

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

MARK SCHEME for the October/November 2015 series

4037 ADDITIONAL MATHEMATICS

4037/23

Paper 2, maximum raw mark 80

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the October/November 2015 series for most Cambridge IGCSE[®], Cambridge International A and AS Level components and some Cambridge O Level components.

® IGCSE is the registered trademark of Cambridge International Examinations.

Page 2	Mark Scheme	Syllabus	Paper
	Cambridge O Level – October/November 2015	4037	23

Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

1	$y = x^3 + 3x^2 - 5x - 7$ $\frac{dy}{dx} = 3x^2 + 6x - 5$ $x = 2 \rightarrow \frac{dy}{dx} = 19$ $y = 3$ eqn of tangent: $\frac{y-3}{x-2} = 19 \rightarrow (y = 19x - 35)$	M1 A1 A1FT B1 A1FT	Differentiate on <i>their</i> $\frac{dy}{dx}$
2	$2x + k + 2 = 2x^2 + (k + 2)x + 8$ $2x^2 + kx + 6 - k = 0$ $b^2 - 4ac = k^2 - 4 \times 2(6 - k)$ $k^2 + 8k - 48 > 0$ $(k + 12)(k - 4) > 0$ $k < -12$ or $k > 4$	M1 A1 M1 DM1 A1 A1	eliminate y or x correct quadratic use discriminant attempt to solve 3 term quadratic $k = -12$ and $k = 4$
3 (a)	$\frac{dy}{dx} = \frac{(2 - x^2)3x^2 - x^3(-2x)}{(2 - x^2)^2} = \left(\frac{6x^2 - x^4}{(2 - x^2)^2} \right)$	M1 A2,1,0	For quotient rule (or product rule on correct y)
(b)	$\frac{dy}{dx} = x \times \frac{1}{2}(4x + 6)^{-0.5} \times 4 + (4x + 6)^{0.5}$ $= \frac{6(x + 1)}{(4x + 6)^{0.5}} \rightarrow k = 6$	M1 A1 A1	product rule
4	$x(4 - \sqrt{3}) = 13$ $x = \frac{13(4 + \sqrt{3})}{(4 - \sqrt{3})(4 + \sqrt{3})}$ $= 4 + \sqrt{3}$ $y = 1 - 2\sqrt{3}$	M1 A1 M1 A1 A1	eliminate y or x simplified rationalisation

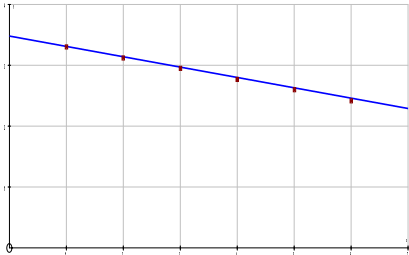
Page 3	Mark Scheme	Syllabus	Paper
	Cambridge O Level – October/November 2015	4037	23

5	$(x-3)(x-3)(x-1) = 0$ $x^3 - 7x^2 + 15x - 9 = 0$ $a = -7$ $b = 15$ $c = -9$	M1 A1 A1 A1	AG for c
6	$\log_x 2 = \frac{\log_2 2}{\log_2 x}$ $2 \log_2 x = \log_2 x^2$ $3 = \log_2 8$ $8x^2 - 29x + 15 (=0)$ $\rightarrow (8x-5)(x-3) (=0)$ $x = \frac{5}{8}$ or $x = 3$	B1 B1 B1 M1 A1	obtain quadratic and attempt to solve
7 (i)	$a = -\frac{20}{(t+2)^3}$ $t = 3 \rightarrow a = -0.16 \text{ m/s}^2$	M1 A1 A1FT	$k(t+2)^{-3}$ oe $k = -20$
(ii)	$\frac{10}{(t+2)^2}$ is never zero.	B1	
(iii)	$s = -\frac{10}{t+2} + 5$	M1 A1 A1	integrate $\frac{k}{t+2}$ $k = -10$ +5
(iv)	$s = \left[-\frac{10}{t+2} \right]_3^8 = -1 + 2$ = 1	M1 A1	insert limits and subtract

Page 4	Mark Scheme	Syllabus	Paper
	Cambridge O Level – October/November 2015	4037	23

8	(i)	$\sec^2 x + \operatorname{cosec}^2 x = \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}$	B1	
		$= \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}$	B1	add fractions
		$= \frac{1}{\sin^2 x \cos^2 x}$	B1	use of $\sin^2 x + \cos^2 x = 1$
		$= \sec^2 x \operatorname{cosec}^2 x$	B1	fully correct solution
	(ii)	$\frac{1}{\cos^2 x \sin^2 x} = 4 \frac{\sin^2 x}{\cos^2 x}$	M1	
		$\rightarrow 4 \sin^2 x = 1$	A1	correct simplified equation
		$\sin x = \pm \frac{1}{\sqrt{2}}$		
		$x = 135^\circ, 225^\circ$	A1, A1	
9	(i)	$f(x) = 3x^2 + 12x + 2 = 3(x+2)^2 - 10$	B1	
		$a = 3$	B1	
		$b = 2$	B1	
		$c = -10$		
	(ii)	minimum $f(x) = -10$	B1FT	
		at $x = -2$	B1FT	
	(iii)	$f\left(\frac{1}{y}\right) = 0 \rightarrow \left(\frac{1}{y}\right) = (\pm)\sqrt{\frac{10}{3}} - 2$	M1	obtain explicit expression for $\frac{1}{y}$ or y
		$y = -5.74, -0.26$	A1, A1	

Page 5	Mark Scheme	Syllabus	Paper
	Cambridge O Level – October/November 2015	4037	23

10	(i) $\frac{d}{dx}(e^{2-x^2}) = -2xe^{2-x^2}$ (ii) $-\frac{3e^{2-x^2}}{2} + c$ (iii) $\left[-\frac{3e^{2-x^2}}{2} \right]_1^{\sqrt{2}} = -\frac{3}{2} + \frac{3}{2}e$ 2.58 (iv) $y = 3xe^{2-x^2}$ $\frac{dy}{dx} = 3x(-2xe^{2-x^2}) + 3e^{2-x^2}$ $\frac{dy}{dx} = 0 \rightarrow x = \pm \frac{1}{\sqrt{2}} = \pm 0.707$ $y = \pm \frac{3}{\sqrt{2}}e^{1.5} = \pm 9.51$	B1 M1 A1FT M1 A1 M1 A1 A1 A1	$k = -2$ De^{2-x^2} $D = \frac{-3}{2}$ or $\frac{3}{k}$ insert limits on <i>their</i> (ii) and subtract product rule both x or a pair both y																					
11	(i) $\log N = \log A - t \log b$ (ii) <table border="1"><tr><td>t</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>$\log N$</td><td>3.30</td><td>3.11</td><td>2.95</td><td>2.77</td><td>2.60</td><td>2.41</td></tr><tr><td>$\ln N$</td><td>7.60</td><td>7.17</td><td>6.79</td><td>6.38</td><td>5.98</td><td>5.56</td></tr></table>  (iii) gradient $= -\log b = \frac{2.415 - 3.3}{5} \rightarrow b = 1.5$ intercept $= \log A = 3.47 \rightarrow A = 2950$ (iv) $t = 10 \rightarrow N = \frac{2950}{1.5^{10}} = 51$ (v) $N = 10 \rightarrow 1.5^t = 295 \rightarrow t = \frac{\log 295}{\log 1.5} = 14 \text{ years}$	t	1	2	3	4	5	6	$\log N$	3.30	3.11	2.95	2.77	2.60	2.41	$\ln N$	7.60	7.17	6.79	6.38	5.98	5.56	B1 M1 M1 A1 DM1 DM1 A1 B1 M1 A1	find logs of N plot $\log N$ or $\ln N$ against t or $-t$ straight line passing through five points set gradient $= -\log b$ and solve set intercept $= \log A$ and solve both values correct substitute $N = 10$, <i>their</i> A, b into given or transformed equation
t	1	2	3	4	5	6																		
$\log N$	3.30	3.11	2.95	2.77	2.60	2.41																		
$\ln N$	7.60	7.17	6.79	6.38	5.98	5.56																		

Page 6	Mark Scheme	Syllabus	Paper
	Cambridge O Level – October/November 2015	4037	23

12	$v_p = \begin{pmatrix} 250 \cos 20^\circ \\ 250 \sin 20^\circ \end{pmatrix}, v_r = \begin{pmatrix} V \cos 30^\circ \\ V \sin 30^\circ \end{pmatrix}, v_w = \begin{pmatrix} 0 \\ w \end{pmatrix}$	B1	
	$v_r = v_p + v_w$ $\begin{pmatrix} V \cos 30^\circ \\ V \sin 30^\circ \end{pmatrix} = \begin{pmatrix} 250 \cos 20^\circ \\ 250 \sin 20^\circ \end{pmatrix} + \begin{pmatrix} 0 \\ w \end{pmatrix}$		
	$V = \frac{250 \cos 20^\circ}{\cos 30^\circ}$ $= 271 \text{ km/hr}$	M1 A1	equate x components and solve
	$w = V \sin 30^\circ - 250 \sin 20^\circ$ $= 50.1 \text{ km/hr}$	M1 A1	equate y components and solve
	OR triangle with sides 250 V w opposite angles 60° 110° 10°	B1	
	sine rule: $\frac{w}{\sin 10^\circ} = \frac{250}{\sin 60^\circ}$ $w = 50.1 \text{ km/hr}$ $\frac{V}{\sin 110^\circ} = \frac{250}{\sin 60^\circ}$ $V = 271 \text{ km/hr}$	M1 A1 M1 A1	apply to correct triangle and solve apply to correct triangle and solve