

Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/32

Paper 3 Further Mechanics

October/November 2020

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

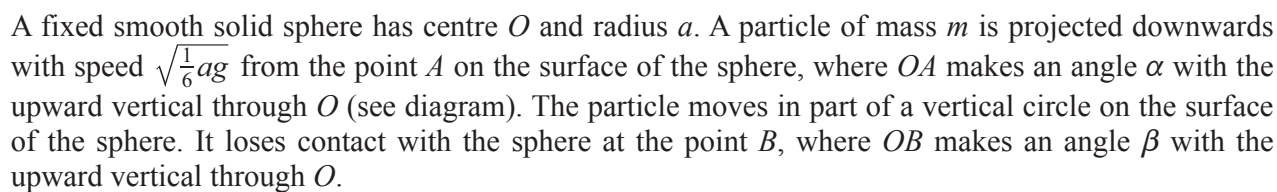
INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 ms^{-2} .

INFORMATION

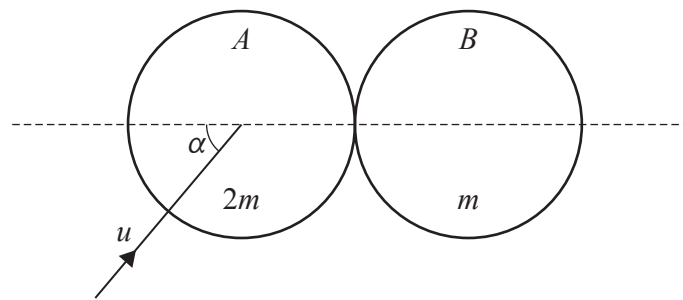
- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Blank pages are indicated.



[5]

[illegible]



Two uniform smooth spheres A and B of equal radii have masses $2m$ and m respectively. Sphere B is at rest on a smooth horizontal surface. Sphere A is moving on the surface with speed u and collides with B . Immediately before the collision, the direction of motion of A makes an angle α with the line of centres of the spheres, where $\tan \alpha = \frac{4}{3}$ (see diagram). The coefficient of restitution between the spheres is $\frac{1}{3}$.

Find the speed of A after the collision.

[5]

[illegible]

- 3** An object consists of a uniform solid circular cone, of vertical height $4r$ and radius $3r$, and a uniform solid cylinder, of height $4r$ and radius $3r$. The circular base of the cone and one of the circular faces of the cylinder are joined together so that they coincide. The cone and the cylinder are made of the same material.
- (a)** Find the distance of the centre of mass of the object from the end of the cylinder that is not attached to the cone. [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The lines are evenly spaced across the entire page, providing a guide for consistent letter formation. There is no text or other markings on the page.

- 4 A particle P of mass m is moving in a horizontal circle with angular speed ω on the smooth inner surface of a hemispherical shell of radius r . The angle between the vertical and the normal reaction of the surface on P is θ .

(a) Show that $\cos \theta = \frac{g}{\omega^2 r}$. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

The plane of the circular motion is at a height x above the lowest point of the shell. When the angular speed is doubled, the plane of the motion is at a height $4x$ above the lowest point of the shell.

(b) Find x in terms of r .

[4]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

- 5** A particle P is projected with speed $u \text{ ms}^{-1}$ at an angle of θ above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t s are denoted by x m and y m respectively.

- (a)** Starting from the equation of the trajectory given in the List of formulae (MF19), show that

$$y = x \tan \theta - \frac{gx^2}{2u^2}(1 + \tan^2 \theta). \quad [1]$$

[illegible]

When $\theta = \tan^{-1} 2$, P passes through the point with coordinates $(10, 16)$.

- (b)** Show that there is no value of θ for which P can pass through the point with coordinates $(18, 30)$. [6]

[illegible]

- Show that $k = 4mg$ and find in terms of a the greatest height above Q reached by P .

[illegible]

- 7 A particle P of mass m kg moves in a horizontal straight line against a resistive force of magnitude mkv^2 N, where v ms⁻¹ is the speed of P after it has moved a distance x m and k is a positive constant. The initial speed of P is u ms⁻¹.

(a) Show that $x = \frac{1}{k} \ln 2$ when $v = \frac{1}{2}u$. [4]

This image shows a full page of a worksheet designed for handwriting practice. It features 20 evenly spaced, horizontal dotted lines across the entire width of the page. The background is plain white, and there are no margins, text, or other markings present.

- $$\frac{1}{3k} \ln \left(\frac{A - ku^3}{B - ku^3} \right),$$

[7]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

Additional Page

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