UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
International General Certificate of Secondary Education

ADDITIONAL MATHEMATICS 0606/02

Paper 2

May/June 2004

2 hours

Additional Materials: Answer Booklet/Paper
Electronic calculator
Graph paper
Mathematical tables

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Write your answers on the separate Answer Booklet/Paper provided.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **5** printed pages and **3** blank pages.

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\, r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$
$$\sec^2 A = 1 + \tan^2 A.$$
$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}.$$
$$a^2 = b^2 + c^2 - 2bc \cos A.$$
$$\Delta = \tfrac{1}{2} bc \sin A.$$

1 The position vectors of the points A and B, relative to an origin O, are $\mathbf{i} - 7\mathbf{j}$ and $4\mathbf{i} + k\mathbf{j}$ respectively, where k is a scalar. The unit vector in the direction of $\overrightarrow{AB}$ is $0.6\mathbf{i} + 0.8\mathbf{j}$. Find the value of k. [4]

2 Given that x is measured in radians and $x > 10$, find the smallest value of x such that

$$10\cos\left(\frac{x+1}{2}\right) = 3. \quad [4]$$

3 Given that $\mathscr{E} = \{\text{students in a college}\}$,
$A = \{\text{students who are over 180 cm tall}\}$,
$B = \{\text{students who are vegetarian}\}$,
$C = \{\text{students who are cyclists}\}$,
express in words each of the following

(i) $A \cap B \neq \varnothing$, **(ii)** $A \subset C'$. [2]

Express in set notation the statement

(iii) all students who are both vegetarians and cyclists are not over 180 cm tall. [2]

4 Prove the identity $(1 + \sec\theta)(\operatorname{cosec}\theta - \cot\theta) \equiv \tan\theta$. [4]

5 The roots of the quadratic equation $x^2 - \sqrt{20}x + 2 = 0$ are c and d. Without using a calculator, show that $\frac{1}{c} + \frac{1}{d} = \sqrt{5}$. [5]

6 **(a)** Find the values of x for which $2x^2 > 3x + 14$. [3]

(b) Find the values of k for which the line $y + kx = 8$ is a tangent to the curve $x^2 + 4y = 20$. [3]

7 Functions f and g are defined for $x \in \mathbb{R}$ by

$$\mathrm{f} : x \mapsto \mathrm{e}^x,$$

$$\mathrm{g} : x \mapsto 2x - 3.$$

(i) Solve the equation $\mathrm{fg}(x) = 7$. [2]

Function h is defined as gf.

(ii) Express h in terms of x and state its range. [2]

(iii) Express h^{-1} in terms of x. [2]

8 Solve

(i) $\log_3(2x + 1) = 2 + \log_3(3x - 11)$, [4]

(ii) $\log_4 y + \log_2 y = 9$. [4]

9 Express $6 + 4x - x^2$ in the form $a - (x + b)^2$, where a and b are integers. [2]

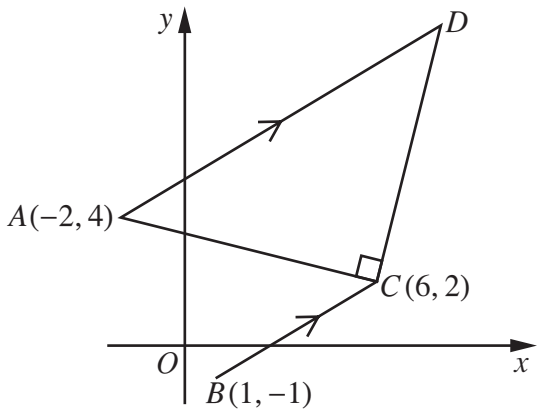
(i) Find the coordinates of the turning point of the curve $y = 6 + 4x - x^2$ and determine the nature of this turning point. [3]

The function f is defined by $\text{f} : x \mapsto 6 + 4x - x^2$ for the domain $0 \leqslant x \leqslant 5$.

(ii) Find the range of f. [2]

(iii) State, giving a reason, whether or not f has an inverse. [1]

10 Solutions to this question by accurate drawing will not be accepted.



In the diagram the points A, B and C have coordinates $(-2, 4)$, $(1, -1)$ and $(6, 2)$ respectively. The line AD is parallel to BC and angle $ACD = 90°$.

(i) Find the equations of AD and CD. [6]

(ii) Find the coordinates of D. [2]

(iii) Show that triangle ACD is isosceles. [2]

11 It is given that $y = (x + 1)(2x - 3)^{3/2}$.

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ can be written in the form $kx\sqrt{2x - 3}$ and state the value of k. [4]

Hence

(ii) find, in terms of p, an approximate value of y when $x = 6 + p$, where p is small, [3]

(iii) evaluate $\displaystyle\int_2^6 x\sqrt{2x - 3}\,\mathrm{d}x$. [3]

12 Answer only **one** of the following two alternatives.

EITHER

A particle moves in a straight line so that, t s after leaving a fixed point O, its velocity, v ms^{-1}, is given by $v = 10(1 - e^{-\frac{1}{2}t})$.

(i) Find the acceleration of the particle when $v = 8$. [4]

(ii) Calculate, to the nearest metre, the displacement of the particle from O when $t = 6$. [4]

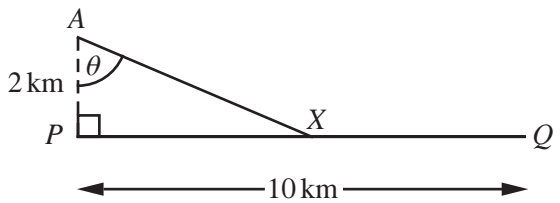
(iii) State the value which v approaches as t becomes very large. [1]

(iv) Sketch the velocity-time graph for the motion of the particle. [2]

OR

(i) By considering sec θ as $(\cos \theta)^{-1}$ show that $\frac{d}{d\theta}(\sec \theta) = \frac{\sin \theta}{\cos^2 \theta}$. [2]

(ii) The diagram shows a straight road joining two points, P and Q, 10 km apart. A man is at point A, where AP is perpendicular to PQ and AP is 2 km. The man wishes to reach Q as quickly as possible and travels across country in a straight line to meet the road at point X, where angle $PAX = \theta$ radians.



The man travels across country along AX at 3 km h^{-1} but on reaching the road he travels at 5 km h^{-1} along XQ. Given that he takes T hours to travel from A to Q, show that

$$T = \frac{2 \sec \theta}{3} + 2 - \frac{2 \tan \theta}{5}.$$ [4]

(iii) Given that θ can vary, show that T has a stationary value when $PX = 1.5$ km. [5]

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