



CANDIDATE
NAME

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CENTRE
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CANDIDATE
NUMBER

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9231/12

May/June 2021

2 hours

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

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- 1 Prove by mathematical induction that $2^{4n} + 31^n - 2$ is divisible by 15 for all positive integers n . [6]

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- 2 (a) Use standard results from the List of formulae (MF19) to find $\sum_{r=1}^n (1-r-r^2)$ in terms of n , simplifying your answer. [3]

[illegible]

(b) Show that

$$\frac{1-r-r^2}{(r^2+2r+2)(r^2+1)} = \frac{r+1}{(r+1)^2+1} - \frac{r}{r^2+1}$$

and hence use the method of differences to find $\sum_{r=1}^n \frac{1-r-r^2}{(r^2+2r+2)(r^2+1)}$. [5]

(c) Deduce the value of $\sum_{r=1}^{\infty} \frac{1-r-r^2}{(r^2+2r+2)(r^2+1)}$. [1]

3 The equation $x^4 - 2x^3 - 1 = 0$ has roots $\alpha, \beta, \gamma, \delta$.

(a) Find a quartic equation whose roots are α^3 , β^3 , γ^3 , δ^3 and state the value of $\alpha^3 + \beta^3 + \gamma^3 + \delta^3$.
[4]

[illegible]

- (b)** Find the value of $\frac{1}{\alpha^3} + \frac{1}{\beta^3} + \frac{1}{\gamma^3} + \frac{1}{\delta^3}$. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (c) Find the value of $\alpha^4 + \beta^4 + \gamma^4 + \delta^4$. [2]

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- 4 The matrix $\mathbf{M}$ represents the sequence of two transformations in the x - y plane given by a rotation of 60° anticlockwise about the origin followed by a one-way stretch in the x -direction, scale factor d ($d \neq 0$).

(a) Find $\mathbf{M}$ in terms of d .

[4]

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(b) The unit square in the x - y plane is transformed by $\mathbf{M}$ onto a parallelogram of area $\frac{1}{2}d^2$ units².

Show that $d = 2$.

[2]

This image shows a full page of white paper with ten horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and extend across the entire width of the page. There is no text or other markings on the paper.

The matrix $\mathbf{N}$ is such that $\mathbf{MN} = \begin{pmatrix} 1 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$.

- (c) Find N . [3]

[illegible]

- (d) Find the equations of the invariant lines, through the origin, of the transformation in the x - y plane represented by **MN**. [5]

[illegible]

- 5 The curve C has polar equation $r = a \cot\left(\frac{1}{3}\pi - \theta\right)$, where a is a positive constant and $0 \leq \theta \leq \frac{1}{6}\pi$.

It is given that the greatest distance of a point on C from the pole is $2\sqrt{3}$.

- (a) Sketch C and show that $a = 2$.

[3]

- (b) Find the exact value of the area of the region bounded by C , the initial line and the half-line $\theta = \frac{1}{6}\pi$. [4]

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- (c) Show that C has Cartesian equation $2(x + y\sqrt{3}) = (x\sqrt{3} - y)\sqrt{x^2 + y^2}$. [3]

[illegible][illegible]

7 The curve C has equation $y = \frac{x^2 + x + 9}{x + 1}$.

(a) Find the equations of the asymptotes of C .

[3]

[illegible]

(b) Find the coordinates of the stationary points on C .

[4]

[illegible]

(c) Sketch C , stating the coordinates of any intersections with the axes.

[3]

.....

(d) Sketch the curve with equation $y = \left| \frac{x^2 + x + 9}{x + 1} \right|$ and find the set of values of x for which $2|x^2 + x + 9| > 13|x + 1|$.

[5]

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