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4037/12

May/June 2018

2 hours

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

DO **NOT** WRITE IN ANY BARCODES.

You are reminded of the need for clear presentation in your answers.

The total number of marks for this paper is 80.

This document consists of **16** printed pages.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

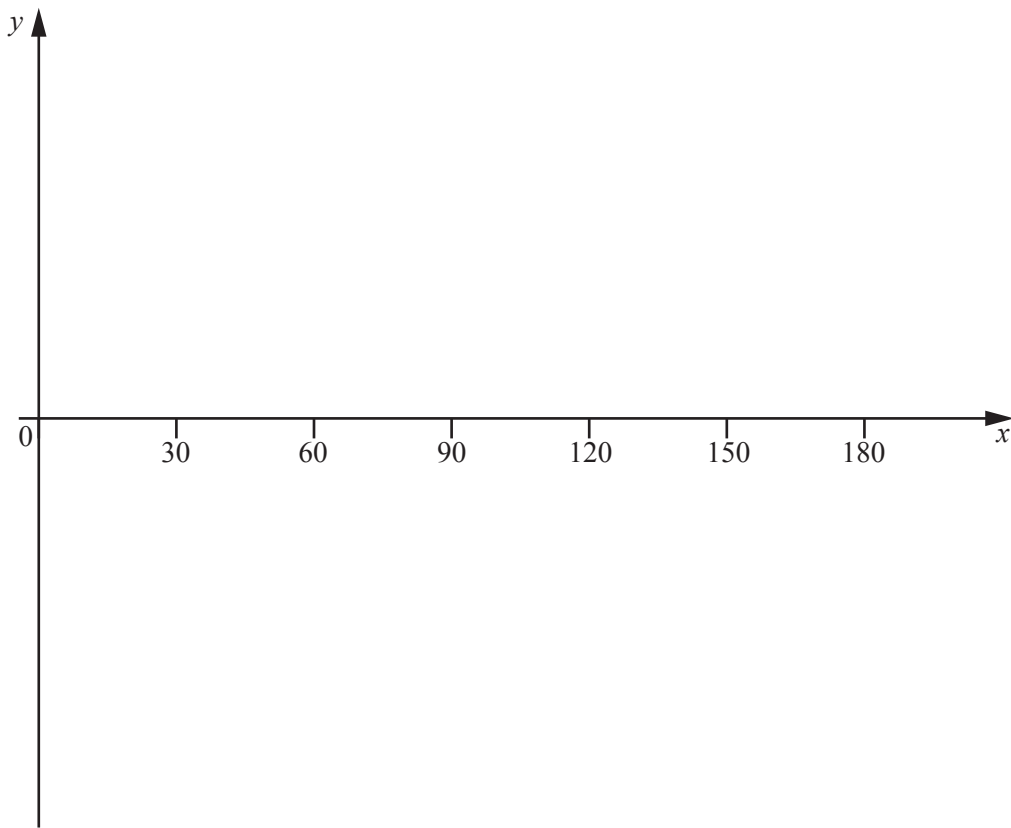
1 It is given that $y = 1 + \tan 3x$.

(i) State the period of y .

[1]

(ii) On the axes below, sketch the graph of $y = 1 + \tan 3x$ for $0^\circ \leq x^\circ \leq 180^\circ$.

[3]



- 2 Find the values of k for which the line $y = 1 - 2kx$ does not meet the curve $y = 9x^2 - (3k + 1)x + 5$.
[5]

- 3 The variables x and y are such that when e^y is plotted against x^2 , a straight line graph passing through the points $(5, 3)$ and $(3, 1)$ is obtained. Find y in terms of x . [5]

- 4 A particle P moves so that its displacement, x metres from a fixed point O , at time t seconds, is given by $x = \ln(5t + 3)$.

(i) Find the value of t when the displacement of P is 3m. [2]

(ii) Find the velocity of P when $t = 0$. [2]

(iii) Explain why, after passing through O , the velocity of P is never negative. [1]

(iv) Find the acceleration of P when $t = 0$. [2]

- 5 (i) The first three terms in the expansion of $\left(3 - \frac{1}{9x}\right)^5$ can be written as $a + \frac{b}{x} + \frac{c}{x^2}$. Find the value of each of the constants a , b and c . [3]

- (ii) Use your values of a , b and c to find the term independent of x in the expansion of

$$\left(3 - \frac{1}{9x}\right)^5 (2 + 9x)^2. \quad [3]$$

- 6 Find the coordinates of the stationary point of the curve $y = \frac{x+2}{\sqrt{2x-1}}$. [6]

7 A population, B , of a particular bacterium, t hours after measurements began, is given by $B = 1000e^{\frac{t}{4}}$.

(i) Find the value of B when $t = 0$. [1]

(ii) Find the time taken for B to double in size. [3]

(iii) Find the value of B when $t = 8$. [1]

8 **(a)** Solve $3 \cos^2 \theta + 4 \sin \theta = 4$ for $0^\circ \leq \theta \leq 180^\circ$. [4]

(b) Solve $\sin 2\phi = \sqrt{3} \cos 2\phi$ for $-\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}$ radians. [4]

9 (a) (i) Solve $\lg x = 3$. [1]

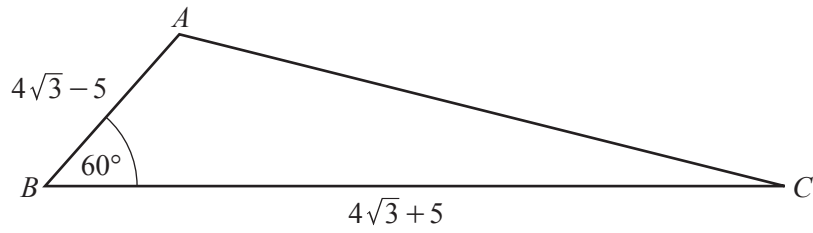
(ii) Write $\lg a - 2 \lg b + 3$ as a single logarithm. [3]

(b) (i) Solve $x - 5 + \frac{6}{x} = 0$. [2]

(ii) Hence, showing all your working, find the values of a such that $\log_4 a - 5 + 6 \log_a 4 = 0$. [3]

10 Do not use a calculator in this question.

All lengths in this question are in centimetres.



The diagram shows the triangle ABC , where $AB = 4\sqrt{3} - 5$, $BC = 4\sqrt{3} + 5$ and angle $ABC = 60^\circ$.

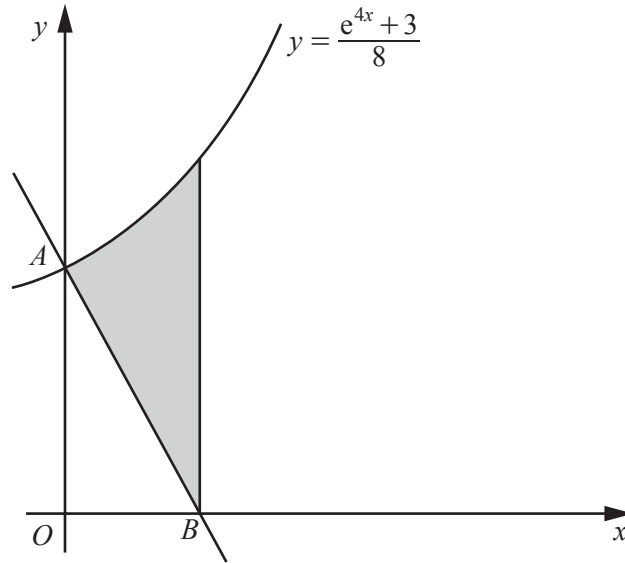
It is known that $\sin 60^\circ = \frac{\sqrt{3}}{2}$, $\cos 60^\circ = \frac{1}{2}$, $\tan 60^\circ = \sqrt{3}$.

(i) Find the exact value of AC .

[4]

- (ii) Hence show that $\operatorname{cosec} ACB = \frac{2\sqrt{p}}{q}(4\sqrt{3} + 5)$, where p and q are integers. [4]

11



The diagram shows the graph of the curve $y = \frac{e^{4x} + 3}{8}$. The curve meets the y -axis at the point A .

The normal to the curve at A meets the x -axis at the point B . Find the area of the shaded region enclosed by the curve, the line AB and the line through B parallel to the y -axis. Give your answer in the form $\frac{e}{a}$, where a is a constant. You must show all your working.

[10]

Question 12 is printed on the next page.

12 Do not use a calculator in this question.

- (a) Given that $\frac{6^p \times 8^{p+2} \times 3^q}{9^{2q-3}}$ is equal to $2^7 \times 3^4$, find the value of each of the constants p and q . [3]

- (b) Using the substitution $u = x^{\frac{1}{3}}$, or otherwise, solve $4x^{\frac{1}{3}} + x^{\frac{2}{3}} + 3 = 0$. [4]

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