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9231/13

May/June 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

1 (a) Show that

$$\tan(r+1) - \tan r = \frac{\sin 1}{\cos(r+1)\cos r}. \quad [2]$$

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Let $u_r = \frac{1}{\cos(r+1)\cos r}$.

(b) Use the method of differences to find $\sum_{r=1}^n u_r$. [3]

[illegible]

- (c) Explain why the infinite series $u_1 + u_2 + u_3 + \dots$ does not converge. [1]

2 The cubic equation $2x^3 - 4x^2 + 3 = 0$ has roots α, β, γ . Let $S_n = \alpha^n + \beta^n + \gamma^n$.

(a) State the value of S_1 and find the value of S_2 . [3]

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(b) (i) Express S_{n+3} in terms of S_{n+2} and S_n . [1]

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(ii) Hence, or otherwise, find the value of S_4 . [2]

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- (c) Use the substitution $y = S_1 - x$, where S_1 is the numerical value found in part (a), to find and simplify an equation whose roots are $\alpha + \beta$, $\beta + \gamma$, $\gamma + \alpha$. [3]

This image shows a full page of white paper with horizontal blue or grey ruling lines. The lines are evenly spaced and run across the width of the page, providing a template for handwriting practice. There are no margins, text, or other markings on the page.

- (d) Find the value of $\frac{1}{\alpha+\beta} + \frac{1}{\beta+\gamma} + \frac{1}{\gamma+\alpha}$. [2]

[illegible]

3 (a) Prove by mathematical induction that, for all positive integers n ,

$$\sum_{r=1}^n (5r^4 + r^2) = \frac{1}{2}n^2(n+1)^2(2n+1). \quad [6]$$

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- (b)** Use the result given in part **(a)** together with the List of formulae (MF19) to find $\sum_{r=1}^n r^4$ in terms of n , fully factorising your answer. [3]

[illegible]

4 The matrices **A**, **B** and **C** are given by

$$\mathbf{A} = \begin{pmatrix} 2 & k & k \\ 5 & -1 & 3 \\ 1 & 0 & 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } \quad \mathbf{C} = \begin{pmatrix} 0 & 1 & 1 \\ -1 & 2 & 0 \end{pmatrix},$$

where k is a real constant.

(a) Find $\mathbf{CAB}$.

[3]

[illegible]

(b) Given that $\mathbf{A}$ is singular, find the value of k .

[3]

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- (c) Using the value of k from part (b), find the equations of the invariant lines, through the origin, of the transformation in the x - y plane represented by **CAB**. [5]

[illegible]

- 5** The curve C has polar equation $r = \frac{1}{\pi - \theta} - \frac{1}{\pi}$, where $0 \leq \theta \leq \frac{1}{2}\pi$.

(a) Sketch C .

[3]

- (b)** Show that the area of the region bounded by the half-line $\theta = \frac{1}{2}\pi$ and C is $\frac{3-4\ln 2}{4\pi}$. [6]

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- 6** The lines l_1 and l_2 have equations $\mathbf{r} = -\mathbf{i} - 2\mathbf{j} + \mathbf{k} + s(2\mathbf{i} - 3\mathbf{j})$ and $\mathbf{r} = 3\mathbf{i} - 2\mathbf{k} + t(3\mathbf{i} - \mathbf{j} + 3\mathbf{k})$ respectively.

The plane Π_1 contains l_1 and the point P with position vector $-2\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$.

- (a) Find an equation of Π_1 , giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b} + \mu\mathbf{c}$. [2]

[illegible]

The plane Π_2 contains l_2 and is parallel to l_1 .

- (b) Find an equation of Π_2 , giving your answer in the form $ax + by + cz = d$. [4]

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This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

[illegible]

7 The curve C has equation $y = \frac{x^2 - x - 3}{1 + x - x^2}$.

(a) Find the equations of the asymptotes of C .

[2]

[illegible]

(b) Find the coordinates of any stationary points on C .

[3]

[illegible]

- (c) Sketch C , stating the coordinates of the intersections with the axes.

[3]

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- (d) Sketch the curve with equation $y = \left| \frac{x^2 - x - 3}{1 + x - x^2} \right|$ and find in exact form the set of values of x for which $\left| \frac{x^2 - x - 3}{1 + x - x^2} \right| < 3$.

[6]

Additional Page

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