9. With respect to a fixed origin O, the line l_1 is given by the equation

$$\mathbf{r} = \begin{pmatrix} 8 \\ 1 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} -5 \\ 4 \\ 3 \end{pmatrix}$$

where μ is a scalar parameter.

The point A lies on l_1 where $\mu = 1$

(a) Find the coordinates of A.

(1)

The point P has position vector $\begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix}$

The line l_2 passes through the point P and is parallel to the line l_1

(b) Write down a vector equation for the line l_2

(2)

(c) Find the exact value of the distance AP.

Give your answer in the form $k\sqrt{2}$, where k is a constant to be found.

(2)

The acute angle between AP and l_2 is θ

(d) Find the value of $\cos\theta$

(3)

A point E lies on the line l_2

Given that $AP = PE$,

(e) find the area of triangle APE,

(2)

(f) find the coordinates of the two possible positions of E.

(5)

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8. Relative to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = \begin{pmatrix} 4 \\ -3 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix} \quad \text{where } \lambda \text{ is a scalar parameter}$$

$$l_2: \quad \mathbf{r} = \begin{pmatrix} 2 \\ 0 \\ -9 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} \quad \text{where } \mu \text{ is a scalar parameter}$$

Given that l_1 and l_2 meet at the point X,

(a) find the position vector of X.

(5)

The point $P(10, -7, 0)$ lies on l_1

The point Q lies on l_2

Given that $\overrightarrow{PQ}$ is perpendicular to l_2

(b) calculate the coordinates of Q.

(5)

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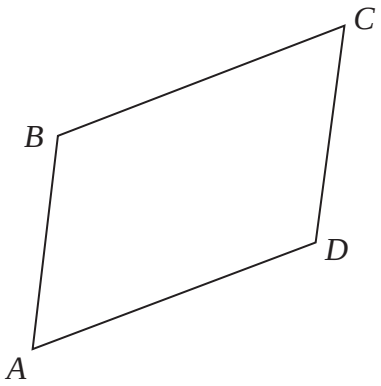


Figure 1

Figure 1 shows a sketch of parallelogram *ABCD*.

Given that $\overrightarrow{AB} = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{BC} = 2\mathbf{i} + 5\mathbf{j} + 8\mathbf{k}$

(a) find the size of angle *ABC*, giving your answer in degrees, to 2 decimal places.

(3)

(b) Find the area of parallelogram *ABCD*, giving your answer to one decimal place.

(2)

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8. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = \begin{pmatrix} -1 \\ 5 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} \qquad l_2: \quad \mathbf{r} = \begin{pmatrix} 2 \\ -2 \\ -5 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -3 \\ b \end{pmatrix}$$

where λ and μ are scalar parameters and b is a constant.

Prove that for all values of $b \neq 7$, the lines l_1 and l_2 are skew.

(6)

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7. Relative to a fixed origin O, the line l has equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ -10 \\ -9 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} \quad \text{where } \lambda \text{ is a scalar parameter}$$

Given that $\overrightarrow{OA}$ is a unit vector parallel to l,

(a) find $\overrightarrow{OA}$

(2)

The point X lies on l.

Given that X is the point on l that is closest to the origin,

(b) find the coordinates of X.

(5)

The points O, X and A form the triangle OXA.

(c) Find the exact area of triangle OXA.

(3)

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9. (i) Relative to a fixed origin O, the points A, B and C have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively.

Points A, B and C lie in a straight line, with B lying between A and C.

Given $AB : AC = 1 : 3$ show that

$$\mathbf{c} = 3\mathbf{b} - 2\mathbf{a}$$

(3)

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7. With respect to a fixed origin O,

- the line l has equation $\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix} + \lambda \begin{pmatrix} -4 \\ -3 \\ 5 \end{pmatrix}$ where λ is a scalar constant
- the point A has position vector $9\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$

Given that X is the point on l nearest to A,

(a) find

(i) the coordinates of X

(ii) the shortest distance from A to l.
Give your answer in the form $\sqrt{d}$, where d is an integer.

(7)

The point B is the image of A after reflection in l.

(b) Find the position vector of B.

(2)

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8. With respect to a fixed origin O the points A and B have position vectors

$$\begin{pmatrix} 6 \\ 6 \\ 2 \end{pmatrix} \text{ and } \begin{pmatrix} 6 \\ 0 \\ 7 \end{pmatrix}$$

respectively.

The line l_1 passes through the points A and B.

(a) Write down an equation for l_1

Give your answer in the form $\mathbf{r} = \mathbf{p} + \lambda\mathbf{q}$, where λ is a scalar parameter.

(2)

The line l_2 has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 5 \\ 9 \end{pmatrix}$$

where μ is a scalar parameter.

(b) Show that l_1 and l_2 do **not** meet.

(4)

The point C is on l_2 where $\mu = -1$

(c) Find the acute angle between AC and l_2

Give your answer in degrees to one decimal place.

(5)

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6. Relative to a fixed origin O,

- the point A has position vector $\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}$
- the point B has position vector $5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$
- the point C has position vector $3\mathbf{i} + p\mathbf{j} - \mathbf{k}$

where p is a constant.

The line l passes through A and B.

(a) Find a vector equation for the line l

(3)

Given that $\overrightarrow{AC}$ is perpendicular to l

(b) find the value of p

(3)

(c) Hence find the area of triangle ABC, giving your answer as a surd in simplest form.

(3)

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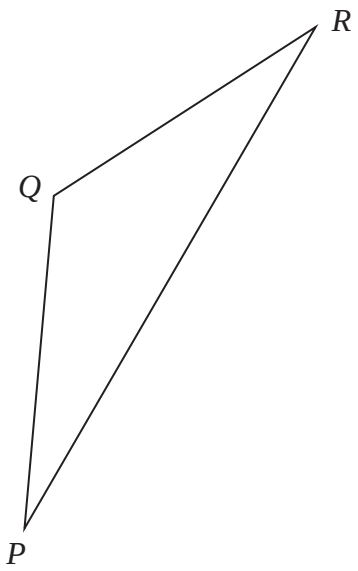


Figure 1

Figure 1 shows a sketch of triangle PQR.

Given that

- $\overrightarrow{PQ} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$
- $\overrightarrow{PR} = 8\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$

(a) Find $\overrightarrow{RQ}$

(2)

(b) Find the size of angle PQR, in degrees, to three significant figures.

(3)

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9. With respect to a fixed origin O, the equations of lines l_1 and l_2 are given by

$$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 8 \\ 10 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$$

$$l_2 : \mathbf{r} = \begin{pmatrix} -4 \\ -1 \\ 2 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 4 \\ 8 \end{pmatrix}$$

where λ and μ are scalar parameters.

Prove that lines l_1 and l_2 are skew.

(5)

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6. Relative to a fixed origin O.

- the point A has position vector $2\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$
- the point B has position vector $8\mathbf{i} + 3\mathbf{j} - 7\mathbf{k}$

The line l passes through A and B.

(a) (i) Find $\overrightarrow{AB}$

(ii) Find a vector equation for the line l

(3)

The point C has position vector $3\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$

The point P lies on l

Given that $\overrightarrow{CP}$ is perpendicular to l

(b) find the position vector of the point P

(5)

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4. Relative to a fixed origin O,

- the point A has position vector $4\mathbf{i} + 8\mathbf{j} + \mathbf{k}$
- the point B has position vector $5\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$
- the point P has position vector $2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

The straight line l passes through A and B.

(a) Find a vector equation for l.

(2)

The point C lies on l so that PC is perpendicular to l.

(b) Find the coordinates of C.

(4)

The point P' is the reflection of P in the line l.

(c) Find the coordinates of P'

(2)

(d) Hence find $\left|\overrightarrow{PP'}\right|$, giving your answer as a simplified surd.

(2)

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6. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$ where λ is a scalar parameter.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -7 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -1 \\ 8 \end{pmatrix}$ where μ is a scalar parameter.

Given that l_1 and l_2 meet at the point P

(a) state the coordinates of P

(1)

Given that the angle between lines l_1 and l_2 is θ

(b) find the value of $\cos\theta$, giving the answer as a fully simplified fraction.

(3)

The point Q lies on l_1 where $\lambda = 6$

Given that point R lies on l_2 such that triangle QPR is an isosceles triangle with $PQ = PR$

(c) find the exact area of triangle QPR

(3)

(d) find the coordinates of the possible positions of point R

(3)

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6. Relative to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (3\mathbf{i} + p\mathbf{j} + 7\mathbf{k}) + \lambda(2\mathbf{i} - 5\mathbf{j} + 4\mathbf{k})$$

$$l_2 : \mathbf{r} = (8\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) + \mu(4\mathbf{i} + \mathbf{j} + 2\mathbf{k})$$

where λ and μ are scalar parameters and p is a constant.

Given that l_1 and l_2 intersect,

(a) find the value of p,

(4)

(b) find the position vector of the point of intersection.

(2)

(c) Find the acute angle between l_1 and l_2

Give your answer in degrees to one decimal place.

(3)

The point A lies on l_1 with parameter $\lambda = 2$

The point B lies on l_2 with $\overrightarrow{AB}$ perpendicular to l_2

(d) Find the coordinates of B

(5)

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2. With respect to a fixed origin, O, the point A has position vector

$$\overrightarrow{OA} = \begin{pmatrix} 7 \\ 2 \\ -5 \end{pmatrix}$$

Given that

$$\overrightarrow{AB} = \begin{pmatrix} -2 \\ 4 \\ 3 \end{pmatrix}$$

(a) find the coordinates of the point B.

(2)

The point C has position vector

$$\overrightarrow{OC} = \begin{pmatrix} a \\ 5 \\ -1 \end{pmatrix}$$

where a is a constant.

Given that $\overrightarrow{OC}$ is perpendicular to $\overrightarrow{BC}$

(b) find the possible values of a.

(4)

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6. With respect to a fixed origin O, the line l_1 is given by the equation

$$\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + \lambda(8\mathbf{i} - \mathbf{j} + 4\mathbf{k})$$

where λ is a scalar parameter.

The point A lies on l_1

Given that $|\overrightarrow{OA}| = 5\sqrt{10}$

(a) show that at A the parameter λ satisfies

$$81\lambda^2 + 52\lambda - 220 = 0$$

(3)

Hence

(b) (i) show that one possible position vector for A is $-15\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$

(ii) find the other possible position vector for A.

(3)

The line l_2 is parallel to l_1 and passes through O.

Given that

- $\overrightarrow{OA} = -15\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$
- point B lies on l_2 where $|\overrightarrow{OB}| = 4\sqrt{10}$

(c) find the area of triangle OAB, giving your answer to one decimal place.

(4)

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8. Relative to a fixed origin O

- the point A has coordinates $(-10, 5, -4)$
- the point B has coordinates $(-6, 4, -1)$

The straight line l_1 passes through A and B.

(a) Find a vector equation for l_1

(2)

The line l_2 has equation

$$\mathbf{r} = \begin{pmatrix} 3 \\ p \\ q \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$$

where p and q are constants and μ is a scalar parameter.

Given that l_1 and l_2 intersect at B,

(b) find the value of p and the value of q.

(3)

The acute angle between l_1 and l_2 is θ

(c) Find the exact value of $\cos\theta$

(3)

Given that the point C lies on l_2 such that AC is perpendicular to l_2

(d) find the exact length of AC, giving your answer as a surd.

(2)

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8. Relative to a fixed origin O, the line l has equation

$$\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 2 \\ -1 \end{pmatrix}$$

where λ is a scalar parameter.

The point A and the point B lie on line l

Given that

- A has coordinates $(-2, a, 4)$
- B has coordinates $(b, 3, 1)$

(a) find the value of the constant a and the value of the constant b.

(2)

(b) Hence find vector $\overrightarrow{AB}$

(2)

The point C has coordinates $(4, 7, -2)$.

(c) Find the size of angle CAB, giving your answer in degrees to one decimal place.

(4)

The point D lies on the line l so that the area of triangle CAD is twice the area of triangle CAB.

(d) Find the coordinates of the two possible positions of D.

(4)

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6. Relative to a fixed origin O,

- the point A has position vector $\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$
- the point B has position vector $5\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

The line l passes through A and B.

(a) Write down a vector equation for l

(2)

Given also that

- the point C has position vector $3\mathbf{i} + \alpha\mathbf{j} + 5\mathbf{k}$ where α is a constant
- the points A, B and C form the triangle ABC
- angle BAC is $45°$

(b) find the exact possible values of α

(6)

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8: Relative to a fixed origin O,

- the point A has position vector $2\mathbf{i} - \mathbf{j} + 5\mathbf{k}$
- the point B has position vector $3\mathbf{i} + 4\mathbf{j} - 6\mathbf{k}$

Given that the line l passes through A and B,

(a) (i) find $\overrightarrow{AB}$

(ii) find a vector equation for l.

(3)

The point C has position vector $p\mathbf{i} + 4\mathbf{j} - \mathbf{k}$, where p is a constant.

Given that $\overrightarrow{CA}$ is perpendicular to $\overrightarrow{CB}$,

(b) find the possible values of p.

(4)

The points A, B and C form a triangle ABC.

Given also that $p > 0$

(c) find the area of triangle ABC, giving your answer to 3 significant figures.

(4)

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5. The line l_1 has equation

$$\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$$ where λ is a scalar parameter

The line l_2 has equation

$$\mathbf{r} = \begin{pmatrix} -1 \\ \beta \\ 6 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -1 \\ 7 \end{pmatrix}$$ where β is a constant and μ is a scalar parameter

Given that l_1 and l_2 intersect at the point P

(i) find the value of β

(ii) find the coordinates of P

(6)

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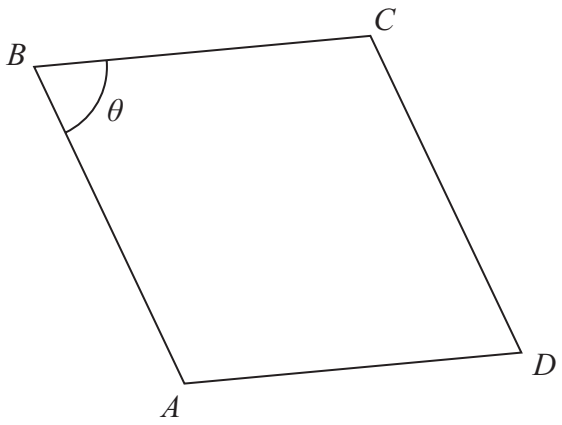


Figure 2

In this question you must show all stages of your working.
Solutions relying entirely on calculator technology are not acceptable.

Figure 2 shows a sketch of a parallelogram $ABCD$.

Given that

- $\overrightarrow{AB} = 6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$
- $\overrightarrow{BC} = -4\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$
- angle $ABC = \theta$, $0 < \theta < 90°$

(a) find the exact value of $\cos\theta$, giving your answer in simplest form.

(3)

(b) Show that the area of parallelogram $ABCD$ is $5\sqrt{k}$, where k is an integer to be found.

(3)

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6. With respect to a fixed origin O, the line l_1 has equation

$$\mathbf{r} = \begin{pmatrix} -3 \\ 0 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$$

where λ is a scalar parameter.

The point P lies on l_1

Given that P is the point on l_1 that is nearest O,

(a) find the coordinates of P.

(4)

With respect to the same fixed origin O, the line l_2 has equation

$$\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ a \\ -2 \end{pmatrix}$$

where a is a constant and μ is a scalar parameter.

Given that l_1 and l_2 intersect at the point Q,

(b) find the value of a,

(4)

(c) find the coordinates of Q.

(2)

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5. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \quad \mathrm{r} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$

$$l_2 : \quad \mathrm{r} = \begin{pmatrix} 10 \\ c \\ 3 \end{pmatrix} + \mu \begin{pmatrix} a \\ b \\ -2 \end{pmatrix}$$

where a, b and c are constants and λ and μ are scalar parameters.

Given that

- l_1 and l_2 meet when $\lambda = -2$
- l_1 and l_2 are perpendicular

find the value of a, the value of b and the value of c.

(6)

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9. *ABCD* is a parallelogram with *AB* parallel to *DC* and *AD* parallel to *BC*. The position vectors of *A*, *B*, *C* and *D* relative to a fixed origin *O* are **a**, **b**, **c** and **d** respectively.

Given that

$$\mathbf{a} = \mathbf{i} + \mathbf{j} - 2\mathbf{k} \qquad \mathbf{b} = 3\mathbf{i} - \mathbf{j} + 6\mathbf{k} \qquad \mathbf{c} = -\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$$

(a) find the position vector **d**,

(3)

(b) find the angle between the sides *AB* and *BC* of the parallelogram,

(4)

(c) find the area of the parallelogram *ABCD*.

(2)

The point *E* lies on the line through the points *C* and *D*, so that *D* is the midpoint of *CE*.

(d) Use your answer to part (c) to find the area of the trapezium *ABCE*.

(2)

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2. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1: \quad \mathbf{r} = \begin{pmatrix} -2 \\ 5 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} \qquad l_2: \quad \mathbf{r} = \begin{pmatrix} 6 \\ 12 \\ -5 \end{pmatrix} + \mu \begin{pmatrix} 4 \\ -3 \\ 10 \end{pmatrix}$$

where λ and μ are scalar parameters.

(a) With reference to the equations, explain why l_1 and l_2 are not parallel.

(1)

(b) Prove by contradiction that l_1 and l_2 do not intersect.

(5)

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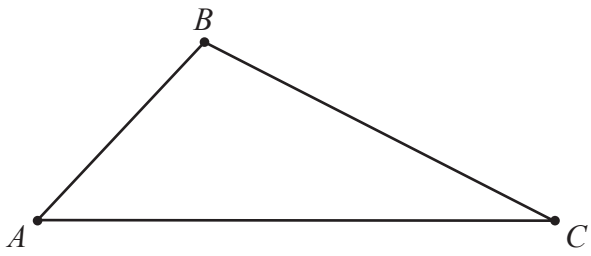


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows a sketch of triangle ABC.

Given that

- $\overrightarrow{AB} = 2\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$
- $\overrightarrow{AC} = 6\mathbf{i} - 2\mathbf{j} - 8\mathbf{k}$

(a) find the size of angle BAC, giving the answer, in degrees, to one decimal place.

(3)

(b) Hence find the area of triangle ABC, giving the answer to one decimal place.

(2)

The line AB is extended to a point D such that angle ADC is a right angle.

(c) Find $\overrightarrow{AD}$

(4)

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7. Relative to a fixed origin O

- the point A has position vector $5\mathbf{i} - 3\mathbf{j} + \mathbf{k}$
- the point B has position vector $3\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$
- the point C has position vector $\alpha\mathbf{i} + \beta\mathbf{j} - 5\mathbf{k}$

where α and β are constants.

The straight line l passes through A and B.

(a) Find a vector equation for l.

(2)

Given that C lies on l

(b) find the value of α and the value of β.

(2)

(c) Find, in degrees to one decimal place, the acute angle between OC and AB.

(3)

The point D lies on l such that OD is perpendicular to l.

(d) Find the exact coordinates of D.

(3)

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