

MATHEMATICS D (CALCULATOR VERSION)

Paper 4024/01

Paper 1

General comments

There were many excellent scripts. Some candidates suffered by having many gaps in their knowledge; it was clear that many had not met prime factors, inequalities, standard form, vectors, upper and lower bounds and so on. Presentation of the work was generally good. There was no evidence that candidates were short of time to answer all the questions they could do.

Question 1

- (a) This was usually done correctly but some were careless when removing brackets. Some did not do the operation in the correct order, first adding the 3 and 2 and producing the answer -5 .
- (b) This was usually well done.

Answer: (a) 1; (b) $\frac{8}{15}$.

Question 2

- (a) Provided they handled the units correctly they were usually successful.
- (b) The majority interpreted 2.45 hours as 2h. 45 m and so reached the answer 165.

Answer: (a) 1.77 kg; (b) 147 minutes.

Question 3

- (a) This was well done but some interpreted $25x^2$ as $(25x)^2$, given the coefficient of 125. Others gave the power of x as 8.
- (b) $-3/2$ and 2 was a common wrong answer, but many were successful.

Answer: (a) $5x^6$; (b) $1\frac{1}{2}$ or -2 .

Question 4

There two percentage calculations were well handled although $\frac{500}{8}$ was sometimes evaluated incorrectly.

Answer: (a) 80%; (b) $62\frac{1}{2}\%$.

Question 5

- (a) This problem was very well done; just a few had two out of place and a few more had them in reverse order.
- (b) This was usually done correctly.

Answer: (a) 0.7^2 , $\frac{7}{11}$, 0.7 , $\frac{7}{9}$; (b) 400.

Question 6

- (a) Weaker candidates could not cope with this, they possibly did not understand the word “difference”. The answers 34 and -34 were both accepted.
- (b) The common error here, following 34 in part (a), was to halve it to get 17. It was not widely realized that an average is half the sum; or, of course, the lower value plus half the difference. Another common wrong answer was 9.

Answer: (a) 34°C ; (b) -9°C .

Question 7

- (a) Many could get as far as $6\frac{1}{2}/9$, but that was not acceptable; doubling top and bottom was a neat way to the required answer.
- (b) Relatively few candidates were able to answer this correctly. Common incorrect answers were 50 (from candidates who got to $50\sqrt{2}$ and then gave $\sqrt{2}$ correct to one significant figure), 70.7 (from 50×1.4) and 7.
- (c) Many candidates knew that 3 to the power zero was 1, but many had difficulty evaluating 4 to the power $3/2$.

Answer: (a) $\frac{13}{18}$; (b) 70; (c) 8.

Question 8

Many were familiar with a ‘factor ladder’ and produced one or more correct string of primes; they had difficulty picking out the primes needed for the three answers.

- (a) They sometimes lost a factor carelessly e.g. $2^2 \times 3^2$.
- (b) A number of candidates gave the HCF, rather than the LCM and a larger number failed to give the answer as the product of prime factors.
- (c) As 360 had been given in the form $2^3 \times 3^2 \times 5$ it can be seen that extra factors 3×5^2 converted it to $2^3 \times 3^3 \times 5^3$. Many could not interpret the question in this way and did much unnecessary work.

Answer: (a) $2^2 \times 3^3$; (b) $2^2 \times 3^3 \times 5$; (c) 75.

Question 9

- (a) Adding 4 to all parts then dividing all parts by 3 produced $-1 \leq x < 2$, sadly many could not do this, or they made slips along the way.
- (b) Provided their end numbers were respectively a negative and a positive integer they could earn the mark by providing the appropriate list of integers for their inequality. Of those who had part (a) correct, many included 2 in their list of integers and others omitted the zero.

Answer: (a) $-1 (\leq x < 2)$; (b) $-1, 0, 1$.

Question 10

- (a) This was well done. It was unfortunate that some candidates did the hard work in getting to 15:6 but then failed to cancel the common factor 3.
- (b) Again many did this well, knowing that 1.5 and 6 could not be added. Some did the work correctly to 21×10^7 but then left it in that form.

Answer: (a) 5:2 ; (b) 2.1×10^8 .

Question 11

Most candidates could see that part (a) needed $1 - \frac{2}{5} - \frac{1}{3}$ and were successful. Far fewer could see that in part (b) \$1600 was $\frac{2}{5} - \frac{1}{3} = \frac{1}{15}$ of the total.

Answer: (a) $\frac{4}{15}$; (b) \$24000.

Question 12

Some candidates showed little or no knowledge of vectors; others were partially successful.

- (a) Some could travel from B to A using $-a - 2c + 4a$ and that earned the mark even when not simplified. A few got $2c - 3a$ or $3a + 2c$.
- (b) To score in this part they certainly needed to have part (a) right. They needed to see that $2\bar{a} - \frac{2}{3}\bar{c} + 4\bar{a}$ was $\frac{2}{3}(3\bar{a} - 2\bar{c})$ or that $3\bar{a} - 2\bar{c}$ was $\frac{3}{2}\left(2\bar{a} - \frac{4}{3}\bar{c}\right)$ or that $3\overline{OP} = 6\bar{a} - 4\bar{c}$ and $2\overline{BA} = 6\bar{a} - 4\bar{c}$. Many did manage one of these more or less convincingly. Writing things like "they have the same basic vector" or "they have the same gradient" did not gain a mark.
- (c) They needed to use the fact that triangle with equal heights have areas proportional to their bases. Many good candidates who had reached $\overline{BA} = \frac{3}{2}\overline{OP}$ in (b) got confused with the rule for similar triangles and gave 9:4 as their answer.

Answer: (a) $3a - 2c$; (b) Establishing $k\overline{OP} = \ell\overline{BA}$; (c) $\frac{3}{2}$.

Question 13

- (a) This was usually drawn correctly.
- (b) The used method was to find the angle sum of a heptagon, 900; subtract 126 and divide by 6. Unfortunately many could not find the 900, often assuming 360 or 540. A few used the half diagram formed by the line of symmetry but again the angle sum of a pentagon defeats many. Other methods were used occasionally and those using exterior angles were often successful if they remembered to subtract from 180, once they had achieved the exterior angle of 51.

Answer: (b) 129° .

Question 14

Just a few good candidates knew anything about upper and lower bounds of numbers given to a stated accuracy; they coped very well and scored full marks. The majority fared very badly with this question.

Answer: (a) 1550 ($\leq d <$) 1650, 5.5 ($\leq$ speed $<$) 6.5; (b) 300 s.

Question 15

- (a) Just a few could perform the algebraic moves and the final simplification to obtain the correct 2×2 matrix.
- (b) Most candidates had met the rules for finding an inverse and many scored 1 or 2 marks. Many knew, or nearly knew how to transpose the elements. Some knew the rule for finding the determinant and some knew what to do with it. The common errors concerning the determinant were to calculate it as 22 or +2 or to omit it altogether.

Answer: (a) $\begin{pmatrix} -4 & 2 \\ -6 & 0 \end{pmatrix}$; (b) $-\frac{1}{2}\begin{pmatrix} 2 & 3 \\ 4 & 5 \end{pmatrix}$

Question 16

Many candidates were unfamiliar with the notation $f(x)$ and $f(-2)$.

- (a) (i) Some were successful and reached $4p + 7$ or its equivalent but many went on to solve $4p + 7 = 0$ not fully understanding the phrase "in terms of p ".
- (ii) They were rarely successful with this, as they interpreted $f(0)$ as, very rarely as 3.
- (b) There was much unproductive algebra done in this part. Just a few managed $(a-1)^2 - 1$ and about half of those could remove the brackets and simplify.

Answer: (a)(i) $4p + 7$, (ii) -1 ; (b) $a^2 - 2a$ or equivalent.

Question 17

- (a) The majority had met $y = mx + c$ but very few could fit the ideas to the diagram, the gradient 2 seemed to be too difficult to calculate despite the scales being equal.
- (b) Most candidates drew correctly $x = 1$ and $y = 3$, though some reversed them. $x + y = 2$ was often correctly drawn. The candidates had great difficulty showing the required region often shading carelessly, or not showing enough shading to indicate that the region continued below the x axis.

Answer: (a) $y = 2x + 3$.

Question 18

Many candidates showed little understanding of Venn diagrams and set notation.

- (a) A small number of candidates interpreted the expression in set notation correctly, the rest did fill-in parts of the diagram but not the right area.
- (b) Some did write $P \cap Q'$, a few who obviously knew a bit about sets wrote $P \cup Q'$ or $P' \cap Q$.
- (c) $36 - 4 = 32$ candidates did history and geography, but the numbers say that 45 did, therefore 13 must do both. This argument appeared a few times. The more formal approach was sometimes used: $(25 - x) + x + (20 - x) + 4 = 36$. This relationship seen as an equation or seen on a Venn diagram often led to the correct answer. It was often misused by having the 4 omitted or by being solved incorrectly.

Answer: (b) $P \cap Q'$; (c) 13.

Question 19

Most candidates recognised the isosceles triangle and got $x = 20$. Next came the testing part : $(360 - 140)/2 = 110$, this was too difficult for many. Some candidates thought that OBCD was a cyclic quadrilateral and got $y = 40$, others that it was a parallelogram hence $y = 140$.

Going back to part **(a)** many candidates could see the alternate angles and got $z = 20$ correctly. Their value of t gained the mark if it was correct or if it was 180 minus (their y + their z).

Good candidates scored 4, many 3 or 2.

Answer: **(a)** 20° ; **(b)** 110° ; **(c)** 20° ; **(d)** 50° .

Question 20

- (a)** A sense of proportion would surely lead to the answer 3:5. Many wrote 5:3 or 5:2 or 3:2.
- (b)** Candidates were awarded this mark provided they squared the parts of their ratio in **(a)**.
- (c)** Many cubed their ratio in **(a)** but that only scored a method mark if 3 and 5 were involved. Most good candidates did find 27:125 and converted it correctly to 27:(125-27) Weak thinking produced the very common wrong answer 27:8.

Answer: **(a)** 3:5; **(b)** 9:25; **(c)** 27:98.

Question 21

- (a) (i)** Many candidates know they needed the gradient of the line at $t = 16$ and so they wrote down $7\frac{1}{2} \div 4$. They then had great difficulty converting that to $15/8$ or equivalents. Some made the more serious error $7\frac{1}{2} \div 16$.
- (a) (ii)** Many knew they needed the area under the curve and they successfully calculated 95, although a few gave the area under the triangle (60).
- (b)** Correct distance/time graphs were quite rare, it was not appreciated that in an acceleration stage the line would be curved.

Answer: **(a)(i)** $\frac{15}{8} \text{ m/s}^2$, **(ii)** 95 m.

Question 22

- (a) (i)** Very few candidates realised that if $AC = 5$ was taken as the base then the height was 6 and the area therefore 15. Instead much hard work was done with the area split up in various ways; some counted squares, not an easy method on this figure.
 - (ii)** $(-2, -1)$ was a common wrong answer, the cyclical order of ABCP not being appreciated. Poor drawing of CP and AP led to coordinates (10, 10) or (9, 9).
 - (iii)** Many struggled with this area as in part **(i)** those who realised that it was just double triangle ABC were given the mark whatever their previous answer.
 - (iv)** Very few candidates dropped a perpendicular from B to AC produced and very few could apply the basic definition of tan to the diagram.
- (b)** Again it would have been most helpful to the candidate if the construction suggested in **(a) (iv)** had been drawn. Some did see that $5/k$ was needed; very few knew that it was negative.

Answer: **(a)(i)** 15, **(ii)** (10, 9), **(iii)** 30, **(iv)** $\frac{3}{5}$; **(b)** $-\frac{5}{k}$ or $\frac{k^2 - 111}{10k}$.

Question 23

- (a) Nearly all candidates correctly drew the focus round point L.
- (b) Many candidates drew lines parallel to the coastline but some were unsure about the focus as it went round the corner off point B. Some made no attempt at an arc; some had the correct arc but also intersecting parallels and unless they convinced the Examiner that the arc was the focus they lost a mark.
- (c) Some did manage to score for the extreme bearings. They were credited with a mark whatever their focus off point B, provided it was consistent with their stated bearing.

Question 24

- (a) Rays drawn from O through the triangle to form the enlarged image should have been relatively straightforward, but many could not do that. Some who did use rays managed to be a whole square out with one of the points, usually (10, 2) was (9, 2).
- (b) There were many good attempts to describe this transformation. They nearly all knew the title, rotation, but giving the centre and angle and sense of rotation proved more testing.
- (c) Those who saw that the matrix moved (x, y) to (−x, y) were totally successful. Those who multiplied the coordinate of triangle T by the matrix were successful if they were careful with the arithmetic and with the plotting. Many could not do this problem.

Answer: **(a)** Triangle (4, 4), (8, 4) and (10, 2); **(b)** Rotation, 90° CW, centre (0, 0) **(c)** Triangle (−2, 2), (−4, 2) and (−5, 1).

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Paper 2

General comments

The general standard of the scripts produced in response to this paper was similar to that seen in previous years. There were enough testing questions to enable the strongest candidates to demonstrate their ability but also enough less demanding questions to encourage the less able. The majority of scripts were well presented and explained, but once again a small number of candidates set out their work in two columns, either formally or, worse, by putting parts of solutions beside one another in any space on the page. Such candidates risk having some of their work overlooked and thus losing credit. Only a handful of candidates had their calculators set in the wrong trigonometric mode this year and only a small number gave answers with less than three significant figures.

Most candidates attempted both of the graph questions in **Section B**, with attempts at the other three questions in roughly equal numbers.

There was some excellent work on **Questions 5** and **6**. The worst response was that to the geometry question, number 4, where too many failed to demonstrate the congruence efficiently and produced arguments that started by assuming the result required in the last two parts.

Comments on specific questions

Section A

Question 1

Although most candidates spotted the common factor in the first part and did complete the factorisation, some produced incomplete solutions, or $5(x + 4)(x - 4)$. Many made the simplification more difficult by failing to extract the common factor in the denominator and there was some illegal cancellation. A large number “lost” the 10 completely, but many correct solutions were seen. The question on algebraic fractions was generally well done, though some sign errors were observed. Many errors were noted in the last part. Some started by trying to subtract 2π from both sides of the equation, and others tried to transfer the square root from one side of the equation to the other. A few misread the question as the 2π th root. Many did start sensibly by trying to square the given equation, but too many obtained $2\pi^2$ in place of $4\pi^2$. Other errors occurred when trying to find an expression for g starting from an expression for L/g . Too often the reciprocal of the correct expression appeared.

Answer: **(a)(i)** $5(x + 2)(x - 2)$ **(ii)** $\frac{x - 2}{2(x - 1)}$ **(b)** $\frac{y + 29}{(y - 3)(y + 5)}$ **(c)** $\frac{4\pi^2 L}{T^2}$

Question 2

This question was well answered in most cases. A few halved the differences between the coordinates in the first part. The main error in part **(b)** was finding DO in place of OD . Very few showed evidence of the use of vectors or a sketch, which might have helped in the last part where parallelogram was a common wrong answer.

Answer: **(a)(i)** (9, 6) **(ii)** $\frac{3}{4}$ **(iii)** 10 **(b)(i)** (–12, 2) **(ii)** Trapezium

Question 3

This question was quite well answered in most cases. Some marks were lost where candidates did not quote explicit statements for $\cos QPR$ or $\sin PSR$ when correct answers were not obtained. The cosine formula was usually used correctly, but some sign errors were seen, and a few quoted the complement of the correct answer. The majority used the sine formula to find angle PSR and then the angle sum of the triangle, but some long methods were seen. The main error in the area was to use 68° as the included angle in place of the answer to the previous part.

Answer: (a) 101.7° (b) 70.4° (c) 1850 m^2

Question 4

Most candidates obtained the correct answer to the first part. The better were able to complete the congruence, but did not always quote any reasons. Weaker candidates tended to quote equal elements which were not yet justified, including $DX = XY$. The last two parts caused more trouble, with many quoting as known facts statements that needed to be shown. It was anticipated that angle DXB would be added to the angles AXD and BXY of the congruent triangles to justify the 60° and then to consider the isosceles triangle DXY with an angle of 60° to justify that it is an equilateral triangle. Some candidates effectively reversed the last two parts by stating that triangles CDY , XAD and XPY are all congruent so the three sides of triangle DXY are equal and the angle follows. This is valid and was accepted, of course.

Answer: (a) 150°

Question 5

This question was very well answered, with many completely correct solutions seen. Part (a)(iii) was well done, but wrong combinations of 1.65 and 1.44, even $1.65 - 1.44$, were seen and a few quoted only 691 euros. A small number of candidates stopped at \$17 496 in part (b)(ii) and a further small number thought the answer to (b)(iii) ought to be $3 \times 8 = 24\%$. Perhaps a smaller proportion of candidates than usual used 88% of \$46 480 in the last part.

Answer: (a)(i) \$825 (ii) £625 (iii) 691.2 euros
(b)(i) \$16 200 (ii) \$18 895.68 (iii) 26.0% (c) \$41 500

Question 6

The majority of candidates used the formula for the quadratic equation correctly, though a few completed the square. Some lost some credit by failing to quote the answers to the required degree of accuracy, with 0.661 by no means uncommon. Most candidates formed the simultaneous equations correctly, and many were able to complete the solution in spite of the awkward numbers. It was clear that the substitution method was more difficult than the elimination method in this case. Candidates should be encouraged to know both methods and use the more appropriate one.

Answer: (a) 0.66 and -2.38 (b)(ii) $3x + 5y = 654$ (iii) $x = 78$ $y = 84$

Section B

Question 7

This question was quite well answered on the whole, but perhaps less well than might have been expected. The area sometimes lost credit by using $4\pi r^2$ or $3\pi r^2$ (with a further πr^2 for the base) for the hemisphere or $\pi r^2 h$ or $\pi r h$ for the cylinder. The capacity of the tank caused problems. Too many used the volume of a complete sphere, or used $2\pi r^2 h$ for the cylinder. A large proportion of candidates failed to express the answer in litres as required. The time needed to empty the tank was then likely to be absurdly large. The units of time caused some difficulties, with 1.41 minutes being quoted as 1 minute 41 seconds. Attempts at the length of the bath often used a correct method, dividing the volume by the area of a trapezium, and many coped with the mixed units quite well. The most successful expressed everything in centimetres (or metres) first. Some of the quoted answers were obviously absurdly long or short as lengths of a bath.

Answer: (a) $21\,700 \text{ cm}^2$ (b)(i) 254 litres (ii) 1 minute 25 s (iii) 1.70 m

Question 8

The majority of candidates managed to answer some of the parts of this question well, but other parts proved to be a real test more for all but the most able. The continuation of the sequence was usually correct and most were able to use the given formula when n is 7 and 100. Surprisingly many did not realise that the sum of the multiples of 5 could be found by multiplying the previous answer by 5. Only the best candidates saw that the answer to (a)(v) was found by using T_{500} – the answer to (iv). These two parts were often omitted, but most started on part (b) with some success, though some did not notice that the required sum is S_{20} . Some candidates noticed that the values for S_3 , S_4 and T_4 had been given, thereby gaining a fairly easy mark, but missing the hint that had been intended for the last part. Too often a particular further example was given in that part, which was not accepted, but some used acceptable algebraic methods. It was hoped that they would follow the pattern established in the question to reach $(n + 1) + n + (n - 1) + \dots + 1$.

Answer: (a)(i) 21, 28 (iii) 5050 (iv) 25 250 (v) 100 000
(b)(i) 56, 84 (iii) 1540

Question 9

The majority of candidates used Pythagoras to obtain the first result, but a few used trigonometry. The angle of depression was badly done, with 65° a very common wrong answer. Several found angles in triangle ABF , which were not needed. In the remainder of the question the use of 28.6 or 28.56... was accepted. Most of those who used $100 \tan 25 - (28.56 \text{ or } 28.6)$ for BF reached a correct answer, though some lost some credit by only showing two significant figures. Other methods were used, but they were more demanding. Candidates found it easier to decide which lengths were needed to calculate angle BCN than angle DBA . Some did more work than was necessary, for example finding both CN and BC when calculating angle BCN . They might find it easier if they sketched relevant plane intersections first.

Answer: (b)(i) 25° (ii) 18.0 or 18.1 m (c)(i) 13.7° or 13.8° (ii) 50.8°

Question 10

There were many excellent graphs drawn, though a few who gave the answer 8.03 in (a) plotted at the point (6, 8.3). Most candidates saw that the answer to (c) lies where $y = 4$ on the graph, and obtained values in the given ranges, though a few drew another graph. The tangent and gradient were usually well done and a ruled straight line that was long enough to cut the curve twice was drawn. Some lost a mark because at least one of the readings was out of range however. The better candidates had little difficulty in forming the equation, but some tried putting their two values into the equation in a , b and c . They could not be successful.

Answer: (a) 8 or 8.03 (c) 1.35 to 1.45 and 3.55 to 3.70 (d) 1.20 to 1.40
(e)(ii) 1.45 to 1.55 and 4.55 to 4.65 (iii) $2x^3 - 5x^2 - 30x + 50 = 0$

Question 11

The majority of those who attempted this question did understand the concept of a histogram and scored well on this question. Follow through marks were awarded where the histogram was not correctly interpreted but the cumulative frequency curve did result from the answer given to the first part, provided a possible curve did result. Weaker candidates sometimes used interval widths in the first table leading to a straight line for the cumulative frequency curve. The median was usually correct but some candidates had difficulty in reading off the lower quartile at about 37.5. A value just below 150 was expected. The first probability was usually simplified and correct, though a few gave the probability of a height less than 160 cm. Although questions similar to the last part have often been set in the past, producing many correct answers, the response this year was most disappointing. Even good candidates were apt to forget the factor 2 or to use a denominator of 150×150 in place of 150×149 . Some made both errors.

Answer: (a)(i) 30 20 30 35 20 5 (ii) 40 60 90 125 145 150
(c)(i) 157 to 158 (ii) 149 to 149.6 (iii) 16.0 to 17.0 (d) $2/5$ (e) $10/449$