



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/43

Paper 4 Further Probability & Statistics

May/June 2021

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages.

- 1** Farmer A grows apples of a certain variety. Each tree produces 14.8 kg of apples, on average, per year. Farmer B grows apples of the same variety and claims that his apple trees produce a higher mass of apples per year than Farmer A 's trees. The masses of apples from Farmer B 's trees may be assumed to be normally distributed.

A random sample of 10 trees from Farmer *B* is chosen. The masses, x kg, of apples produced in a year are summarised as follows.

$$\Sigma x = 152.0 \quad \Sigma x^2 = 2313.0$$

Test, at the 5% significance level, whether Farmer B 's claim is justified. [6]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 2 A company is developing a new flavour of chocolate by varying the quantities of the ingredients. A random selection of 9 flavours of chocolate are judged by two tasters who each give marks out of 100 to each flavour of chocolate.

Chocolate	A	B	C	D	E	F	G	H	I
Taster 1	72	86	75	92	98	79	87	60	62
Taster 2	84	72	74	95	85	87	82	75	68

Carry out a Wilcoxon matched-pairs signed-rank test at the 10% significance level to investigate whether, on average, there is a difference between marks awarded by the two tasters. [7]

[illegible]

- 3 The heights, x m, of a random sample of 50 adult males from country A were recorded. The heights, y m, of a random sample of 40 adult males from country B were also recorded. The results are summarised as follows.

$$\Sigma x = 89.0 \quad \Sigma x^2 = 159.4 \quad \Sigma y = 67.2 \quad \Sigma y^2 = 113.1$$

Find a 95% confidence interval for the difference between the mean heights of adult males from country A and adult males from country B . [8]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 4 X is a discrete random variable which takes the values $0, 2, 4, \dots$. The probability generating function of X is given by

$$G_X(t) = \frac{1}{3-2t^2}.$$

- (a) Find $E(X)$ and $\text{Var}(X)$. [5]

[illegible]

(b) Find $P(X = 4)$.

[3]

[illegible]

- 5 Chai packs china mugs into cardboard boxes. Chai's manager suspects that breakages occur at random times and that the number of breakages may follow a Poisson distribution. He takes a small sample of observations and finds that the number of breakages in a one-hour period has a mean of 2.4 and a standard deviation of 1.5.

(a) Explain how this information tends to support the manager's suspicion. [2]

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The manager now takes a larger sample and claims that the numbers of breakages in a one-hour period follow a Poisson distribution. The numbers of breakages in a random sample of 180 one-hour periods are summarised in the following table.

Number of breakages	0	1	2	3	4	5	6	7 or more
Frequency	21	33	46	31	23	16	10	0

The mean number of breakages calculated from this sample is 2.5.

(b) Use the data from this larger sample to carry out a goodness of fit test, at the 10% significance level, to test the claim. [8]

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- 6 The continuous random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{8} & 0 \leq x < 1, \\ \frac{1}{28}(8-x) & 1 \leq x \leq 8, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Find the cumulative distribution function of X . [3]

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- (b) Find the value of the constant a such that $P(X \leq a) = \frac{5}{7}$. [3]

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The random variable Y is given by $Y = \sqrt[3]{X}$.

- (c) Find the probability density function of Y . [5]

[illegible]

[illegible]

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