

# ADDITIONAL MATHEMATICS

Paper 4037/11

Paper 11

## General comments

The majority of candidates were able to make a reasonable attempt at the paper. Marks obtained covered the whole range, with only a few, who were clearly unprepared for the paper, obtaining scores in single figures.

Candidates should be reminded of the importance of showing all their working clearly, as the omission of necessary working can lead to the loss of marks. Marks may also be awarded for correct working when the final answer is incorrect.

Centres should note that, from June 2011, candidates will be required to answer on the question paper, rather than in a separate answer booklet. New specimen papers have been produced and can be found on the CIE Teacher Support site.

## Comments on specific questions

### Question 1

- (i) Most candidates were able to convert the square root to a power. The need to use the chain rule to differentiate was usually recognised.
- (ii) The product rule was used by the majority of candidates, but some had difficulty with the differential of  $\sin 2x$ , often missing out the negative sign and sometimes giving just  $\sin 2$ . Attempts at simplification were often inaccurate,  $\sin 4x^2$  being common.

Answers: (i)  $\frac{3x^2}{2}(1+x^2)^{-\frac{1}{2}}$  (ii)  $2x \cos 2x - 2x^2 \sin 2x$

### Question 2

- (i) Most candidates were able to gain full marks in this part of the question, although a number of candidates gave 45 as the coefficient of  $x^2$ . A small minority left their answer in terms of  $\binom{n}{r}$  and failed to complete their calculations.
- (ii) There were many correct solutions to this part, most candidates appreciating the need for three terms from the expansion. Those who wrote out the whole expansion to pick out the relevant terms were more likely to make errors, as well as use up time.

Answers: (i)  $1 + 18x + 135x^2$  (ii) 76

### Question 3

Most candidates used the discriminant to get to the critical values, but only a few went on to give the range of values required. Weaker candidates expanded the expression and then attempted to solve it using the quadratic formula.

Answers:  $k \leq 2$  and  $k \geq 10$



#### Question 4

- (a) Many candidates obtained the correct solution for part (i) but found (ii) and (iii) more difficult.
- (b) Answers here were sometimes incorrectly given as inequalities, for example  $9 \leq x \leq 14$ . A common incorrect answer to (iii) was  $\{0\}$ .

Answers: (b)(i)  $\{9, 10, 11, 12, 13, 14\}$  (ii)  $\{5, 6, 7, \dots, 20\}$  (iii)  $\emptyset$

#### Question 5

This question was attempted well by most candidates. There were many completely correct solutions. A few candidates factorised the equation, but then stopped at that point, without solving it.

Answers:  $x = -2, -4$ , or  $\frac{1}{3}$

#### Question 6

- (i) This question was well done by most candidates, although  $x^{\frac{1}{2}} + y^2$  was a common incorrect answer.
- (ii) Most candidates appreciated that this answer required the difference of two logarithms, although some did not then realise that  $\log_8 8 = 1$ .
- (iii) This proved to be the most difficult part of the question. Many candidates managed to get to 6, but the change of base was too difficult for some and a completely correct answer was only obtained by the better candidates.

Answers: (i)  $\frac{1}{2}x + 2y$  (ii)  $y - 1$  (iii)  $6 + 3x$

#### Question 7

- (i) Many candidates calculated  $-3$ , but then failed to state the range.
- (ii) Most realised that they needed to “interchange”  $x$  and  $y$ , although some answers were left in terms of  $y$  rather than  $x$ . A number of candidates expanded the brackets prior to any other work, and then tried to rearrange the resulting expression, often leading to answers in terms of both  $x$  and  $y$ .
- (iii) Most candidates were able to apply the functions in the correct order to obtain an equation in  $x$ . Of those who simplified this equation to a quadratic, a number failed to notice that  $-2.2$  should be rejected, as it is not in the domain. Those who chose the simpler method of taking a square root often got one solution by taking only the positive square root, without justifying the rejection of the other solution from the negative square root.

Answers: (i)  $f \geq -3$  (ii)  $f^{-1} = \frac{\sqrt{x+3}-1}{2}$  (iii)  $x = 1$

#### Question 8

- (a) There were many good solutions to this question. Of those candidates who did not gain full marks, the most common error was to multiply, rather than add, the powers of 2 on the left hand side of the equation.



- (b)(i) Most candidates could work out that  $\sqrt{108} = 6\sqrt{3}$ , but surprisingly, in view of the next part, found  $\frac{12}{\sqrt{3}}$  much more difficult.
- (ii) The majority of candidates knew how to rationalise the denominator, and gained full marks.

Answers: (a)  $x = 5$  (b)(i)  $2\sqrt{3}$  (ii)  $5\sqrt{5} + 11$

### Question 9

- (a)(i) Most candidates were able to differentiate correctly, although  $5 - 4e^{-x}$  was a common incorrect answer.
- (ii) This was usually well done, although some candidates substituted  $x = p$  in  $\frac{dy}{dx}$ , rather than  $x = 0$ .
- (b) Candidates found this the most difficult part of the question, and could make little progress unless they realised the need to find  $\frac{dA}{dx}$  from  $A = x^2$ .

Answers: (a)(i)  $5 + 4e^{-x}$  (ii)  $9p$  (b)  $\frac{1}{12}$

### Question 10

- (i) Most candidates were able to obtain an equation in  $\tan x$  and to solve it. Those who squared the equation to get another equation in either  $\sin x$  or  $\cos x$  usually finished up with extra, invalid solutions.
- (ii) Most candidates used the correct trigonometrical identity to obtain an equation in  $\sin x$ . A number of them then chose to divide by  $\sin x$  and so lost one of the solutions.
- (iii) Although most candidates knew that  $\sec x = \frac{1}{\cos x}$ , a large number were unable to manipulate the equation correctly to obtain  $\cos\left(\frac{z}{3}\right) = \frac{1}{4}$ . Common errors were  $\sec z = 12$  or  $\cos\left(\frac{3}{z}\right) = 4$ .

Answers: (i)  $14.0^\circ, 194.0^\circ$  (ii)  $180^\circ, 199.5^\circ, 340.5^\circ$  (iii)  $3.95$  radians

### Question 11 EITHER

- (i) Most candidates recognised the need to differentiate using the quotient rule, and to equate  $\frac{dy}{dx}$  with 0, and this was generally well done. Few, however, were able to find the exact coordinates of the stationary point, and decimal answers were common.
- (ii) Candidates who simplified  $\frac{dy}{dx}$  in part (i) found this question to be reasonably straightforward, but those who failed to cancel by  $x$  found this second differentiation to be very difficult, often failing to realise that there was a product within the numerator of their  $\frac{dy}{dx}$ .

- (iii) Most candidates knew how to determine the nature of a stationary point by considering the sign of  $\frac{d^2y}{dx^2}$  at that point.

Answers: (i)  $x = e^{\frac{1}{2}}, y = \frac{1}{2e}$  (ii)  $\frac{-5 + 6\ln x}{x^4}$  (iii) Maximum

**Question 11 OR**

- (i) There were few good solutions, many candidates being unable to integrate correctly. Answers such as  $6 \sin\left(x^2 + \frac{\pi}{2}\right)$  were common, and the constant of integration was often omitted. Some candidates found a straight line equation for the curve using the given derivative as the gradient.
- (ii) Most candidates tried to solve  $\frac{dy}{dx} = 0$ , but failed to find all three possible values of  $x$ .
- (iii) Few candidates were able to get to the correct solution. Many were able to find a perpendicular gradient, but used the coordinates given in part (i) to find the equation of the normal.

Answers: (i)  $y = 3 \sin\left(2x + \frac{\pi}{2}\right) + 5$  (ii)  $0, \frac{\pi}{2}, \pi$  (iii)  $y = -\frac{1}{6}x + 5.39$

# ADDITIONAL MATHEMATICS

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Paper 4037/12

Paper 12

## General comments

Most candidates were able to make a good attempt at the paper, showing a fair understanding of the syllabus content. Most showed the ability to apply mathematical techniques both appropriately and correctly. Candidates are reminded of the need to present their work clearly; this also means showing the relevant steps involved in the solution of a problem, not merely writing down the answer. Too many candidates lost marks by failing to show the methods involved in their solutions. Candidates are also reminded to work to the appropriate degree of accuracy. Answers, if not exact, are to be to 3 significant figures which means prior workings must be to a greater degree of accuracy. Again, too many candidates lost marks by not working to this required level of accuracy.

Centres should note that, from June 2011, candidates will be required to answer on the question paper, rather than in a separate answer booklet. New specimen papers have been produced and can be found on the CIE Teacher Support site.

## Comments on specific questions

### Question 1

Most candidates realised that a quadratic equation in one variable was to be obtained. There were many arithmetic errors which resulted in incorrect equations. However, most were able to make a correct attempt to solve their quadratic equation, but there were some who 'lost' the solution  $x = 0$ .

Answer:  $(0,1), \left(\frac{1}{4}, -2\right)$

### Question 2

Whilst most candidates made the appropriate substitutions there were again arithmetic errors which meant that the required value of  $a = 8$  could not be found. Candidates are advised to check through their work when this happens to try to identify errors.

Answer:  $a = 8, b = 13$

### Question 3

(i) Finding  $\overrightarrow{AB}$  was usually done correctly. However, although most candidates found the magnitude of  $\overrightarrow{AB}$  there are still those who do not know how to find a unit vector.

(ii) Problems with this part usually involved a sign error between relevant vectors.

Answer: (i)  $\frac{1}{29} \begin{pmatrix} 21 \\ -20 \end{pmatrix}$  (ii)  $\begin{pmatrix} 46 \\ -35 \end{pmatrix}$

#### Question 4

- (i) Most candidates realised that the straight line took the form of  $y^2 = -2 \sec x + c$ , having correctly found the gradient. Errors occurred when it came to the calculation of  $+c$  with candidates misunderstanding which values to substitute into their equation.
- (ii) Correct solutions were in the minority. The question asked for the exact value of  $\cos x$ . There were many decimal answers and many candidates actually tried to solve their equation to find  $x$ .

Answer: (i)  $y^2 = -2 \sec x + 6.4$       (ii)  $\frac{5}{6}$

#### Question 5

If candidates did not realise that calculus was involved then they did not progress far. Most found the coordinates of the point A and those that did attempt differentiation were usually able to obtain the required gradient, the equation of the normal and hence make an attempt at the coordinates where the normal meets the  $x$ -axis.

Answer:  $\left(\frac{14}{3}, 0\right)$

#### Question 6

- (a) (i) Most candidates were able to do a reasonable sketch of the curve  $y = \cos x$ , although some attempts appeared to be more straight lines and V-shaped than was necessary. While many were able to apply both the appropriate transformations to their curve, some were only able to apply one of these transformations.
- (ii) Only candidates with completely correct curves were able to obtain the mark for this part. Some chose to try to solve the equation, misunderstanding the demands of the question.
- (b) (i) There were few incorrect answers for this part.
- (ii) Many candidates incorrectly gave an answer of 3, misunderstanding the nature of the question.

Answers: (a)(ii) 4      (b)(i) 5      (ii)  $\frac{2\pi}{3}$

#### Question 7

- (i) Candidates were usually able to produce a reasonable graph; however, many used awkward scales which then caused errors in reading from their graph in the later part of the question. Most graphs were drawn with the axes in the correct position.
- (ii) Although candidates had been instructed as to what to plot, many failed to make the connection between the gradient of their graph and the value of  $n$ . Of those that did, quite a few failed to realise that the gradient was negative. Poor plotting and scales often led to inaccuracies for those that did try to find the gradient.

Candidates had been asked to use their graph to find the value of  $n$ , but there were still those that chose to use the original equation and make calculations from this and the original points. These solutions did not score at all.

- (iii) Many of those who failed to find  $n$  were able to use their graph effectively to find a value for  $p$ , although many forgot that they had to use a power of ten in addition to the use of their graph. Again poor plotting and scales led to inaccuracies that produced solutions that were outside the acceptable range.

Answers: (ii)  $-1.42 \leq n \leq -1.32$       (iii)  $25 \leq p \leq 32$



### Question 8

- (i) Apart from the odd arithmetic slip, most of the solutions offered were correct.
- (ii) There were the occasional errors in either the determinant or the adjoint matrix, but most solutions were correct.
- (iii) The question was written to test the candidates' knowledge of matrix algebra and it was hoped that most candidates would realise that  $\mathbf{X} = \mathbf{AB}$  and all that would be required would be a straightforward case of matrix multiplication. Whilst many candidates did solve the problem in this way, others chose to find  $\mathbf{B}^{-1}$  and use this together with a matrix containing 4 unknown elements and simultaneous equations to find  $\mathbf{X}$ . Although this is a perfectly acceptable but longer method, there were many arithmetic slips which caused candidates to lose marks. Others attempted erroneously to try to divide 2 matrices thus showing a complete misunderstanding of the topic.

Answers: (i)  $\begin{pmatrix} 16 & 9 \\ 1 & -2 \end{pmatrix}$  (ii)  $\frac{1}{5} \begin{pmatrix} 2 & -3 \\ -1 & 4 \end{pmatrix}$  (iii)  $\begin{pmatrix} -5 & 12 \\ 0 & 8 \end{pmatrix}$

### Question 9

- (i) Although there were many correct solutions, some candidates had difficulty forming a correct expression for the perimeter and were thus unable to obtain a correct value for the angle.
- (ii) Most candidates used the correct formula for the area of a sector of a circle. Some candidates lost marks as they omitted to find the correct areas to form a ratio, obtaining the answer 9:64 instead. Many candidates lost marks by not giving their answer in the required form.

Answer: (i) 0.5 (ii) 9:55

### Question 10

- (i) Very few candidates obtained incorrect answers, choosing correctly to use combinations rather than permutations.
- (ii) Most solutions seen were correct. The use of permutations was rare.
- (iii) Occasional solutions were seen where the candidate was attempting to choose from 7 and other solutions did not consider all the cases required. However the great majority of solutions seen were correct.

Answers: (i) 120 (ii) 36 (iii) 100

### Question 11

This question was probably the most poorly done question on the paper, with some candidates not attempting it.

- (i) Too many candidates did not know how to deal with the logarithmic term in the equation.  $4 = \ln 2t + \ln 3$  was a common error. Some candidates gave an inaccurate answer of 26.
- (ii) The need for differentiation was realised by most candidates (apart from a few who insisted on trying to use linear motion equations). Many candidates were unable to differentiate correctly, often including logarithmic terms in their answers (usually from an incorrect attempt at differentiating a product) or forgetting the factor of 12.
- (iii) This part of the question was badly done in the same way as part (ii), with similar errors compounding those already made. Occasionally candidates omitted the negative sign from their answer although this was usually the result of poor differentiation.

Answers: (i) 25.8 (ii) 4.8 (iii) -1.92



### Question 12 EITHER

This was less popular than the **OR** option.

- (i) Most candidates realised that they had to integrate and include an arbitrary constant in order to show what was required. Some adopted an approach of verification but only went as far as differentiating the given equation, not showing that the coordinates satisfied the given equation.
- (ii) Correct attempts at solutions were common, but many candidates chose to give only one solution to their equation. Those that chose to give their solutions in decimal form in radians often did not give the required level of accuracy which then had a detrimental effect on later work.
- (iii) The area of the rectangle was often forgotten, but most candidates were able to make a reasonable attempt at integration to find the area under the curve. Some adopted an all in one approach which usually tended to be more successful.

Answers: (i)  $y = 4 \sin 2x + 3$       (ii)  $\frac{\pi}{12}, \frac{5\pi}{12}$       (iii) 1.37

### Question 12 OR

- (i) There were many correct solutions although there were candidates who forgot to include an arbitrary constant. This then had a detrimental effect in the next part of the question. Other candidates mistakenly assumed that the value of the arbitrary constant was 1 as the curve went through the point (0, 1).
- (ii) Most candidates were able to make a reasonable attempt at finding the coordinates of the turning point, recognising that they already had the necessary derivative already given in the first part of the question. Problems occurred when it came to the accuracy of the required coordinates. Too many candidates did not give their answers to the required significant figures, 0.23 being the most common solution. This then caused problems when calculating the y-coordinate using prematurely approximated values. Exact values for the coordinates were acceptable.
- (iii) Usually done well; most candidates were able to produce a correct method for determining the nature of the turning point. Some, however, did not state the reason for their conclusion which was an essential part of the method required.
- (iv) Apart from those candidates who calculated the equation of the normal rather than the tangent, the great majority of the solutions seen were correct.

Answers: (i)  $y = 2e^{3x} - 12x - 1$       (ii) (0.231, 0.227)      (iii) minimum point      (iv)  $\left(\frac{1}{6}, 0\right)$



# ADDITIONAL MATHEMATICS

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Paper 4037/21

Paper 21

## General comments

Some candidates produced high quality work displaying wide ranging mathematical skills, with well presented, clearly organised answers. Overall however, the performance was weak, with a substantial number of candidates scoring fewer than 20 marks. The majority of candidates attempted the majority of questions.

Candidates found that **Questions 5, 6, and 8** were most straightforward, and most likely to yield full marks. It was very common for a score of zero to be obtained on **Question 10** and many candidates omitted this question altogether. It was pleasing to see the almost universal improvement in the manipulation of matrices. In **Question 4(a)** calculators were used too readily, the requirement of the question for exact values being ignored. **Question 7** saw the use of some long methods, especially when calculating the area having found appropriate coordinates, with more efficient methods proving more successful.

Centres should note that, from June 2011, candidates will be required to answer on the question paper, rather than in a separate answer booklet. New specimen papers have been produced and can be found on the CIE Teacher Support site.

## Comments on specific questions

### Question 1

This question was not answered well, with many candidates failing to appreciate that the axes were labelled  $\frac{y}{x^2}$  and  $x^3$ . Many candidates, whilst knowing they had to find the gradient and intercept, did not know what to do afterwards. A common final answer was  $y = -2x + 15$ . Several of those who did manage to arrive at a relation of the form  $\frac{y}{x^2} = mx^3 + c$  then proceeded to spoil their answer by writing  $y = mx^6 + cx^2$ .

Answer:  $y = -2x^5 + 15x^2$

### Question 2

Most candidates were able to gain some marks for this question but full marks were not often seen. Part (i) was found to be the most straightforward although 8 was a not infrequent response. Candidates often got one or other of parts (ii) and (iii) correct with the main errors involving multiplying factorials or leaving answers incomplete using C or P notation.

Answers: (i) 40320      (ii) 56      (iii) 60



### Question 3

Although most candidates were able to obtain the required relation between  $b$  and  $a$  in part (i), this was often not used in part (ii). Instead, it was common to see the remainders equated to zero and then simultaneous equations solved for  $a$  and  $b$ . Others attempted to solve  $f(3) = 2f(2)$ , or incorrectly  $2f(3) = f(2)$ , usually with no reference to part (i) again.

Many candidates did not use the Remainder Theorem in either part but instead used the long division method which proved less successful, as candidates often failed to reach a remainder in  $b$  and  $a$  only.

Answer: (i)  $5 - a$

### Question 4

(a) This question was not answered well by the majority of candidates. Almost none appreciated the significance of the phrase 'exact values' so that approximate decimals appeared for both answers. Those who made attempts at applying Pythagoras' theorem were often spoilt by omitting brackets and writing  $p^2 + 2p^2 = 1$ .

(b) While some candidates proved the trigonometric identities with relative ease the proof seldom gained full marks, with dubious algebra and inaccurate identities often evident. Frequently the only mark gained was for removing the brackets correctly. Many attempts lasted for several pages, the preferred approach usually being to work with sines and cosines. Even the very best candidates often lost marks on this question.

Answers: (i)  $p = \frac{1}{\sqrt{5}}$ ,  $\operatorname{cosec} x = \sqrt{5}$

### Question 5

This question was well answered by a large number of candidates, with many achieving full marks. The overall method for solution was seen as straightforward by the majority of candidates. The differentiation was generally successful, but solving the equation  $2 - 16x^{-1.5} = 0$  was often done incorrectly. Many candidates resorted to using logarithms and although this was occasionally and laboriously completed successfully, it generally led to poor or muddled algebra.

Answer: (4,144)

### Question 6

This was a routine question for almost all candidates, and a good source of marks for many. Candidates demonstrated secure understanding of solving quadratic and linear simultaneous equations. Most were able to gain some marks for this question, although weaker candidates invariably arrived at  $4x^2 + 20x + 17 = 0$  or similar, instead of the correct quadratic equation for  $x$ , the negative  $y^2$  mainly causing their undoing.

Answer:  $\sqrt{32}$  or  $4\sqrt{2}$  or 5.66



### Question 7

Although several methods were suitable, it was essential to first find the coordinates of  $D$ . The bulk of the work should have taken place in doing this but some candidates attempted to find the area without finding it, gaining no credit as a result. However, most candidates were able to obtain some credit, usually for identifying the gradient of  $BC$  and its perpendicular. Not all saw the importance of the midpoint however, giving the equation of an incorrect bisector.

The latter part of this question frequently covered several sides of paper, most of which gained no marks at all and involved finding a variety of lengths and angles. Many candidates made the false assumption that  $AB$  and  $BC$  were perpendicular, which meant that they tried to calculate the area of a trapezium. Relatively few candidates arrived at the correct area of 15 square units. There were some elegant solutions involving the difference of two triangles but by far the simplest and most efficient solution involved using the array method for calculating an area.

Answer: 15

### Question 8

Almost all candidates were able to gain some marks on this matrix question. However, a large number of careless arithmetical slips were made and there were many candidates who thought that it was not possible to calculate the product  $\mathbf{BA}$  asked for in part (a)(iii). The inverse of matrix  $\mathbf{C}$  was usually found correctly, but the product  $\mathbf{C}^{-1}\mathbf{D}$  was often attempted in part (b)(ii). It was pleasing to see so many candidates using the inverse matrix, rather than simultaneous equations, even though they had not been instructed to do so.

$$\text{Answers: (a)(i)} \begin{pmatrix} 4 & 6 & 14 \\ 2 & -10 & 8 \end{pmatrix} \quad \text{(ii)} \begin{pmatrix} 12 & 8 \\ 64 & 44 \end{pmatrix} \quad \text{(iii)} \begin{pmatrix} 5 & 1 & 18 \\ 22 & -6 & 80 \end{pmatrix}$$
$$\text{(b)(i)} \quad \mathbf{C}^{-1} = \frac{1}{5} \begin{pmatrix} 6 & -1 \\ -7 & 2 \end{pmatrix} \quad \text{(ii)} \quad \frac{1}{5} \begin{pmatrix} 3 & 2 \\ -5 & 0 \end{pmatrix}$$

### Question 9

Performance overall on this question was disappointing and full marks were seldom awarded. Those candidates who knew the basic calculus commonly obtained partial credit from obtaining the correct forms for the integral in part (i) and the derivative in part (ii). However, in part (i) the need to find the time at which the particle next comes to rest was almost universally ignored and very few candidates were able to obtain the value  $t = \frac{\pi}{2}$ . Very few candidates remembered the constant of integration in part (i). In fact it was sometimes assumed that  $-2\cos 2t$  was sufficient for an answer. Very few remembered to use radian mode on their calculator in part (ii).

The very weakest candidates used no calculus, and made attempts using equations of constant speed or constant acceleration.

Answers: (i) 4      (ii) 7.68

### Question 10

Only a small proportion of candidates managed to secure full marks on this question. Many candidates were able to arrive at the given answer of 25 km in the first part, but there were very few attempts at the second part seen and many omitted the question altogether. Some of those who had successfully completed part (i) seemed to think that the 7 and 24 which had been used somehow had to be used in part (ii). Of those who did work their way through part (ii) and successfully solved the quadratic equation many omitted to calculate the length of time as required and gave a final answer of  $t = 2$  or  $6$  or  $2 < t < 6$ . There was also confusion in some minds with what appeared to be matrix methods applied with large values appearing from some form of multiplication.

Answer: (ii) 4



### Question 11 EITHER

Some good attempts at this question were seen, although invariably some candidates used the obtuse angle where the reflex angle was required. It was pleasing to see that there was hardly any confusion between the perimeter and area formulae, and that in part (i) very few included the perimeter of the whole triangle in their calculation. Errors were often made in the misuse of  $\pi$ , either including it in formulae incorrectly or overusing it in other cases. Another error made by several candidates was to assume triangle  $OAD$  could be split into two right-angled triangles by subdividing  $0.8\pi$  in a number of arbitrary ways. In part (ii) marks were sometimes lost by not working to sufficient accuracy. This particularly applied to those scripts where all the angles in radians had been converted to slightly inaccurate angles in degrees.

Answers: (i) 45.8      (ii) 89.0

### Question 11 OR

This option was reasonably well done, with fairly routine marks available from finding the derivative in part (i) and for integrating to find the area under the curve in part (ii). In part (i), many correctly found the equation of the tangent then solved simultaneously with the equation of the curve, and solved the resulting cubic for all three roots, realising that  $(x - 1)$  was a factor. In part (ii) the most common approach was to subtract the area under the curve from a trapezium, rather than using the difference of the line and curve. In general, errors here tended to be errors of detail rather than principle, though some candidates seem to have considered the solution to be complete having found the area under the curve.

Answer: (ii) 6.75



# ADDITIONAL MATHEMATICS

Paper 4037/22

Paper 22

## General comments

There were many excellent, well presented scripts seen. These demonstrated a clear understanding of the principles involved in the topics examined. There were a considerable number of candidates who scored full marks, but a similar number who failed to score full marks because of careless reading of the paper. Examples of this were a failure to “show the coordinates of the points where the graph meets the axes” in **Question 7**, a failure to give “answers correct to 2 decimal places” in **Question 11 part (i)** and using the form  $a(x + b)^2 + c$  rather than  $(ax + b)^2 + c$  in **Question 12 OR part (ii)**.

Centres should note that, from June 2011, candidates will be required to answer on the question paper, rather than in a separate answer booklet. New specimen papers have been produced and can be found on the CIE Teacher Support site.

## Comments on specific questions

### Question 1

A very straightforward start for most candidates, with most picking up at least the first two marks. The final term caused the major problems with a variety of answers offered by the weaker candidates. A small but significant number of candidates attempted to combine the three terms before integration and found it difficult to proceed. Others tried to combine the terms after integration and again got into difficulties, but this time they did not lose marks already awarded for correct integration.

Answer:  $2x + \frac{5x^2}{2} + \frac{1}{x-2} (+c)$

### Question 2

The success rate for the whole question was mixed. The diagram in part **(a)** was reasonably well done, although quite a number of candidates had little idea of how to handle Venn diagrams. The most frequent error was to leave  $A \cap C$  unshaded. A very small number decided to include  $(A \cup B \cup C)'$ .

The majority of candidates were able to express the region correctly in part **(b)** although quite a number gave an expression that was far more complex than necessary. The simplest form was  $X' \cup Y$  but many gave this incorrectly as  $X' \cap Y$ . Some candidates included the Universal set – e.g.  $\xi \cup (X' \cup Y)$  making it completely wrong.

The success or otherwise in part **(c)** did not seem to be affected by whether the candidates set up a formal equation or not. Most of the better candidates were able to correctly calculate 6 without showing any working. Candidates who did attempt to set up an equation frequently forgot to include the variable that they were trying to find, “x”, after including “16 – x” and “18 – x”. Also frequently forgotten was the “2”.

Answers: **(b)**  $X' \cup Y$ ,  $(X \cap Y)'$ ,  $X' \cup (X \cap Y)$  or equivalent      **(c)** 6



### Question 3

This question was generally well done with many candidates scoring full marks. Weaker candidates often reached  $4\pi^2$ , but did not know how to proceed further.

Answer:  $144 \text{ cm}^3 \text{s}^{-1}$

### Question 4

Part (i) was done less well than part (ii). Most candidates were able to set up the correct expression for  $\tan \theta$  but were unable to rationalise correctly. Many simply inverted the fraction to give  $\frac{7\sqrt{6}}{16}$ . Others showed that they knew what to attempt but failed completely to handle the arithmetic.

Part (ii) was very well done with the vast majority gaining full marks. Almost all candidates were able to correctly apply Pythagoras and almost all of these went on to correctly arrive at  $\sqrt{275}$ . At this stage however, a number of candidates were unable to deal with the surds.

Answers: (i)  $\frac{8\sqrt{6}}{21}$  (ii)  $5\sqrt{11}$

### Question 5

Full marks were scored by the majority of candidates. Most found the first factor from the remainder theorem and then went on to find the quadratic factor, usually by synthetic division or algebraic division, and then factorised this factor. A few found all 3 factors using the remainder theorem. One mark was often lost because one or more roots were not listed, with candidates just giving the factors. Very few candidates scored less than 5 marks; these tended to be the lowest achievers overall.

Answer:  $-2$  and  $\frac{1}{2}$  and  $3$

### Question 6

In general good candidates scored very well on this question but weaker candidates made little or no progress.

Almost all candidates were able to write down the equation for the area as  $A = xy$ , but many were unable to proceed further because they were unable to link it with the equation for the line  $y = 12 - 2x$ . Being unable to write  $A$  in terms of  $x$ , made it virtually impossible for candidates to progress further. Those who were able to set up the correct expression for  $A$  generally went on to obtain 7 or 8 marks. Quite a number failed to give the value for  $A$  required in part (iii) and a small number failed to determine the nature of the stationary value.

Answers: (i)  $A = 12x - 2x^2$  (ii)  $3$  (iii)  $18$ , maximum

### Question 7

This was another high scoring question, with most candidates scoring 6 or 7 marks. Most commonly, a mark was lost on the graph for not clearly labelling where the line cut the  $y$ -axis. Most candidates were successful in drawing the graph of the modulus function correctly, although a few joined the two straight lines with a small curve. Part (iii) was relatively well answered with many candidates working through the algebraic equation to find the values of  $x$ . Those candidates who read from the graph did not always score well as their diagram was only a sketch and therefore not accurate.

Answer (iii)  $-1.5$  and  $-3.75$



### Question 8

On the whole, most candidates scored full marks in part (i) with only minor mistakes being seen by candidates who did not. There were mixed results in part (ii), with many candidates not fully understanding how to obtain a coefficient of  $x$  from a series. Candidates who wrote out the series were able to deduce the correct coefficient with only a few losing marks at the calculation stage.

Part (iii) proved problematic for weaker candidates, some of whom wrote out the series but did not indicate the correct term. Some stronger candidates realised that they needed to find  $r$  from  ${}^nC_r$ , finding that  $r = 6$ . However some candidates who had correctly found  $r = 6$ , then merely stated that the independent term was the seventh term but did not perform the required calculation.

Answers: (i)  $1 - 21x + 189x^2 - 945x^3$  (ii)  $-4375$  (iii)  $5376$

### Question 9

Most candidates realised that they were required to differentiate using the product or quotient rule in part (i) but only the better candidates were able to make solid progress through the whole question. Weaker candidates often mixed up the two methods, usually writing it in the form for product rule and applying the quotient rule, which was sometimes quoted incorrectly. Handling the indices was also a significant problem for many. Having said that, a great many candidates did collect the first three marks for getting the derivative in an unsimplified form, but arranging it into the form required proved beyond many. The algebraic manipulation was frequently of doubtful accuracy but  $k = 2$  was the most popular outcome whether justified or not. The able candidates who realised the link between the two parts had little difficulty arriving at the correct answer in part (ii) although a small number chose to multiply their integral by 2 rather than to divide. Weaker candidates attempted to integrate the expression as given and got themselves into great difficulties.

Answers: (i)  $k = 2$  (ii)  $\frac{9}{16}$

### Question 10

A significant number of candidates were not confident in handling logarithms or indeed powers. However, the use of a calculator enabled many to answer part (b). This was an easy question for those who did know the laws of logarithms.

Candidates with a complete understanding of log theory had little difficulty in achieving the correct answer in part (i) from  $2\log X - \log Y$ . Many candidates, however, lacked this understanding and seemed to assume that  $X = 6$  and  $Y = 4$  to substitute into the original expression to give  $\log(36 \div 4)$  and gave 9 as the answer. A smaller number showed a partial understanding to start with  $\log(2X \div Y)$  which led to  $\log(12 \div 4) = \log 3$ .

Again in (ii) the many candidates who understood what to do had little difficulty in achieving the correct answer. A significant number rewrote  $\log_{\sqrt{Y}}(X)$  as  $(\log Y \div \log X)$  rather than  $(\log X \div \log Y)$ . Others realised that in logarithm theory "divide" meant "subtract" and gave an answer of  $6 - 4 = 2$ .

In part (iii), good candidates had little difficulty in obtaining the correct answer. Weaker candidates showed very little understanding of the theory of indices and consequently rarely scored any marks. A significant number of candidates calculated a value for  $z$  as 6.585 and then raised 2 to this power. If performed accurately this did lead to 96, however answers such as 95.7 were very common; these answers lost 1 mark.

In the final part most candidates started to break down 512 into multiples of 2 or 4. A great many arrived at  $2 \cdot 2^{4.5}$  or  $4^{4.5}$  but the  $\sqrt{512}$  was a complication that defeated the vast majority and they were unable to translate this into the required form with mixed answers such as  $4^2 \times \sqrt{2}$  being common.

Answers: (a)(i) 8 (ii) 1.5 (b) 96 (c)  $4^{\frac{9}{4}}$



### Question 11

There was a range of marks in part **(a)** although a number of candidates did not give the answer to 2 decimal places and so lost a mark. Quite a few did not find the second solution at all or did not find it correctly. A few found the solutions in degrees. There were quite a lot of candidates scoring full marks in part **(b)**. Most earned a mark for knowing how to deal with  $\cot y$  and  $\operatorname{cosec} y$ , and went on to express their equation in  $\cos y$ . In rearranging the equation some lost the factor of 6 or a minus sign. Others introduced a factor of  $\sin y$  and ended up with extra solutions. Where the equation was correct, most went on to get all 4 solutions although sometimes marks were lost for incorrect rounding. A few found solutions in the wrong quadrants – e.g. 60 and 300.

Answers: **(a)** 0.85 and 2.29      **(b)** 48.2, 120, 240 and 311.8

### Question 12 EITHER

This question on the whole was well answered with candidates substituting  $f(x)=3$  to obtain the correct quadratic. Attempting to factorise the quadratic proved problematic for some candidates who were unable to find that  $k = 4$  and 12, others found the correct values for  $k$  but did not identify the correct range of values. There were a minority who ignored the 3 and consequently found values of 0 and 16 for  $k$ .

In part **(ii)** many candidates lost marks by not reading the question which stated that the answer needed to be in the form  $(ax+b)^2+c$ ; a final answer of  $4\left(x + \frac{5}{4}\right)^2 + \frac{15}{4}$  was written by many candidates, thus losing them marks. In part **(iii)** many candidates scored full marks, despite losing marks in part **(ii)**, with only a minority losing marks by again not reading the question which asked for the minimum to be stated. Those who just gave the coordinates of the turning point without identifying which was the least value of  $f(x)$  were penalised.

Answers: **(i)**  $4 < k < 12$       **(ii)**  $a = 2, b = 2.5$  and  $c = 3.75$       **(iii)** 3.75 at  $x = -1.25$

### Question 12 OR

There were very few correct answers to part **(i)**, even from good candidates. Mostly they had little idea how to start and many simply left this part blank. Part **(ii)**, however, was very well done. Wrong answers were rare and, generally, only from the very weakest of candidates. Part **(iii)** was very well answered by most candidates. There was an occasional error in calculating  $g^{-1}(15) = 10$  and a small number of candidates correctly achieved the correct expression for  $4x^2 - 20x + 26$  but divided by 2 to give  $2x^2 - 10x + 13$  before putting it equal to 10. This obviously led to an incorrect equation that needed the use of the quadratic formula and therefore an incorrect solution. There were very few correct graphs seen in the last part. Quite a number of candidates showed the right idea but restricted the graph to the first quadrant.

Answers: **(i)**  $f \geq 1$       **(ii)** 15      **(iii)** 1 and 4

