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4037/22

May/June 2018

2 hours

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

DO **NOT** WRITE IN ANY BARCODES.

You are reminded of the need for clear presentation in your answers.

The total number of marks for this paper is 80.

This document consists of **15** printed pages and **1** blank page.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \dots + \binom{n}{r} a^{n-r} b^r + \dots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

2. TRIGONOMETRY*Identities*

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

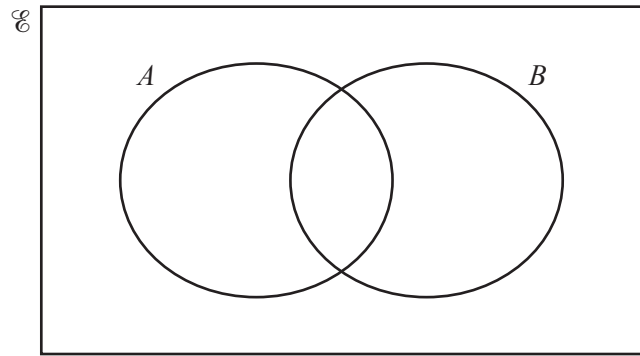
1 (i) Show that $\cos \theta \cot \theta + \sin \theta = \operatorname{cosec} \theta$.

[3]

(ii) Hence solve $\cos \theta \cot \theta + \sin \theta = 4$ for $0^\circ \leq \theta \leq 90^\circ$.

[2]

- 2 (a) On the Venn diagram below, shade the region that represents $A \cap B'$.

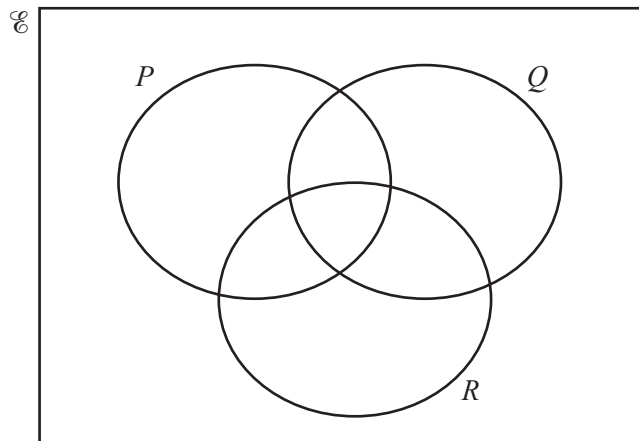


[1]

- (b) The universal set $\mathcal{E}$ and sets P , Q and R are such that

$$\begin{array}{lll} (P \cup Q \cup R)' = \emptyset, & P' \cap (Q \cap R) = \emptyset, & \\ n(Q \cap R) = 8, & n(P \cap R) = 8, & n(P \cap Q) = 10, \\ n(P) = 21, & n(Q) = 15, & n(\mathcal{E}) = 30. \end{array}$$

Complete the Venn diagram to show this information and state the value of $n(R)$.

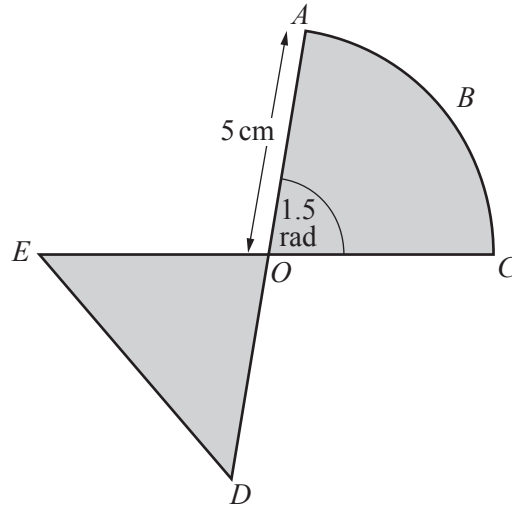


$n(R) = \dots\dots\dots$ [4]

- 3 It is given that $x + 3$ is a factor of the polynomial $p(x) = 2x^3 + ax^2 - 24x + b$. The remainder when $p(x)$ is divided by $x - 2$ is -15 . Find the remainder when $p(x)$ is divided by $x + 1$. [6]

- 4 Find the coordinates of the points where the line $2y - 3x = 6$ intersects the curve $\frac{x^2}{4} + \frac{y^2}{9} = 5$. [5]

- 5** **(a)** Four parts in a play are to be given to four of the girls chosen from the seven girls in a drama class. Find the number of different ways in which this can be done. [2]
- (b)** Three singers are chosen at random from a group of 5 Chinese, 4 Indian and 2 British singers. Find the number of different ways in which this can be done if
- (i)** no Chinese singer is chosen, [1]
- (ii)** one singer of each nationality is chosen, [2]
- (iii)** the three singers chosen are all of the same nationality. [2]



In the diagram, ABC is an arc of the circle centre O , radius 5 cm, and angle AOC is 1.5 radians. AD and CE are diameters of the circle and DE is a straight line.

(i) Find the total perimeter of the shaded regions. [3]

(ii) Find the total area of the shaded regions. [3]

7 Vectors $\mathbf{i}$ and $\mathbf{j}$ are vectors parallel to the x -axis and y -axis respectively.

Given that $\mathbf{a} = 2\mathbf{i} + 3\mathbf{j}$, $\mathbf{b} = \mathbf{i} - 5\mathbf{j}$ and $\mathbf{c} = 3\mathbf{i} + 11\mathbf{j}$, find

(i) the exact value of $|\mathbf{a} + \mathbf{c}|$, [2]

(ii) the value of the constant m such that $\mathbf{a} + m\mathbf{b}$ is parallel to $\mathbf{j}$, [2]

(iii) the value of the constant n such that $n\mathbf{a} - \mathbf{b} = \mathbf{c}$. [2]

8 **(a)** $\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 1 & -3 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 0 & -2 \\ 3 & -5 \end{pmatrix}$. Find $(\mathbf{BA})^{-1}$. [4]

(b) The matrix $\mathbf{X}$ is such that $\mathbf{XC} = \mathbf{D}$, where $\mathbf{C} = \begin{pmatrix} -2 & 5 & 3 \\ 0 & 10 & 4 \end{pmatrix}$ and $\mathbf{D} = \begin{pmatrix} -4 & 5 & 4 \end{pmatrix}$.

(i) State the order of the matrix $\mathbf{C}$. [1]

(ii) Find the matrix $\mathbf{X}$. [2]

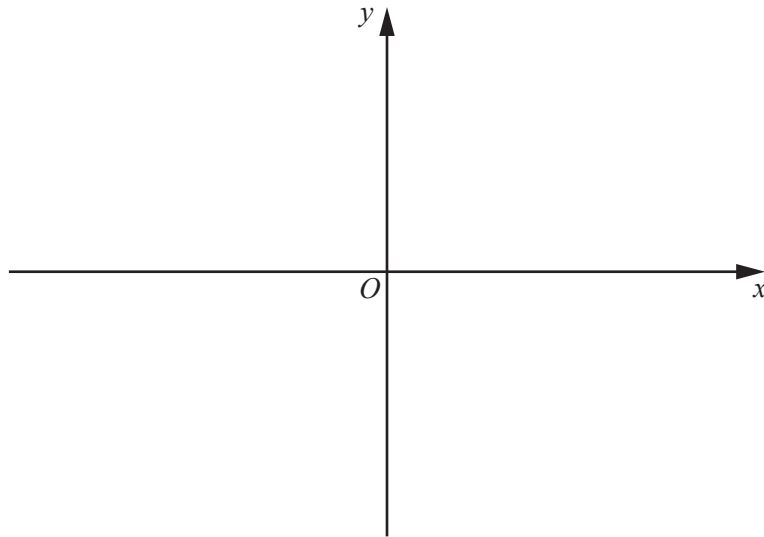
9 (i) Differentiate $x^4(\sqrt{\sin x})$ with respect to x .

[4]

(ii) Hence find $\int \left(x + \frac{x^4 \cos x}{\sqrt{\sin x}} + 8x^3(\sqrt{\sin x}) \right) dx$.

[3]

- 10 (a) (i) On the axes below, sketch the graph of $y = |(x + 3)(x - 5)|$ showing the coordinates of the points where the curve meets the x -axis. [2]



- (ii) Write down a suitable domain for the function $f(x) = |(x + 3)(x - 5)|$ such that f has an inverse. [1]

- (b) The functions g and h are defined by

$$\begin{aligned} g(x) &= 3x - 1 && \text{for } x > 1, \\ h(x) &= \frac{4}{x} && \text{for } x \neq 0. \end{aligned}$$

- (i) Find $hg(x)$. [1]

- (ii) Find $(hg)^{-1}(x)$. [2]

- (c) Given that $p(a) = b$ and that the function p has an inverse, write down $p^{-1}(b)$. [1]

11 (a) Find $\int \sqrt[3]{2x-1} \, dx$. [2]

(b) (i) Find $\int \sin 4x \, dx$. [2]

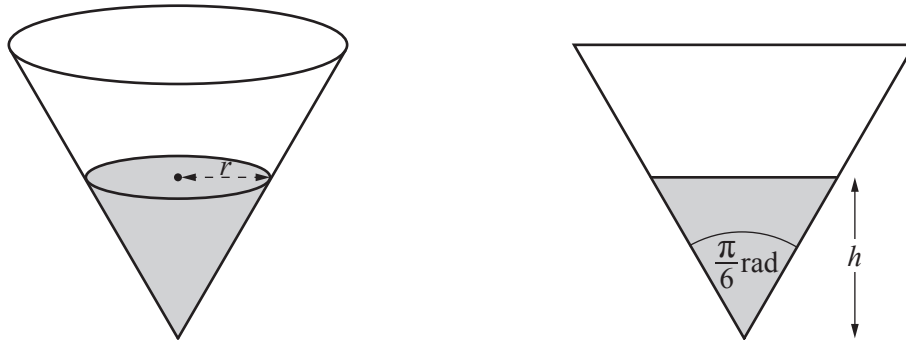
(ii) Hence evaluate $\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \sin 4x \, dx$. [2]

(c) Show that $\int_0^{\ln 8} e^{\frac{x}{3}} \, dx = 3$. [5]

12 In this question all lengths are in centimetres.

The volume of a cone of height h and base radius r is given by $V = \frac{1}{3}\pi r^2 h$.

It is known that $\sin \frac{\pi}{12} = \frac{\sqrt{6} - \sqrt{2}}{4}$, $\cos \frac{\pi}{12} = \frac{\sqrt{6} + \sqrt{2}}{4}$, $\tan \frac{\pi}{12} = 2 - \sqrt{3}$.



A water cup is in the shape of a cone with its axis vertical. The diagrams show the cup and its cross-section. The vertical angle of the cone is $\frac{\pi}{6}$ radians. The depth of water in the cup is h . The surface of the water is a circle of radius r .

(i) Find an expression for r in terms of h and show that the volume of water in the cup is given by

$$V = \frac{\pi(7 - 4\sqrt{3})h^3}{3}. \quad [4]$$

- (ii) Water is poured into the cup at a rate of $30 \text{ cm}^3 \text{ s}^{-1}$. Find, correct to 2 decimal places, the rate at which the depth of water is increasing when $h = 5$. [4]

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