JUNE 2004

GCE ORDINARY LEVEL

MARK SCHEME

MAXIMUM MARK: 80

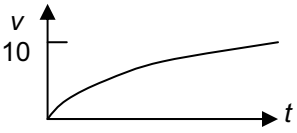
SYLLABUS/COMPONENT: 4037/02

ADDITIONAL MATHEMATICS
Paper 2

UNIVERSITY of CAMBRIDGE
International Examinations

1	[4]	$(\mathbf{i} - 7\mathbf{j}) + \lambda(0.6\mathbf{i} + 0.8\mathbf{j}) = 4\mathbf{i} + k\,\mathbf{j}$	M1 A1
		$1 + 0.6\lambda = 4 \quad \Rightarrow \quad \lambda = 5$	
		$-7 + 0.8\lambda \quad \Rightarrow \quad -7 + 0.8 \times 5 = -3 = k$	M1 A1
2	[4]	Attempt at $\cos^{-1} 0.3 \quad \Rightarrow$ [72.5° A0] $= 1.266$ [5.017, 7.549] accept 1.3	M1 A1
		$x + 1 = 2.532, 10\,034, 15.098 \quad \Rightarrow \quad x = 14.1$ or better	M1 A1
3	[4]	**(i)** Some vegetarians in the college are over 180 cm tall [or equivalent]	B1
		(ii) No cyclists in the college are over 180 cm tall [or equivalent]	B1
		(iii) $B \cap C \subset A'$ [or equivalent]	B1 B1
4	[4]	$\left(1 + \frac{1}{\cos\theta}\right)\left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right) \Rightarrow \frac{1 - \cos^2\theta}{\cos\theta \sin\theta}$	M1 M1
		$1 - \cos^2\theta \equiv \sin^2\theta \qquad \frac{\sin^2\theta}{\cos\theta \sin\theta} \rightarrow \tan\theta$	B1 A1
		Must be useful use of Pythagoras	
5	[5]	$x = \frac{\sqrt{20} \pm \sqrt{20 - (4 \times 2)}}{2} = \sqrt{5} \pm \sqrt{3}$ or $\frac{\sqrt{20} \pm \sqrt{12}}{2}$	M1 A1
		$\frac{1}{\sqrt{5} + \sqrt{3}} + \frac{1}{\sqrt{5} - \sqrt{3}}$ [or $\frac{2}{\sqrt{20} + \sqrt{12}} + \frac{2}{\sqrt{20} - \sqrt{12}}$]	M1
		rationalising each fraction or bringing to common denominator	
		Denominator = 2 [or 8] $\Rightarrow \quad \frac{1}{c} + \frac{1}{d} = \sqrt{5}$	A1 A1
6	[6]	**(a)** $2x^2 - 3x - 14 = 0 \Rightarrow (2x - 7)(x + 2) = 0 \Rightarrow x = -2, 3.5$	M1 A1
		$\{x : x < -2\} \cup \{x : x > 3.5\}$	A1
		(b) Eliminate $y \Rightarrow x^2 + 4(8 - kx) = 20$ [or $x \Rightarrow \left(\frac{8 - y}{k}\right)^2 + 4y = 20$]	M1
		$x^2 - 4kx + 12 = 0$ [or $y^2 + (4k^2 - 16)y + (64 - 20k^2) = 0$]	
		Apply " $b^2 = 4ac$ " $16k^2 = 48$ [or $16k^4 = 48k^2$] $\Rightarrow k = \pm\sqrt{3}$	M1 A1

7 [6]	**(i)** e^{2x-3} (= 7) $\Rightarrow$ x = ½ (3 + ln 7) ≈ 2.47 ~ 2.48 (not 2.5)	M1 A1
	(ii) h = $2e^x - 3$ (x, y or) h > – 3 accept ≥	B1 B1
	(iii) h^{-1} (or y) = ln {½ (x + 3)} or ln(x + 3) – ln2 or lg{½(x + 3)}/lge but ln{½(y + 3)} M1 A0 lg (or log) {½(x + 3)} M1 A0	M1 A1 (M1 for logs taken in valid way
8 [8]	**(i)** $\log_3(2x+1) - \log_3(3x-11) = \log_3 \dfrac{2x+1}{3x-11}$ [Or, later, give M1 for log + log=log(product)	M1
	$\log_3(\ ) = 2 \Rightarrow (\ ) = 3^2$	B1
	$2x + 1 = 9(3x - 11) \Rightarrow x = 4$	DM1 A1
	(ii) $\log_4 y = \dfrac{\log_2 y}{\log_2 4} = \dfrac{1}{2}\log_2 y$ [or $\log_2 y = \dfrac{\log_4 y}{\log_4 2} = 2\log_4 y$]	M1 A1
	½ $\log_2 y + \log_2 y = 9$ [or $\log_4 y + 2\log_4 y = 9$] $\Rightarrow$ $y = 2^6$ or $4^3 = 64$	DM1 A1
9 [8]	$6 + 4x - x^2 \equiv 10 - (x-2)^2$	M1 A1
	(i) x = 2 y = 10 Maximum	B1√B1√B1
	(ii) f(0) = 6, f(2) = 10, f(5) = 1 $\Rightarrow$ 1 ≤ f ≤10 [alternatively 1 ≤ B1, ≤ 10 B1]	M1 A1
	(iii) f has no inverse; it is not 1:1	B1
10 [10]	**(i)** m_{BC} = 3/5 Equation of AD is $y - 4 = 3/5(x + 2)$	B1 M1 A1
	m_{AC} = – ¼ Equation of CD is $y - 2 = 4(x - 6)$	B1 M1 A1
	(ii) Solve x = 8, y = 10	M1 A1
	(iii) Length of AC = Length of CD = $\sqrt{68}$	M1 A1
11 [10]	**(i)** d/dx $(2x-3)^{3/2} = (2x-3)^{1/2}$ × 3/2 × 2	M1 A1
	dy/dx = 1 × $(2x-3)^{3/2}$ + (x + 1) × { candidate's d/dx $(2x-3)^{3/2}$ }	M1
	= $\sqrt{2x-3}\{(2x-3)+3(x+1)\} = 5x\sqrt{2x-3} \Rightarrow k = 5$	A1
	(ii) $\delta y \approx dy/dx \times \delta x = (dy/dx)_{x=6} \times p = 90p$	M1 A1
	$(y)_{x=6+p} = (y)_{x=6} + \delta y = 189 + 90p$	A1√
	(iii) $\int x\sqrt{2x-3}\,dx$ = 1/5 (x + 1)($2x-3)^{3/2}$	M1
	$[\ ]_2^6$ = 1/5 (189 – 3) =37.2	DM1 A1

12 [11] **EITHER**	**(i)** $a = \mathrm{d}v/\mathrm{d}t = 5e^{-1/2\,t}$	M1 A1
	$v = 8 = 10(1 - e^{-1/2\,t}) \Rightarrow e^{-1/2\,t} = 0.2 \Rightarrow a = 1$	M1 A1
	(ii) $s = \int v\,\mathrm{d}t = \int\left(10 - 10\,e^{-t/2}\right)\mathrm{d}t = 10t + 20e^{-t/2}$	M1 A1
	$[\ ]_0^6 = (60 + 20e^{-3}) - (20) \approx 41$	DM1 A1
	(iii) 10 **(iv)** 	B1 B2,1,0
12 [11] **OR**	**(i)** $\mathrm{d}/\mathrm{d}\theta\{(\cos\theta)^{-1}\} = -(\cos\theta)^{-2}(-\sin\theta) = \sin\theta/\cos^2\theta$	M1 A1
	(ii) $AX = 2\sec\theta$ $PX = 2\tan\theta$	B1 B1
	$T = \frac{2\sec\theta}{3} + \frac{10 - 2\tan\theta}{5}$	M1 A1
	(iii) $\frac{\mathrm{dT}}{\mathrm{d}\theta} = \frac{2}{3}\frac{\sin\theta}{\cos^2\theta} - \frac{2}{5}\sec^2\theta$	B1 B1√
	$= 0$ when $5\sin\theta = 3 \Rightarrow \sin\theta = 3/5$	M1 A1
	$PX = 2\tan\theta = 2 \times ¾ = 1.5$	A1