CAMBRIDGE
INTERNATIONAL EXAMINATIONS

JUNE 2003

INTERNATIONAL GCSE

MARK SCHEME

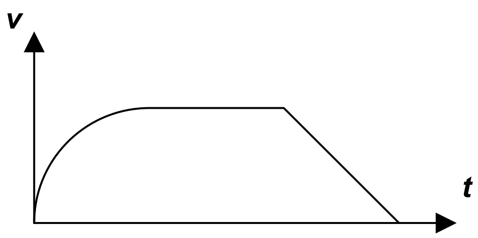
MAXIMUM MARK: 80

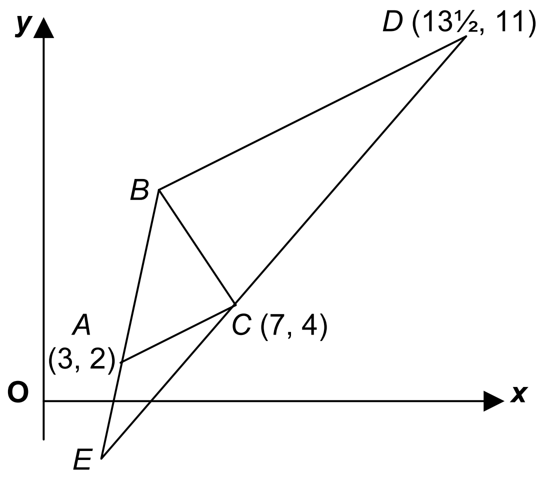
SYLLABUS/COMPONENT: 0606/02

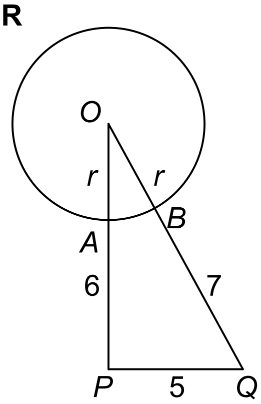
ADDITIONAL MATHEMATICS
Paper 2

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1	Put $x = -b/2$ (or synthetic or long division to remainder) $\Rightarrow 3b^3 + 7b^2 - 4 = 0$ AG	M1 A1
	Search $\Rightarrow b = -1$ [or $b = -2$] (1st root or factor)	M1 A1
	Attempt to divide $\Rightarrow 3b^2 + 4b - 4$ (or $3b^2 + b - 2$) or further search $\Rightarrow b = -2$ [or $b = -1$]	M1
[7]	Factorise (or formula) [3 term quadratic] or method for 3rd value $\Rightarrow b = -2, -1$ or $^2/_3$	DM1 A1
2 (i)	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \pm(9\mathbf{i} + 12\mathbf{j})$	M1
	Unit vector = $\overrightarrow{AB} \div \sqrt{9^2 + 12^2} = \pm(0.6\mathbf{i} + 0.8\mathbf{j})$ [Accept any equivalent unsimplified version of column vectors, $\pm\begin{pmatrix}9\\12\end{pmatrix}$, $\pm\begin{pmatrix}0.6\\0.8\end{pmatrix}$]	M1 A1
(ii)	$\overrightarrow{AC} = {}^2/_3\overrightarrow{AB} = 6\mathbf{i} + 8\mathbf{j}$ (or $\overrightarrow{CB} = {}^1/_3\overrightarrow{AB} = 3\mathbf{i} + 4\mathbf{j}$)	M1
[6]	$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$ (or $\overrightarrow{OB} - \overrightarrow{CB}$) $= 12\mathbf{i} + 5\mathbf{j}$ (or equivalent)	M1 A1
3	$\int(3x^{0.5} + 2x^{-0.5})\,dx = 3x^{1.5}/1.5 + 2x^{0.5}/0.5$ (one power correct sufficient for M mark)	M1 A1 A1
	$\int_1^8 = (2 \times 8\sqrt{8} + 4\sqrt{8}) - (2 + 4)$ Must be an attempt at integration	M1
[6]	Putting $\sqrt{8} = 2\sqrt{2}$ (i.e. one term converted $\sqrt{}$ to $k\sqrt{2}$) $\Rightarrow -6 + 40\sqrt{2}$	B1√ A1
4	$16^{x+1} = 2^{4x+4}$ or 16×2^{4x} or 16×4^{2x} or 16×16^x $20(4^{2x}) = 20(2^{4x})$ or $5(2^{4x+2})$ or 20×16^x	B1 B1
	$2^{x-3}\,8^{x+2} = 2^{x-3}\,2^{3x+6} = 2^{4x+3}$ or 8×2^{4x} or 8×4^{2x} or 8×16^x	B1
[4]	Cancel 2^{4x+2} or 2^{4x} and simplify $\Rightarrow 4.5$ or equivalent	B1
5 (i)	$f(0) = ½$ $f^2(0) = f(½) = (\sqrt{e} + 1)/4 \approx 0.662$ (accept 0.66 or better)	B1 M1 A1
(ii)	$x = (e^y + 1)/4$ $\Rightarrow e^y = 4x - 1$ $\Rightarrow f^{-1} : x \mapsto \ln(4x - 1)$	M1 A1
(iii) [7]	Domain of f^{-1} is $x \geq ½$ Range of f^{-1} is $f^{-1} \geq 0$	B1 B1

6 (i)	$x^2 - 8x + 12 = 0$ Factorise or formula $\Rightarrow$ Critical values $x = 2, 6$ $x^2 - 8x + 12 > 0$ $\Rightarrow \{x : x < 2\} \cup \{x : x > 6\}$	M1 A1 A1
(ii)	$x^2 - 8x = 0$ $\Rightarrow$ Must be an attempt to find *2* solutions $x^2 - 8x < 0$ $\Rightarrow \{x : 0 < x < 8\}$	M1 A1
(iii) [7]	Solution set of $\lvert x^2 - 8x + 6 \rvert < 6$ is combination of **(i)** and **(ii)** $\{x : 0 < x < 2\}$ $\{x : 6 < x < 8\}$	B1 B1 (one for each range)
7 (i)	6! = 720	B1
(ii)	M … $\Rightarrow$ 5! = 120	M1 A1
(iii)	4! 48	M1 A1
(iv)	6!/4! 2! = 15 Accept ${}_6C_4$ or ${}_6C_2$ = 15	B1
(v) [8]	5!/3! 2! = 10 (or, answer to (iv) less ways M can be omitted) (Listing – ignoring repeats $\geqslant$ 8 [M1] $\Rightarrow$ 10 [A1])	M1 A1
8 (i)	Collect sin *x* and cos x $\Rightarrow$ sin *x* = 5 cos *x* Divide by cos *x* $\Rightarrow$ tan *x* = 5 (accept $^1/_5$ – for M only) x = 78.7° or (258.7°) i.e. 1st solution + 180°	M1 M1 A1 A1√
(ii) [8]	Replace $\cos^2 y$ by $1 - \sin^2 y$ $3\sin^2 y + 4\sin y - 4 = 0$ Factorise (or formula) (3 term quadratic) $\Rightarrow$ sin *y* = $^2/_3$ (or -2) y = 0.730 (accept 0.73 or better) or (2.41) i.e. π (or $\frac{22}{7}$) less 1st solution	B1 M1 A1 A1√
9 (i)	$\int (12t - t^2)\,dt = 6t^2 - {}^1/_3 t^3$ From *t* = 0 to *t* = 6 distance = $\int_0^6$ = 144 Max. speed = 36 $\Rightarrow$ from *t* = 6 to *t* = 12 distance = 36 x 6 (= 216) During deceleration distance = $(0^2 - 36^2) \div 2(-4) = 162$ Area of Δ is fine for M mark but value of *t* must be from *constant* acceleration *not* $12 - 2t = \pm 4$ Total distance = 144 + 216 + 162 = 522	M1 A1 A1 B1 M1 A1
(ii) [8]		B2, 1, 0

10 (i)	$\frac{dy}{dx} = \frac{(x-2)2-(2x+4)1}{(x-2)^2} = \frac{-8}{(x-2)^2} \Rightarrow k = -8$	M1 A1
	Must be correct formula for M mark (accept $\frac{-8}{(x-2)^2}$ as answer)	
(ii)	When $y = 0$, $x = -2$ (B mark is for *one* solution only) NB. $x = 0$, $y = -2$	B1
	$m_{tangent} = -8/16 = -1/2 \Rightarrow m_{normal} = +2$ (M is for use of $m_1 m_2 = -1$, whether numeric or algebraic)	M1
	Equation of normal is $y - 0 = 2(x + 2)$ (candidate's m_{normal} and $[x]_{y=0}$ for M mark)	M1 A1
(iii)	When $y = 6$, $x = 4$	B1
	$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} = \frac{-8}{(x-2)^2} \times 0.05 = \frac{-8}{4} \times 0.05 = -0.1$ (accept $\pm$)	M1 A1√
	i.e. $\left[\frac{dy}{dx}\right]_{x=4}$ x 0.05 for M mark.	
[9]	√ is for error in k only. (Condone S $\approx \frac{dy}{dx}$ x S)	
11	**EITHER** 	
	(i) $m_{AC} = (4 - 2)/(7 - 3) = ½$	B1
	$m_{BD} = ½$	B1√
	$m_{BC} = -2$	B1√
	Equation of *BD* is $y - 11 = ½(x - 13.5)$ i.e. $4y = 2x + 17$	M1
	Equation of *BC* is $y - 4 = -2(x - 7)$ i.e. $y = -2x + 18$	M1
	Solving $y = 7$, $x = 5.5$	M1 A1

[10]	**(ii)** $\dfrac{\Delta EBD}{\Delta EAC}$ = (ratio of corresponding sides or *x*- or *y*- steps)^2 = 4/1 Quadrilateral *ABDC*/ Δ *EBD* = 3/4 [Or, find *E*(1/2, -3) and then use array method to find *one* of: area quadrilateral *ABDC* = 22.5 area Δ *EBD* = 30 Find other area and hence ratio = 3/4 or equivalent]	M1 A1 A1 M1 A1 A1
11	**OR**  **(i)** $(r + 6)^2 + 5^2 = (r + 7)^2$ Solve $\Rightarrow$ r = 6 tan *AOB* = 5/12 *AOB* = 0.395 or 22.6° Length of arc *AB* = 6 x 0.395 = 2.37 or better **(ii)** Sector *AOB* = ½ x 6^2 x 0.395 = 7.11 Shaded area = ½ x 5 x 12 - 7.11 All figures in sector and triangle correct √	 M1 M1 A1 M1 M1 A1 M1 M1 A1√
[10]	22.9 or better	A1