



Cambridge International AS & A Level

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

FURTHER MATHEMATICS

9231/23

Paper 2 Further Pure Mathematics 2

October/November 2020

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Blank pages are indicated.



- 1 (a)** By differentiating e^{-x^2} , find the Maclaurin's series for e^{-x^2} up to and including the term in x^2 . [5]

[illegible]

- (b) Deduce an approximation to $\int_0^{\frac{1}{5}} e^{-x^2} dx$, giving your answer as a rational fraction in its lowest terms. [2]

[illegible]

2 The variables x and y are related by the differential equation

$$9\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + y = 3x^2 + 30x.$$

(a) Find the general solution for y in terms of x .

[6]

[illegible]

(b) State an approximate solution for large positive values of x .

$$[1]$$

.....

.....

.....

.....

- 3 (a) Show that the system of equations

$$\begin{aligned}x - 2y - 4z &= 1, \\x - 2y + kz &= 1, \\-x + 2y + 2z &= 1,\end{aligned}$$

where k is a constant, does not have a unique solution. [2]

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Given that $k = -4$, show that the system of equations in part (a) is consistent. Interpret this situation geometrically. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (c) Given instead that $k = -2$, show that the system of equations in part (a) is inconsistent. Interpret this situation geometrically. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (d) For the case where $k \neq -2$ and $k \neq -4$, show that the system of equations in part (a) is inconsistent. Interpret this situation geometrically. [2]

.....

.....

.....

.....

.....

.....

.....

.....

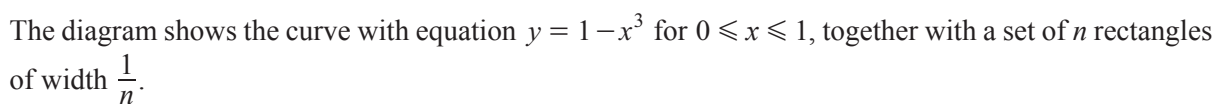
.....

.....

.....

.....

.....


$$\int_0^1 (1-x^3) dx \leq \frac{3n^2+2n-1}{4n^2}. \quad [4]$$
This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

[illegible]

5 It is given that

$$x = \sinh^{-1} t, \quad y = \cos^{-1} t,$$

where $-1 < t < 1$.

(a) By differentiating $\cos y$ with respect to t , show that $\frac{dy}{dt} = -\frac{1}{\sqrt{1-t^2}}$. [4]

[illegible]

(b) Find $\frac{d^2y}{dx^2}$ in terms of t , simplifying your answer.

[5]

[illegible]

- 6 (a)** Use de Moivre's theorem to show that $\sin^4 \theta = \frac{1}{8}(\cos 4\theta - 4 \cos 2\theta + 3)$. [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b) Find the solution of the differential equation

$$\frac{dy}{d\theta} + y \cot \theta = \sin^3 \theta$$

for which $y = 0$ when $\theta = \frac{1}{2}\pi$.

[6]

[illegible]

7 The matrix $\mathbf{P}$ is given by

$$\mathbf{P} = \begin{pmatrix} 1 & 4 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 2 \end{pmatrix}.$$

(a) State the eigenvalues of $\mathbf{P}$. [1]

.....

.....

.....

.....

.....

(b) Use the characteristic equation of $\mathbf{P}$ to find $\mathbf{P}^{-1}$. [4]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 8 (a) Sketch the graph of $y = \coth x$ for $x > 0$ and state the equations of the asymptotes. [2]

- (b) Starting from the definitions of $\coth$ and cosech in terms of exponentials, prove that

$$\coth^2 x - \operatorname{cosech}^2 x = 1. \quad [3]$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The curve C has equation $y = \ln \coth\left(\frac{1}{2}x\right)$ for $x > 0$.

- (c) Show that $\frac{dy}{dx} = -\operatorname{cosech} x$. [3]

[illegible]

- (d)** It is given that the arc length of C from $x = a$ to $x = 2a$ is $\ln 4$, where a is a positive constant.

Show that $\cosh a = 2$ and find, in logarithmic form, the exact value of a . [7]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

[illegible]

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.