

Cambridge O Level

CANDIDATE
NAME

CENTRE
NUMBER

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CANDIDATE
NUMBER

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ADDITIONAL MATHEMATICS

4037/12

Paper 1

October/November 2023

2 hours

You must answer on the question paper.

No additional materials are needed.

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 80.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.

Mathematical Formulae**1. ALGEBRA***Quadratic Equation*

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + b^n$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$

Arithmetic series $u_n = a + (n-1)d$

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series $u_n = ar^{n-1}$

$$S_n = \frac{a(1-r^n)}{1-r} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r} \quad (|r| < 1)$$

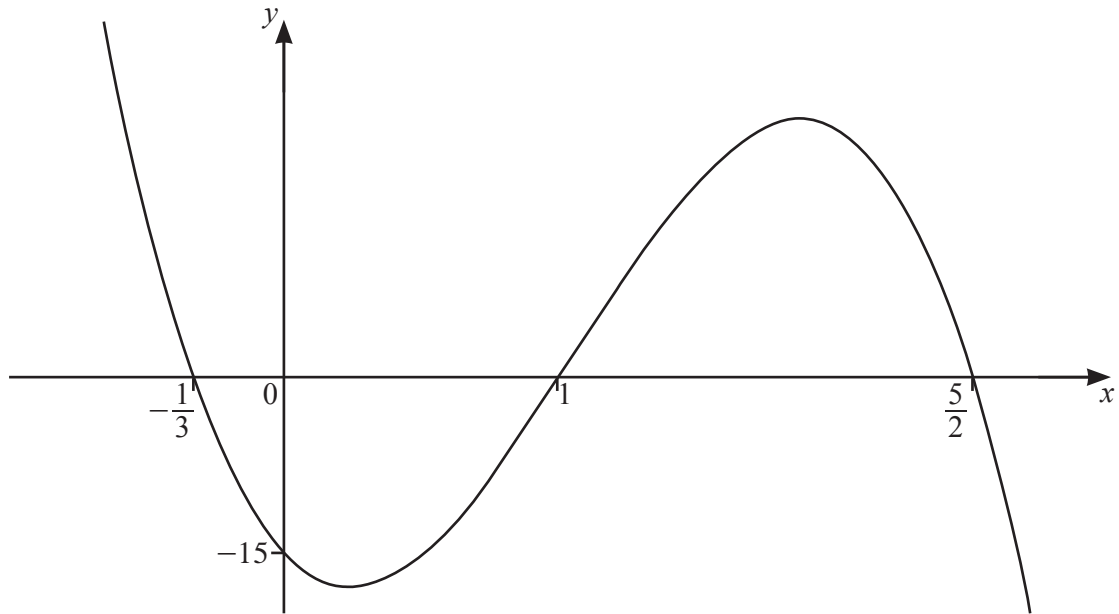
2. TRIGONOMETRY*Identities*

$$\begin{aligned}\sin^2 A + \cos^2 A &= 1 \\ \sec^2 A &= 1 + \tan^2 A \\ \operatorname{cosec}^2 A &= 1 + \cot^2 A\end{aligned}$$

Formulae for $\triangle ABC$

$$\begin{aligned}\frac{a}{\sin A} &= \frac{b}{\sin B} = \frac{c}{\sin C} \\ a^2 &= b^2 + c^2 - 2bc \cos A \\ \Delta &= \frac{1}{2}bc \sin A\end{aligned}$$

1



The diagram shows the graph of the cubic polynomial $y = f(x)$.

- (a) Find an expression for $f(x)$ in factorised form. Write each linear factor with its coefficients as integers. [3]

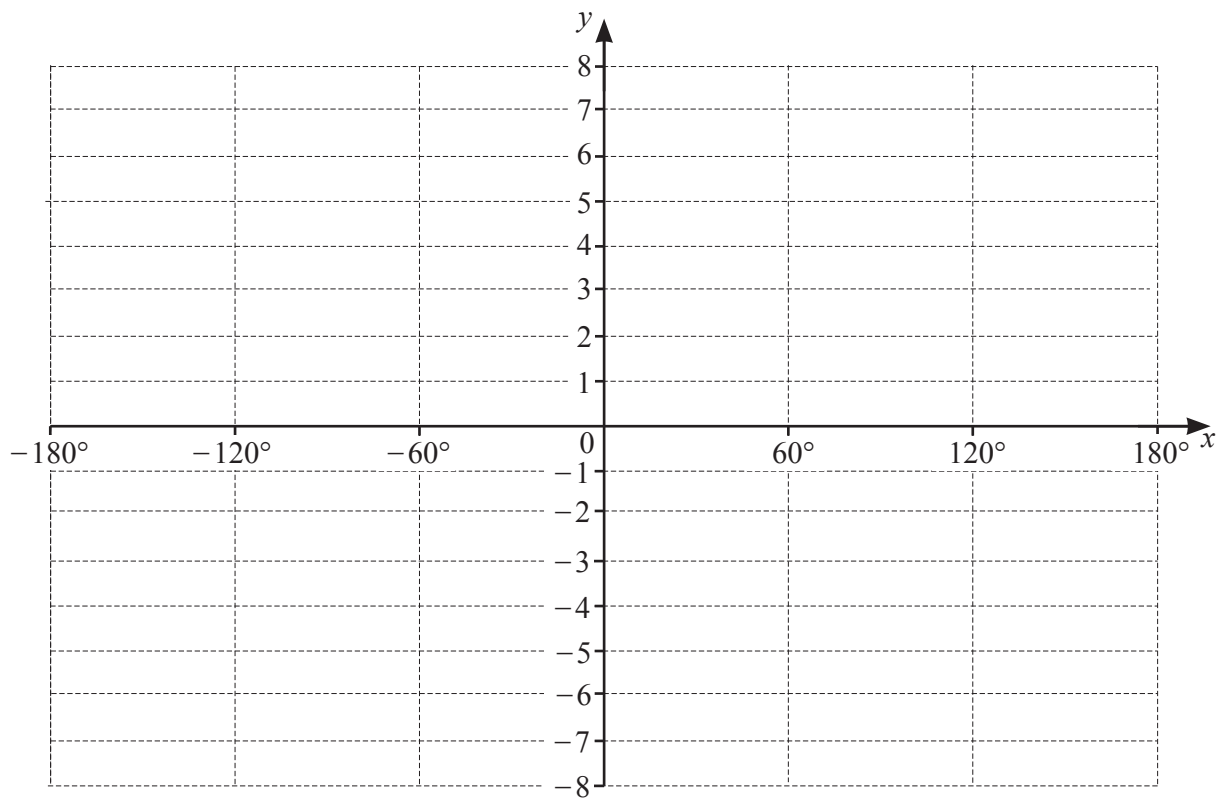
- (b) Write down the values of x such that $f(x) < 0$. [2]

2 The function g is defined by $g(x) = 5 \sin \frac{3x}{4} - 2$ for all values of x .

(a) Write down the amplitude of g . [1]

(b) Write down the period of g in degrees. [1]

(c) On the axes, sketch the graph of $y = g(x)$, for $-180^\circ \leq x \leq 180^\circ$. [3]



- 3 When $\ln(y+2)$ is plotted against x^2 a straight line graph is obtained. The line passes through the points (2.25, 9.37) and (4.75, 3.92). Find y in terms of x . [5]

- 4 (a) It is given that the first four terms, in ascending powers of x , in the expansion of $\left(1 - \frac{x}{2}\right)^n$ can be written in the form $1 - 8x + px^2 + qx^3$, where n, p and q are integers. Find the values of n, p and q .
[5]

- (b) Find the term independent of x in the expansion of $\left(\frac{2}{x^2} + \frac{x}{3}\right)^6$, giving your answer as a rational number.
[2]

- 5 Solve the equation $3 \sec^2\left(2\theta + \frac{\pi}{6}\right) = 4$ for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, giving your answers in terms of π . [5]

- 6 The polynomial $p(x)$ is such that $p(x) = ax^3 + bx^2 + cx - 5$, where a , b and c are integers. It is given that $p'(0) = 12$. It is also given that $p(x)$ has a factor of $3x - 1$ and a remainder of 95 when divided by $x - 2$.

(a) Find the values of a , b and c .

[7]

(b) Show that the equation $p(x) = 0$ has only one real root.

[3]

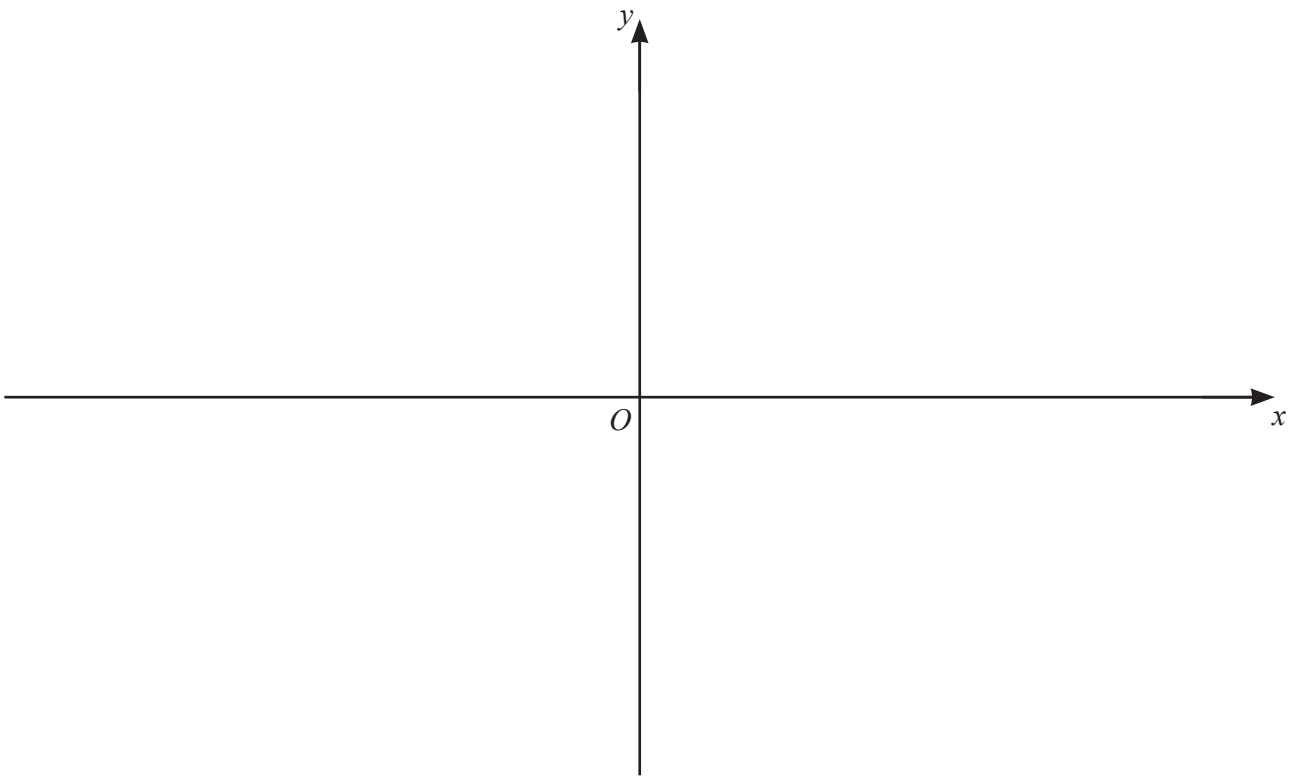
- 7 (a) A 6-digit number is to be formed using the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Each digit can be used only once in any 6-digit number. A 6-digit number cannot start with 0.
- (i) Find how many 6-digit numbers can be formed. [1]
- (ii) Find how many of these 6-digit numbers are divisible by 5. [3]
- (b) A committee of 7 people is to be chosen from 6 doctors, 10 nurses and 8 dentists.
- (i) Find the number of committees that can be chosen. [1]
- (ii) Find the number of committees that can be chosen if all the doctors have to be on the committee. [1]
- (iii) Find the number of committees that can be chosen if there has to be at least one dentist on the committee. [2]

8 (a) It is given that $f : x \rightarrow (3x+1)^2 - 4$ for $x \geq a$, and that f^{-1} exists.

(i) Find the least possible value of a . [1]

(ii) Using this value of a , write down the range of f . [1]

(iii) Using this value of a , sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the axes, stating the intercepts with the coordinate axes. [4]



(b) It is given that $g(x) = \ln(2x^2 + 5)$ for $x \geq 0$,

$$h(x) = 3x - 2 \text{ for } x \geq 0.$$

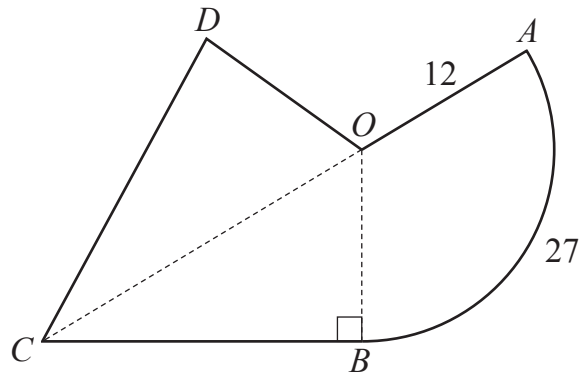
Solve the equation $hg(x) = 4$ giving your answer in exact form.

[3]

9 Solve the equation $12x^{\frac{2}{3}} - 5x^{-\frac{2}{3}} - 11 = 0$ for $x > 0$. Give your answer correct to one decimal place.

[4]

10 In this question all lengths are in centimetres and all angles are in radians.



The diagram shows a badge which consists of a minor sector, OAB , of the circle with centre O and radius 12, and a kite $OBCD$, where $OB = OD$ and $CD = CB$. The arc AB has length 27. The line OB is perpendicular to the line CB , and COA is a straight line.

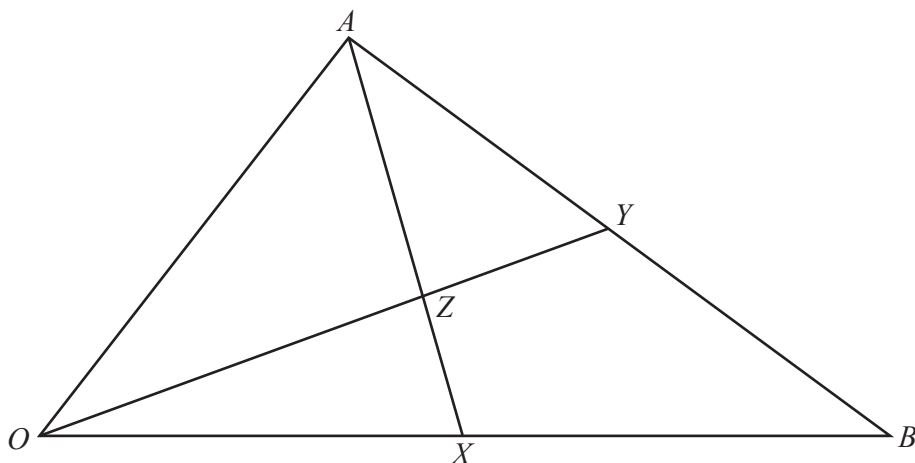
(a) Find the perimeter of the badge.

[4]

(b) Find the area of the badge.

[3]

11



In the triangle OAB , $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The mid-point of the line OB is X , and the mid-point of the line AB is Y . The lines OY and AX intersect at the point Z . It is given that $\overrightarrow{AZ} = \lambda \overrightarrow{AX}$ and $\overrightarrow{OZ} = \mu \overrightarrow{OY}$ where λ and μ are rational numbers.

(a) Find $\overrightarrow{OZ}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and λ . [3]

(b) Find $\overrightarrow{OZ}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and μ . [2]

(c) Find the values of λ and μ .

[3]

(d) Hence find $\overrightarrow{OZ}$ in terms of **a** and **b** only.

[1]

Question 12 is printed on the next page.

12 A curve has equation $y = \frac{\sqrt{5x-2}}{x-3}$.

(a) Explain why the curve does not exist when $x < \frac{2}{5}$. [1]

(b) Show that $\frac{dy}{dx}$ can be written in the form $\frac{-(Ax+B)}{2(x-3)^2\sqrt{5x-2}}$, where A and B are positive integers. [5]

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