



CANDIDATE
NAME

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CENTRE
NUMBER

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CANDIDATE
NUMBER

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9231/21

May/June 2020

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Blank pages are indicated.

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1 Find the solution of the differential equation

$$\frac{dy}{dx} + 5y = e^{-7x}$$

for which $y = 0$ when $x = 0$. Give your answer in the form $y = f(x)$.

[6]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

2 It is given that $y = 2^x$.

(a) By differentiating $\ln y$ with respect to x , show that $\frac{dy}{dx} = 2^x \ln 2$. [3]

[illegible]

(b) Write down $\frac{d^2y}{dx^2}$. [1]

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(c) Hence find the first three terms in the Maclaurin's series for 2^x . [3]

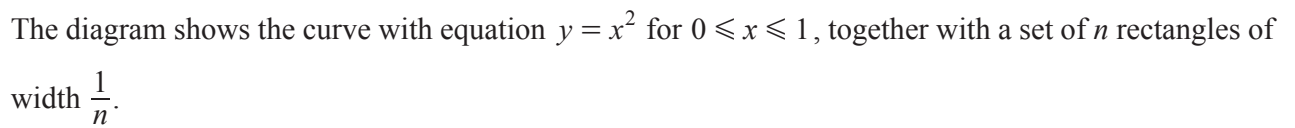
This image shows a full page of primary-ruled handwriting practice paper. It contains ten identical horizontal rows. Each row is defined by three parallel dashed lines: a solid top line, a dashed midline, and a solid bottom line. The rows are evenly spaced across the entire page, leaving margins at the top and bottom. There is no text or other markings on the paper.

- 3 (a) Find the roots of the equation $z^3 = -1 - i$, giving your answers in the form $re^{i\theta}$, where $r > 0$ and $0 \leq \theta < 2\pi$. [5]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Let $w = z_1^{3k} + z_2^{3k} + z_3^{3k}$, where k is a positive integer and z_1, z_2, z_3 are the roots of $z^3 = -1 - i$.

- (b) Express w in the form $Re^{i\alpha}$, where $R > 0$, giving R and α in terms of k . [3]



- $$\int_0^1 x^2 dx < \frac{2n^2 + 3n + 1}{6n^2}. \quad [4]$$

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (b) Use a similar method to find, in terms of n , a lower bound for $\int_0^1 x^2 \, dx$. [4]

5 The curves $C_1 : y = \cosh x$ and $C_2 : y = \sinh 2x$ intersect at the point where $x = a$.

(a) Find the exact value of a , giving your answer in logarithmic form.

[4]

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(b) Sketch C_1 and C_2 on the same diagram.

[2]

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6 The integral I_n , where n is an integer, is defined by $I_n = \int_0^{\frac{1}{2}} (1-x^2)^{-\frac{1}{2}n} dx$.

(a) Find the exact value of I_1 .

[2]

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(b) By considering $\frac{d}{dx} \left(x(1-x^2)^{-\frac{1}{2}n} \right)$, or otherwise, show that

$$nI_{n+2} = 2^{n-1}3^{-\frac{1}{2}n} + (n-1)I_n.$$

[5]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

[illegible]

7 It is given that $x = t^3y$ and

$$t^3 \frac{d^2 y}{dt^2} + (4t^3 + 6t^2) \frac{dy}{dt} + (13t^3 + 12t^2 + 6t)y = 61e^{\frac{1}{2}t}.$$

(a) Show that

$$\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 13x = 61e^{\frac{1}{2}t}. \quad [4]$$

[illegible]

[illegible]

- 8 (a)** Find the values of a for which the system of equations

$$3x + y + z = 0,$$

$$ax + 6y - z = 0,$$

$$ay - 2z = 0,$$

does not have a unique solution.

[3]

[illegible]

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 3 & 1 & 1 \\ 0 & 6 & -1 \\ 0 & 0 & -2 \end{pmatrix}.$$

- (b)** Use the characteristic equation of $\mathbf{A}$ to find the inverse of $\mathbf{A}^2$.

[4]

[illegible]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

[illegible]

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