CAMBRIDGE
INTERNATIONAL EXAMINATIONS

November 2003

INTERNATIONAL GCSE

MARK SCHEME

MAXIMUM MARK: 80

SYLLABUS/COMPONENT: 0606/01

ADDITIONAL MATHEMATICS
Paper 1

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1. $x + 3y = k$ and $y^2 = 2x + 3$ Elimination of x or y $\rightarrow y^2 + 6y - (2k+3) = 0$ or $\rightarrow x^2 - (2k + 18)x + (k^2 - 27) = 0$ Uses $b^2 - 4ac$ $\rightarrow k < -6$	M1 A1 M1 A1 [4]	x or y must go completely, but allow for simple arithmetic or numeric slips co Any use of b^2-4ac, even if =0 or >0 co
2. $8^{-x} = 2^{-3x}$ $4^{½x} = 2^x$ Attempts to link powers of 2 $\rightarrow x - 3 - (-3x) = 5 - (x)$ $\rightarrow x = 1.6$ or 8/5 etc [$\log 8^{-x} = -3x\log 2$, $\log 4^{½x} = x\log 2$ equate coefficients of log 2]	B1 B1 M1 A1 [4] [B1B1 M1A1]	Wherever used Needs to use $x^a \div x^b = x^{a-b}$ co
3. $x^3 + ax^2 + bx - 3$ Puts x=3 $\rightarrow 27+9a+3b-3=0$ Puts x=−2 $\rightarrow -8+4a-2b-3=15$ $(9a+3b=-24$ and $4a-2b=26)$ Sim equations $\rightarrow$ a = 1 and b = −11	 M1A1 M1A1 A1 [5]	 Needs x=3 and =0 for M mark Needs x=−2 and =15 for M mark (A marks for unsimplified) co
4. $(\sqrt{3}-\sqrt{2})^2 = 5 - 2\sqrt{6}$ or $5-2\sqrt{2}\sqrt{3}$ Divides volume by length² $\frac{4\sqrt{2}-3\sqrt{3}}{5-2\sqrt{6}} \times \frac{5+2\sqrt{6}}{5+2\sqrt{6}}$ Denominator = 1 Numerator = $20\sqrt{2}-15\sqrt{3}+8\sqrt{12}-6\sqrt{18}$ But $\sqrt{12} = 2\sqrt{3}$ and $\sqrt{18} = 3\sqrt{2}$ $\rightarrow 2\sqrt{2} + \sqrt{3}$	B1 M1 M1 M1 A1 [5]	Co anywhere V÷l² used × by denominator with sign changed Correct simplification somewhere with either of these co
5 y=0 when $3x + \frac{1}{4}\pi = \pi$ $\rightarrow x = \frac{1}{4}\pi$ $\int 6\sin(3x+\pi/4)dx = -6\cos(3x+\pi/4) \div 3$ Between 0 and π/4 $\rightarrow 2 + \sqrt{2}$ or 3.41	 B1 M1 A2,1 DM1 A1 [6]	 Co. Allow 45° Knows to integrate. Needs "cos". All correct, including ÷3, ×6 and −ve Uses limits correctly – must use x=0 In any form – at least 3sf
6 Wind 50**i** − 70**j** V(still air) = 280**i** −40**j** (i) Resultant velocity = $\mathbf{v_{air} + w}$ $\rightarrow$ 330**i** − 110**j** $\tan^{-1}(110/330) = 18.4°$ $\rightarrow$ Bearing of Q from P = 108° (ii) Resultant speed = $\sqrt{(330^2+110^2)}$ Time = 273 ÷ resultant speed = 47 minutes Scale drawings are ok.	 M1 A1 DM1 A1 M1 A1√ [6]	 Connecting two vectors (allow −) Co (Could get these 2 marks in (ii)) For use of tangent (330/110 ok) co Use of Pythagoras with his components For 273 ÷ √(a²+b²)

7 $(0.6 \quad 0.2 \quad 0.5)\times\begin{pmatrix}8&6&6&5\\5&4&3&2\\3&3&2&2\end{pmatrix}\times\begin{pmatrix}40\\50\\50\\60\end{pmatrix}$ $=(7.3 \quad 5.9 \quad 5.2 \quad 4.4)\times\begin{pmatrix}40\\50\\50\\60\end{pmatrix}$ or $(0.6 \quad 0.2 \quad 0.5)\times\begin{pmatrix}1220\\670\\490\end{pmatrix}$ → \$1111	B2,1,0 M1 A1 M1 B1 [6]	Wherever 3 matrices come – as row or column matrices – as 3 by 4 or 4 by 3 – independent of whether they are compatible for multiplication or not. Correct method for multiplying any 2 of the 3 - co for A mark. Correct method for remaining two. Co – even if from arithmetic.
8 (i) d/dx(lnx) = 1/x $\frac{dy}{dx}=\frac{(2x+3)\times\frac{1}{x}-(\ln x)\times 2}{(2x+3)^2}$ (ii) δy = (dy/dx) × δx = 0.2p (iii) dy/dt = dy/dx × dx/dt → dx/dt = 0.6	B1 M1A1√ M1A1 M1 A1√ [7]	Anywhere, even if not used in "u/v" Uses correct formula. All ok. Could use product formula. A mark unsimplified. Allow if δy mixed with dy/dt. M mark given for algebraic dy/dx × p. Allow if dy/dt mixed with δy √ for 0.12 ÷ his dy/dx. Condone use of δx etc
9 (a) Uses $\sec^2x = 1+\tan^2x$ → quad in sec or ×c² then uses s²+c²=1 → quad in cos → 4sec²x+8secx−5=0 → −5cos²x+8cosx+4=0 →secx = −2.5 (or0.5) or cosx=−0.4 (or2) → x = 113.6° or 246.4° (b) tan(2y+1) = 16/5 = 3.2 Basic angle associated with 3.2 = 1.27 Next angle = π + 1.27 and 2π + 1.27 (Value − 1) ÷ 2 → 3.28 (others are 0.134 and 1.705)	B1 M1 A1A1√ B1 M1 M1A1 [8]	Co. Sets to 0 and uses correct method for solution of a 3 term quadratic in sec or cos. A1 co. A1√ for 360°−"first ans" only. Anywhere (allow 72.6°) Realising the need to add on π and/or 2π Correct order used ie −1, then ÷2 for any correct value. Allow if all 3 values are given, providing none are over 4. (degrees – max 2/4 B1, M0, M1, A0)

10 $f(x) = 5-3e^{\frac{1}{2}x}$		
(i) Range is <5	B1	Allow ≤ or <
(ii) $5-3e^{\frac{1}{2}x} = 0 \rightarrow e^{\frac{1}{2}x} = 5/3$ Logs or calculator $\rightarrow x = 1.02$	M1A1	Normally 2,0 but if working shown, can get M1 if appropriate
(iii)	B1	Shape in 1st quadrant.
(1.02, 0) and (0, 2)	B1√	Both shown or implied by statement.
(iv) $e^{\frac{1}{2}x} = (5-y)\div 3$	M1	Reasonable attempt $e^{\frac{1}{2}x}$ as the subject.
$x/2 = \ln[(5-y)/3]$	M1	Using logs.
$f^{-1}(x) = 2\ln[(5-x)/3]$	A1	All ok, including x, y interchanged.
	[8]	
11		
(i) $y=\frac{1}{2}x$ and $y=3x-15$ $\rightarrow$ C(6,3)	M1 A1	Soln of simultaneous eqns Co (or step method if B done first)
OB=OC+CB	M1	Vectors, step or soln of $y=\frac{1}{2}x+5$ and $y=3x-15$
$\rightarrow$ B(8,9)	A1√	From his C
m of OC = ½, m of AD = −2 eqn of AD is $y-6=-2(x-2)$ or $y=-2x+10$	M1 A1	use of $m_1m_2=-1$ (M0 if perp to $y=3x$) Co – unsimplified.
Soln of $y=\frac{1}{2}x$ and eqn of AD $\rightarrow$ D(4,2)	M1A1	Sol of simultaneous eqns. co.
(ii) Length OC = $\sqrt{45}$, OA = $\sqrt{40}$ Perimeter of OABC = $2(\sqrt{45}+\sqrt{40})$ $\rightarrow$ 26.1	M1 M1 A1	Once. Adding OA,AB,BC,CO Co.
	[11]	
12 **EITHER**		
(i) $125 = \pi r + 2x + 2(5r/4)$ $\rightarrow x = \frac{1}{2}(125 - \pi r - 5r/2)$	M1 A1	Attempt at 4/5 lengths. Co.
$h = 3r/4$	M1	Anywhere in the question – independent of any other working
Area of triangle = $\frac{1}{2} \times 2r \times 3r/4 = 3r^2/4$	M1	Use of ½bh with h as function of r
$A = \frac{1}{2}\pi r^2 + 2rx + \ldots..$ $= 125r - \frac{1}{2}\pi r^2 - 7r^2/4$	B1 A1	Correct $\frac{1}{2}\pi r^2 + 2rx$. Answer given – beware fortuitous ans.
(ii) $dA/dr = 125 - \pi r - 7r/2$	M1A1	Any attempt to differentiate. Co.
Solved = 0 to give	DM1	Setting his differential to 0.
$\rightarrow$ $r = 250 / (2\pi + 7)$ or 18.8	A1	Any correct form.
	[10]	

12 **OR** (i) $h / (12-r) = 30 / 12$ $\rightarrow h = 5(12-r) / 2$ Uses $V=\pi r^2 h$ to give $\rightarrow V = \pi(30r^2 - 5r^3/2)$ (ii) $dV/dr = \pi(60r - 15r^2/2)$ $= 0$ when $r = 8 \rightarrow h = 10$ $\rightarrow V = 640\pi$ or 2010 Volume of cone = $\frac{1}{3}\pi \times 12^2 \times 30$ $\rightarrow 1440\pi$ or 4520 Ratio of 4 : 9 or 1 : 2.25 (3 sf)	M1 A1 M1 A1 M1A1 DM1 A1 M1 A1 [10]	Use of similar triangles – needs ¾ lengths correct. Correct in any form – needs h as subject Needs correct formula Beware fortuitous answers (AG) Any attempt to differentiate. co Setting his dV/dr to 0 + attempt. Correct to 3 or more sig figures Anywhere Exactly 4:9 or 2.25 to 3 sig figures
DM1 for quadratic equation (1) Formula. Sets the equation to 0 Formula must be correct and correctly used. Condone simple slips in sign.		(2) Factors Sets the equation to 0 Attempts to obtain brackets Solves each bracket to 0.