, the x-axis and the lines with equations $x = 2$ and $x = 5$

The region R is rotated through 2π radians about the x-axis to form a solid of revolution.

(b) Using the result from part (a), find the exact value of the volume of the solid generated.

(2)

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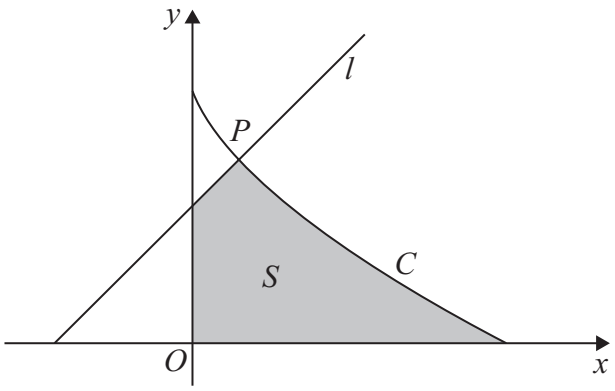


Figure 1

In this question you must show all stages of your working.
Solutions relying on calculator technology are not acceptable.

Figure 1 shows a sketch of the curve C with parametric equations

$$x = 8\cos^3\theta \qquad y = 6\sin^2\theta \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

Given that the point P lies on C and has parameter $\theta = \frac{\pi}{3}$

(a) find the coordinates of P.

(2)

The line l is the normal to C at P.

(b) Find an equation for l in the form $y = mx + c$, where m and c are constants.

(5)

The finite region S, shown shaded in Figure 1, is bounded by the curve C, the line l, the y-axis and the x-axis.

(c) Show that the area of S is given by

$$4 + 144\int_0^{\frac{\pi}{3}} (\sin\theta\cos^2\theta - \sin\theta\cos^4\theta)\,\mathrm{d}\theta$$

(6)

(d) Hence, using algebraic integration, find the exact area of S.

(3)

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7. (i) Use integration by parts to find

$$\int e^{2x} \cos 4x \, dx$$

(6)

(ii) Using the substitution $2x = \sin u$ and algebraic integration, find the exact value of

$$\int_{\frac{1}{4}}^{\frac{\sqrt{3}}{4}} \frac{2x^3}{\sqrt{1-4x^2}} \, dx$$

giving the answer in simplest form.

(*Solutions relying on calculator technology are not acceptable.*)

(7)

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8. The growth of algae on the surface of a pond is being monitored.

The surface area of the pond covered by algae, $S\ \text{m}^2$, is modelled by the equation

$$S = 140 - \frac{k}{0.3t + 8}$$

where t days is the time after monitoring began and k is a constant.

Given that $18\ \text{m}^2$ of the surface area of the pond was covered by algae when monitoring began,

(a) find, according to this model,

(i) the value of k,

(ii) the value of $\frac{\text{d}S}{\text{d}t}$ when $t = 10$

(*Solutions relying entirely on calculator technology are not acceptable.*)

(3)

In an improved model, S increases with respect to t according to the differential equation

$$\frac{\text{d}S}{\text{d}t} = \frac{128 - S}{50}$$

(b) Solve the differential equation, with $S = 18$ when $t = 0$, to find an equation for S in terms of t.

(6)

Given that the algae grows until it covers the entire surface of the pond,

(c) find, according to the **improved** model, the maximum possible area of the pond.

(1)

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2. Given that

$$\frac{2x^2 - x + 11}{(x+2)(2x-3)} \equiv A + \frac{B}{x+2} + \frac{C}{2x-3}$$

(a) find the value of each of the constants A, B and C.

(4)

(b) Hence find

$$\int \frac{2x^2 - x + 11}{(x+2)(2x-3)} \mathrm{d}x$$

(3)

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5. Use the substitution $u = \sqrt{2x-1}$ to show that

$$\int \frac{x}{\sqrt{2x-1}}\,\mathrm{d}x = A(2x-1)^{\frac{1}{2}}(x+B) + k$$

where A and B are constants to be found, and k is an arbitrary constant.

(5)

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8. (a) Use integration to show that

$$\int x\cos 2x\,\mathrm{d}x = Ax\sin 2x + B\cos 2x + c$$

where A and B are constants to be found, and c is an arbitrary constant.

(3)

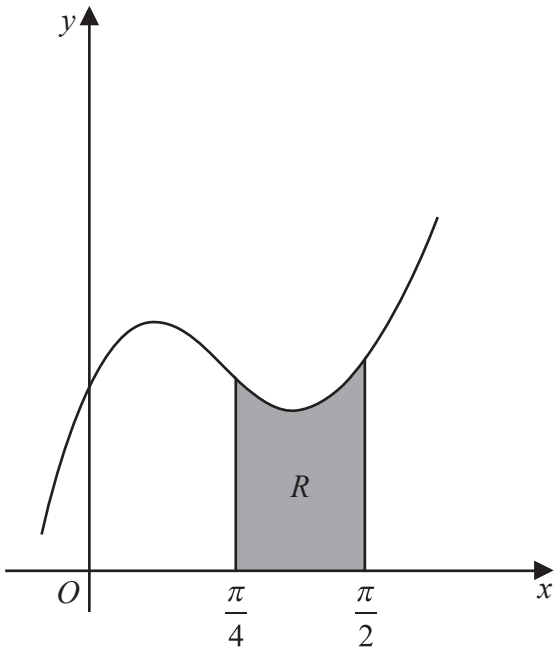


Figure 1

Figure 1 shows a sketch of part of the curve with equation

$$y = 4x + 3\cos 2x$$

The finite region R, shown shaded in Figure 1, is bounded by the curve, the line with equation $x = \frac{\pi}{2}$, the x-axis and the line with equation $x = \frac{\pi}{4}$

The region R is rotated through 2π radians about the x-axis to form a solid of revolution S.

(b) Use algebraic integration to find the exact volume of S, giving the answer in the form

$$\pi\left(p\pi^3 + q\pi + r\right)$$

where p, q and r are constants.

(6)

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9. **In this question you must show all stages of your working.**
Solutions relying entirely on calculator technology are not acceptable.

A drink is put in a freezer to cool.

The temperature, $\theta°\text{C}$, of the drink, t minutes after it is put in the freezer, is modelled by the differential equation

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -k(\theta + 15)$$

where k is a positive constant.

Given that the temperature of the drink when it was put in the freezer was $21°\text{C}$,

(a) solve the differential equation to show that, according to the model,

$$\theta = A\mathrm{e}^{-kt} + B$$

where A and B are integers to be found.

(5)

Given that 20 minutes after the drink is put in the freezer, its temperature is $12.9°\text{C}$,

(b) find the value of k, giving the answer to 3 significant figures.

(2)

(c) Hence find, according to the model, the temperature of the drink, one hour after it was put in the freezer. Give the answer to one decimal place.

(2)

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10.

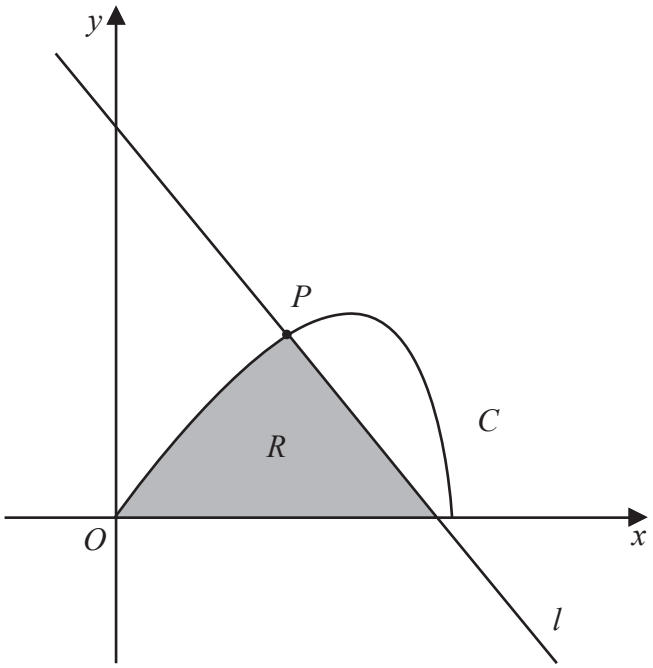


Figure 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 8\cos\theta \qquad y = 5\sin 2\theta \qquad 0 \leqslant \theta \leqslant \frac{\pi}{2}$$

The point P with parameter $\theta = \frac{\pi}{3}$ lies on C.

The line l is the normal to C at P, as shown in Figure 2.

(a) Use calculus to show that l has equation

$$y = p\sqrt{3}\,x + q\sqrt{3}$$

where p and q are rational constants to be found.

(4)

The finite region R, shown shaded in Figure 2, is bounded by C, l and the x-axis.

(b) Using algebraic integration, find the exact area of R, giving the answer in the form $A + B\sqrt{3}$ where A and B are rational constants.

(6)

(Total for Question 10 is 10 marks)

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