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9231/12

October/November 2021

2 hours

You will need: List of formulae (MF19)

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

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[Turn over

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Let $\mathbf{A} = \begin{pmatrix} 3 & 4 \\ 2 & 2 \end{pmatrix}$.

- (b)** The triangle DEF in the x - y plane is transformed by $\mathbf{A}$ onto triangle PQR .

Given that the area of triangle DEF is 13 cm^2 , find the area of triangle PQR . [2]

.....

- (c) Find the matrix $\mathbf{B}$ such that $\mathbf{AB} = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}$. [2]

- (d) Show that the origin is the only invariant point of the transformation in the x - y plane represented by $\mathbf{A}$. [4]

This image shows a full page of white paper with ten horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and extend across the entire width of the page. There is no handwriting or other markings on the paper.

2 It is given that $y = xe^{ax}$, where a is a constant.

Prove by mathematical induction that, for all positive integers n ,

$$\frac{d^ny}{dx^n} = (a^n x + na^{n-1})e^{ax}. \quad [6]$$

[illegible]

3 Let $S_n = \sum_{r=1}^n \ln \frac{r(r+2)}{(r+1)^2}$.

(a) Using the method of differences, or otherwise, show that $S_n = \ln \frac{n+2}{2(n+1)}$. [4]

[illegible]

$$\text{Let } S = \sum_{r=1}^{\infty} \ln \frac{r(r+2)}{(r+1)^2}.$$

(b) Find the least value of n such that $S_n - S < 0.01$.

[3]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

4 The cubic equation $x^3 + 2x^2 + 3x + 3 = 0$ has roots α, β, γ .

(a) Find the value of $\alpha^2 + \beta^2 + \gamma^2$. [2]

[illegible]

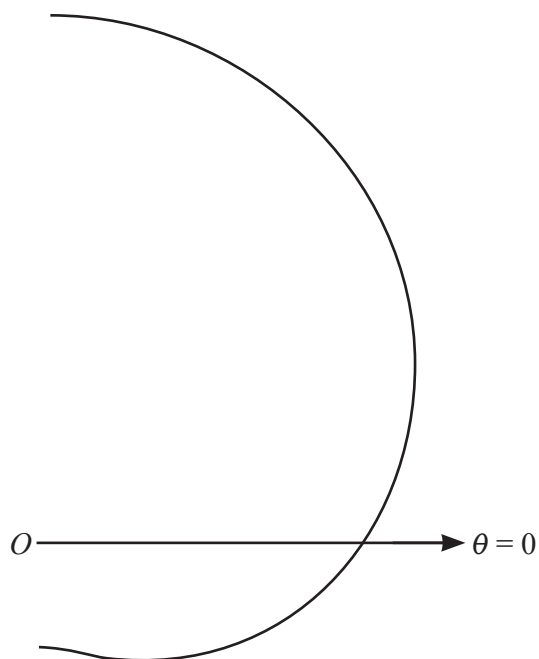
(b) Show that $\alpha^3 + \beta^3 + \gamma^3 = 1$. [2]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 5** The curve C has polar equation $r = 3 + 2 \sin \theta$, for $-\pi < \theta \leq \pi$.

- (a)** The diagram shows part of C . Sketch the rest of C on the diagram.

[1]



The straight line l has polar equation $r \sin \theta = 2$.

- (b)** Add l to the diagram in part (a) and find the polar coordinates of the points of intersection of C and l . [5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

- 6** The curve C has equation $y = \frac{x^2}{x-3}$.

(a) Find the equations of the asymptotes of C .

[3]

[illegible]

(b) Show that there is no point on C for which $0 < y < 12$.

[4]

[illegible]

(c) Sketch C .

[2]

(d) (i) Sketch the graphs of $y = \left| \frac{x^2}{x-3} \right|$ and $y = |x| - 3$ on a single diagram, stating the coordinates of the intersections with the axes. [4]

(ii) Use your sketch to find the set of values of c for which $\left| \frac{x^2}{x-3} \right| \leq |x| + c$ has no solution. [1]

.....

7 The points A, B, C have position vectors

$$2\mathbf{i} + 2\mathbf{j}, \quad -\mathbf{j} + \mathbf{k} \quad \text{and} \quad 2\mathbf{i} + \mathbf{j} - 7\mathbf{k}$$

respectively, relative to the origin O .

(a) Find an equation of the plane OAB , giving your answer in the form $\mathbf{r} \cdot \mathbf{n} = p$. [3]

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

The plane Π has equation $x - 3y - 2z = 1$.

(b) Find the perpendicular distance of Π from the origin. [1]

(d) Find an equation for the common perpendicular to the lines OC and AB . [10]

[illegible]

Additional Page

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