



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/22

Paper 2 Further Pure Mathematics 2

October/November 2020

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Blank pages are indicated.



- 1 Find the Maclaurin's series for $\tan\left(x + \frac{1}{4}\pi\right)$ up to and including the term in x^2 . [5]

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- 2 A curve has equation $y = \cosh x$, for $0 \leq x \leq \frac{1}{2}$.

Find, in terms of π and e, the area of the surface generated when the curve is rotated through 2π radians about the x -axis. [6]

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- 3 Find all the roots of the equation $(w+1)^6 = 1$, giving your answers in the form $x+iy$ where x and y are real and exact. [4]

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4 Find the solution of the differential equation

$$x \frac{dy}{dx} + 2y = e^x$$

for which $y = 3$ when $x = 1$. Give your answer in the form $y = f(x)$. [8]

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5 The curve C has equation

$$y^2 + (xy + 1)^2 = 5.$$

- (a) Show that, at the point $(1, 1)$ on C , $\frac{dy}{dx} = -\frac{2}{3}$. [3]

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- (b) Find the value of $\frac{d^2y}{dx^2}$ at the point $(1, 1)$. [5]

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6 Find the particular solution of the differential equation

$$\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 15x = 102 \cos 3t,$$

given that, when $t = 0$, $x = 1$ and $\frac{dx}{dt} = 0$. [11]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

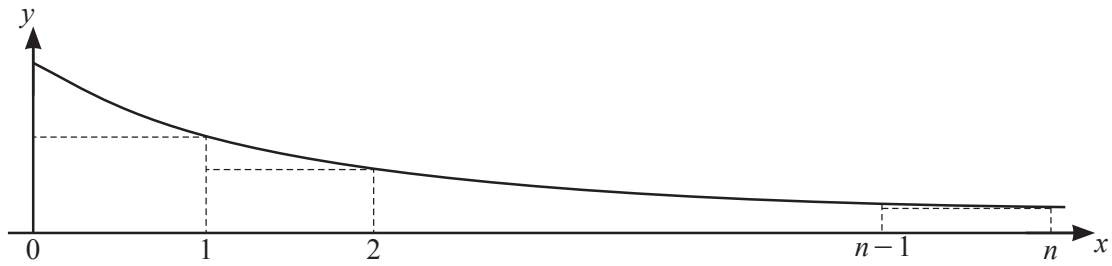
7 (a) Show that $\sum_{r=1}^n z^{2r} = \frac{z^{2n+1} - z}{z - z^{-1}}$, for $z \neq 0, 1, -1$. [2]

This image shows a full page of a document template. It consists of approximately 28 horizontal dotted lines spaced evenly down the page, providing a guide for handwriting or typing. The background is plain white, and there are no margins, headers, or footers visible.

(b) By letting $z = \cos \theta + i \sin \theta$, show that, if $\sin \theta \neq 0$,

$$1 + 2 \sum_{r=1}^n \cos(2r\theta) = \frac{\sin(2n+1)\theta}{\sin \theta}. \quad [5]$$

[illegible]



The diagram shows the curve $y = \frac{1}{\sqrt{x^2 + x + 1}}$ for $x \geq 0$, together with a set of n rectangles of unit width. By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^n \frac{1}{\sqrt{r^2+r+1}} < \ln\left(\frac{1}{3} + \frac{2}{3}n + \frac{2}{3}\sqrt{n^2+n+1}\right). \quad [10]$$

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- 9** It is given that a is a positive constant.

- (a)** Show that the system of equations

$$ax + (2a + 5)y + (a + 1)z = 1,$$

$$-4y = 2,$$

$$3y - z = 3,$$

has a unique solution and interpret this situation geometrically. [3]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} a & 2a+5 & a+1 \\ 0 & -4 & 0 \\ 0 & 3 & -1 \end{pmatrix}.$$

- (b) Show that the eigenvalues of $\mathbf{A}$ are a , -1 and -4 . [2]

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- (c) Find a matrix $\mathbf{P}$ such that

$$\mathbf{A} = \mathbf{P} \begin{pmatrix} a & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -4 \end{pmatrix} \mathbf{P}^{-1}. \quad [5]$$

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This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

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