**3.** A curve has equation

$$y = \frac{x^3}{16} - 4\sqrt{x} + \frac{8}{x}, \quad x > 0$$

(a) Find $\frac{dy}{dx}$ giving your answer in its simplest form.

**(3)**

The point $P(4, -2)$ lies on the curve.

(b) Use the answer to part (a) to find the equation of the normal to the curve at $P$, writing your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers to be found.

**(4)**

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**1.** Given that $y = 4x^3 - \frac{5}{x^2}$, $x \neq 0$, find in their simplest form

(a) $\frac{dy}{dx}$,

**(3)**

**9.**

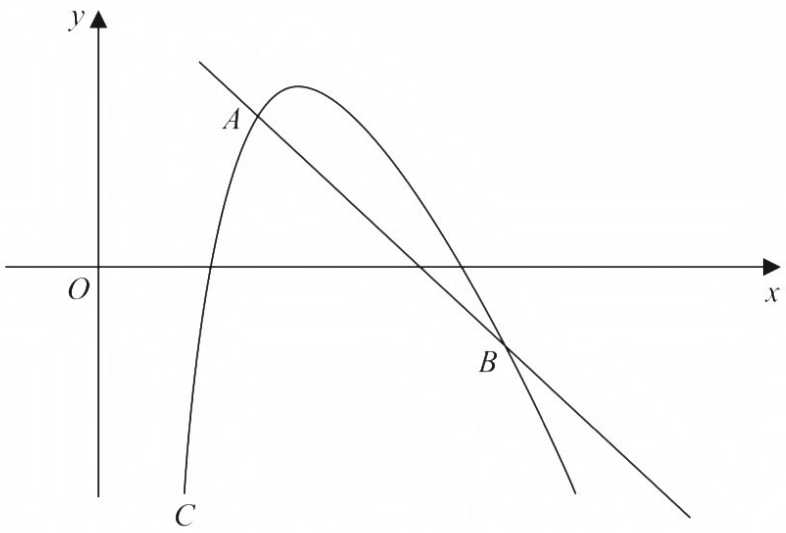



**Figure 3**

A sketch of part of the curve $C$ with equation

$$y = 20 - 4x - \frac{18}{x}, \qquad x > 0$$

is shown in Figure 3.

Point $A$ lies on $C$ and has $x$ coordinate equal to 2

(a) Show that the equation of the normal to $C$ at $A$ is $y = -2x + 7$.

**(6)**

The normal to $C$ at $A$ meets $C$ again at the point $B$, as shown in Figure 3.

(b) Use algebra to find the coordinates of $B$.

**(5)**

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**6.** (*Solutions based entirely on graphical or numerical methods are not acceptable.*)

Given

$$f(x) = 2x^{\frac{5}{2}} - 40x + 8 \qquad x > 0$$

(a) solve the equation $f'(x) = 0$

**(4)**

(b) solve the equation $f''(x) = 5$

**(3)**

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**1.** The curve $C$ has equation $y = \frac{1}{8}x^3 - \frac{24}{\sqrt{x}} + 1$

(a) Find $\frac{dy}{dx}$, giving the answer in its simplest form.

**(3)**

The point $P(4, -3)$ lies on $C$.

(b) Find the equation of the tangent to $C$ at the point $P$. Write your answer in the form $y = mx + c$, where $m$ and $c$ are constants to be found.

**(3)**

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**10.** A curve has equation $y = f(x)$, where

$$f(x) = (x - 4)(2x + 1)^2$$

The curve touches the $x$-axis at the point $P$ and crosses the $x$-axis at the point $Q$.

(a) State the coordinates of the point $P$.

**(1)**

(b) Find $f'(x)$.

**(4)**

(c) Hence show that the equation of the tangent to the curve at the point where $x = \frac{5}{2}$ can be expressed in the form $y = k$, where $k$ is a constant to be found.

**(3)**

The curve with equation $y = f(x + a)$, where $a$ is a constant, passes through the origin $O$.

(d) State the possible values of $a$.

**(2)**

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**5.** A curve has equation

$$y = \frac{x^3}{6} + 4\sqrt{x} - 15 \qquad x \geqslant 0$$

(a) Find $\frac{dy}{dx}$, giving the answer in simplest form.

**(3)**

The point $P\left(4, \frac{11}{3}\right)$ lies on the curve.

(b) Find the equation of the normal to the curve at $P$. Write your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers to be found.

**(4)**

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**9.** **In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

A curve has equation

$$y = \frac{4x^2 + 9}{2\sqrt{x}} \qquad x > 0$$

Find the $x$ coordinate of the point on the curve at which $\frac{dy}{dx} = 0$

**(6)**

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**8.** The curve $C$ has equation

$$y = (x - 2)(x - 4)^2$$

(a) Show that

$$\frac{dy}{dx} = 3x^2 - 20x + 32$$

**(4)**

The line $l_1$ is the tangent to $C$ at the point where $x = 6$

(b) Find the equation of $l_1$, giving your answer in the form $y = mx + c$, where $m$ and $c$ are constants to be found.

**(4)**

The line $l_2$ is the tangent to $C$ at the point where $x = \alpha$

Given that $l_1$ and $l_2$ are parallel and distinct,

(c) find the value of $\alpha$

**(3)**

**1.** A curve has equation

$$y = 2x^3 - 5x^2 - \frac{3}{2x} + 7 \qquad x > 0$$

(a) Find, in simplest form, $\frac{dy}{dx}$

**(3)**

The point $P$ lies on the curve and has $x$ coordinate $\frac{1}{2}$

(b) Find an equation of the normal to the curve at $P$, writing your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers to be found.

**(5)**

**7.** **In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

$$f(x) = 2x - 3\sqrt{x} - 5 \qquad x > 0$$

(b) Solve the equation

$$f''(x) = 6$$

**(5)**

**1.** The curve $C$ has equation

$$y = \frac{x^2}{3} + \frac{4}{\sqrt{x}} + \frac{8}{3x} - 5 \qquad x > 0$$

(a) Find $\frac{dy}{dx}$, giving your answer in simplest form.

**(4)**

The point $P(4, 3)$ lies on $C$.

(b) Find the equation of the normal to $C$ at the point $P$. Write your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers to be found.

**(4)**

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**6.** **In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

A curve $C$ has equation $y = f(x)$ where

$$f(x) = 2(x + 1)(x - 3)^2$$

(b) Write $f(x)$ in the form $ax^3 + bx^2 + cx + d$, where $a$, $b$, $c$ and $d$ are constants to be found.

**(3)**

(c) Hence, find the equation of the tangent to $C$ at the point where $x = \frac{1}{3}$

**(4)**

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**7.**

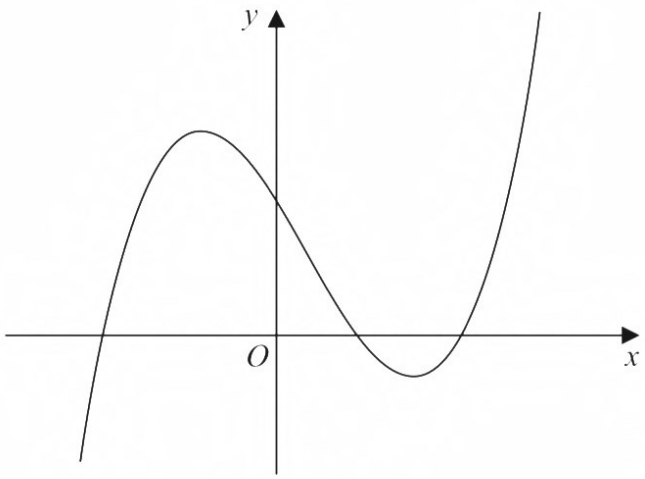



**Figure 3**

Figure 3 shows a sketch of part of the curve with equation $y = f(x)$, where

$$f(x) = (x + 4)(x - 2)(2x - 9)$$

(c) Find $f'(x)$.

**(4)**

(d) Hence find the range of values of $x$ for which the gradient of the curve with equation $y = f(x)$ is less than $-18$

**(3)**

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**10.**

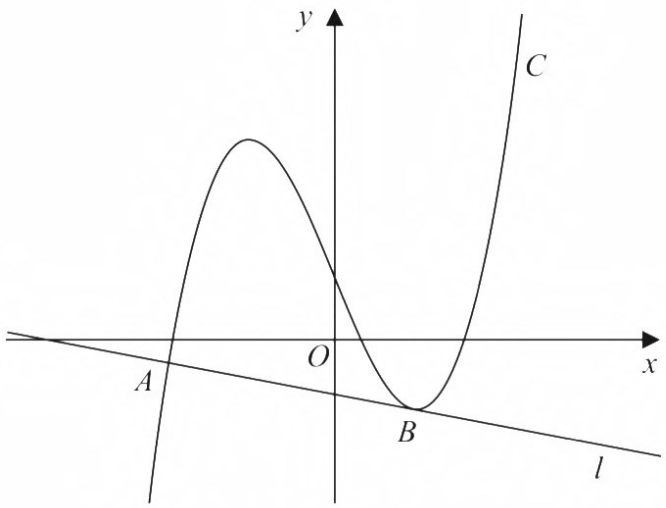



**Figure 5**

Figure 5 shows a sketch of the curve $C$ with equation

$$y = \frac{2}{7}x^3 + \frac{1}{7}x^2 - \frac{5}{2}x + k$$

where $k$ is a constant.

(a) Find $\frac{dy}{dx}$

**(2)**

The line $l$, shown in Figure 5, is the normal to $C$ at the point $A$ with $x$ coordinate $-\frac{7}{2}$

Given that $l$ is also a tangent to $C$ at the point $B$,

(b) show that the $x$ coordinate of the point $B$ is a solution of the equation

$$12x^2 + 4x - 33 = 0$$

**(4)**

(c) Hence find the $x$ coordinate of $B$, justifying your answer.

**(2)**

Given that the $y$ intercept of $l$ is $-1$

(d) find the value of $k$.

**(4)**

**1.** The curve $C$ has equation

$$y = \frac{x^3}{4} - x^2 + \frac{17}{x} \qquad x > 0$$

(a) Find $\frac{dy}{dx}$, giving your answer in simplest form.

**(3)**

The point $R\left(2, \frac{13}{2}\right)$ lies on $C$.

(b) Find the equation of the tangent to $C$ at the point $R$. Write your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers to be found.

**(3)**

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**1.** A curve $C$ has equation

$$y = 2 + 10x^{\frac{1}{2}} - 2x^{\frac{3}{2}} \qquad x > 0$$

(a) Find $\dfrac{dy}{dx}$ giving your answer in simplest form.

**(3)**

(b) Hence find the exact value of the gradient of the tangent to $C$ at the point where $x = 2$ giving your answer in simplest form.

(*Solutions relying on calculator technology are not acceptable.*)

**(2)**

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**4.** **In this question you must show all stages of your working.**

(a) Write

$$y = \frac{5x^2 + \sqrt{x^3}}{\sqrt[3]{8x}}$$

in the form

$$y = Ax^p + Bx^q$$

where $A$, $B$, $p$ and $q$ are constants to be found.

**(4)**

(b) Hence find $\frac{dy}{dx}$ giving each coefficient in simplest form.

**(3)**

**8.** **In this question you must show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(a) Find the equation of the tangent to the curve with equation

$$y = \frac{1}{4}x^3 - 8x^{-\frac{1}{2}}$$

at the point $P(4, 12)$

Give your answer in the form $ax + by + c = 0$ where $a$, $b$ and $c$ are integers.

**(5)**

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**1.** Given that

$$y = 5x^3 + \frac{3}{x^2} - 7x \qquad x > 0$$

find, in simplest form,

(a) $\frac{dy}{dx}$

**(3)**

(b) $\frac{d^2y}{dx^2}$

**(2)**

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**3.** The curve $C$ has equation

$$y = \frac{5x^3 - 8}{2x^2} \qquad x > 0$$

(a) Find $\frac{dy}{dx}$ writing your answer in simplest form.

**(4)**

The point $P(2, 4)$ lies on $C$.

(b) Find an equation for the tangent to $C$ at $P$ writing your answer in the form $ax + by + c = 0$, where $a$, $b$ and $c$ are integers.

**(3)**

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**7.** The curve $C$ has equation $y = f(x)$ where

$$f(x) = 2x^3 - kx^2 + 14x + 24$$

and $k$ is a constant.

(a) Find, in simplest form,

(i) $f'(x)$

(ii) $f''(x)$

**(3)**

The curve with equation $y = f'(x)$ intersects the curve with equation $y = f''(x)$ at the points $A$ and $B$.

Given that the $x$ coordinate of $A$ is 5

(b) find the value of $k$.

**(2)**

(c) Hence find the coordinates of $B$.

**(3)**

**3.** **In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

$$y = x^3 + 96\sqrt{x} + 5 \qquad x > 0$$

(a) Find $\frac{dy}{dx}$, giving each term in simplest form.

**(3)**

(b) Find the solution of the equation

$$\frac{d^2y}{dx^2} = 0$$

writing the answer in the form $2^k$ where $k$ is a constant.

**(3)**

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**10.** **In this question you must show all stages of your working.**

**Solutions relying on calculator technology are not acceptable.**

The curve $C$ has equation

$$y = \frac{2}{3}x^3 - 25x - \frac{56}{x} + \frac{194}{3} \qquad x > 0$$

The point $P$, which lies on $C$, has coordinates $(2, -8)$

(a) Show that an equation of the tangent to $C$ at $P$ is

$$y = -3x - 2$$

**(5)**

The point $Q$ also lies on $C$.

Given that the tangent to $C$ at $Q$ is parallel to the tangent to $C$ at $P$,

(b) find, using algebra and showing your working, the exact $x$ coordinate of $Q$.

**(5)**

**3.** **In this question you must show all stages of your working.**

$$f(x) = \frac{(x+5)^2}{\sqrt{x}} \qquad x > 0$$

(b) (i) Show that when $f'(x) = 0$

$$3x^2 + 10x - 25 = 0$$

(ii) Hence state the value of $x$ for which

$$f'(x) = 0$$

**(5)**

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**9.**

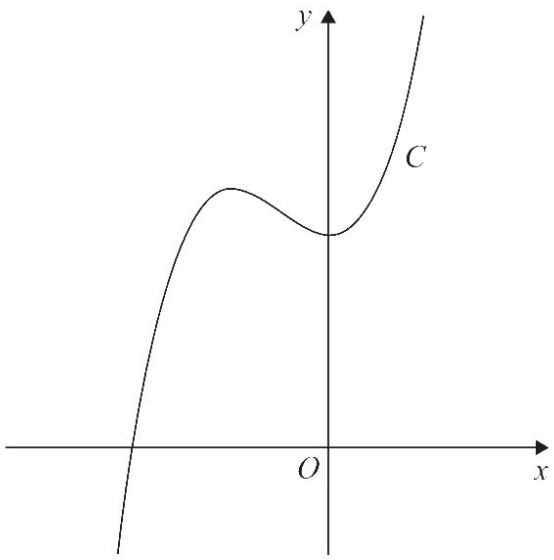



**Figure 4**

Figure 4 shows a sketch of the curve $C$ with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (x + 5)(3x^2 - 4x + 20)$$

(a) Deduce the range of values of $x$ for which $\mathrm{f}(x) \geqslant 0$

**(1)**

(b) Find $\mathrm{f}'(x)$ giving your answer in simplest form.

**(3)**

The point $R(-4, 84)$ lies on $C$.

Given that the tangent to $C$ at the point $P$ is parallel to the tangent to $C$ at the point $R$

(c) find the $x$ coordinate of $P$.

**(4)**

(d) Find the point to which $R$ is transformed when the curve with equation $y = \mathrm{f}(x)$ is transformed to the curve with equation,

(i) $y = \mathrm{f}(x - 3)$

(ii) $y = 4\mathrm{f}(x)$

**(2)**

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**5.** **In this question you must show all stages of your working.**
**Solutions relying on calculator technology are not acceptable.**

The curve $C$ has equation

$$y = 4x^3 + \frac{2}{x} + 9 \qquad x > 0$$

(a) Find $\frac{dy}{dx}$, giving your answer in simplest form.

**(2)**

Given that

- the point $P$ lies on $C$
- the line with equation $y = k \quad 5x$, where $k$ is a constant, is the tangent to $C$ at $P$

(b) show that the $x$ coordinate of $P$ satisfies the equation

$$12x^4 + 5x^2 - 2 = 0$$

**(2)**

(c) Hence find the value of $k$.

**(4)**