

Cambridge International AS & A Level

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--



FURTHER MATHEMATICS

9231/43

Paper 4 Further Probability & Statistics

October/November 2021

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

BLANK PAGE

- 1 The times taken for students at a college to run 200 m have a normal distribution with mean μ s. The times, x s, are recorded for a random sample of 10 students from the college. The results are summarised as follows, where $\bar{x}$ is the sample mean.

$$\bar{x} = 25.6 \qquad \Sigma(x - \bar{x})^2 = 78.5$$

- (a) Find a 90% confidence interval for μ . [4]

[illegible]

A test of the null hypothesis $\mu = k$ is carried out on this sample, using a 10% significance level. The test does not support the alternative hypothesis $\mu < k$.

- (b) Find the greatest possible value of k . [3]

[illegible]

$$F(x) = \begin{cases} 0 & x < -1, \\ \frac{1}{2}(1+x)^2 & -1 \leq x \leq 0, \\ 1 - \frac{1}{2}(1-x)^2 & 0 < x \leq 1, \\ 1 & x > 1. \end{cases}$$

-
- This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

-
- This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(c) Find $E(X^2)$.

[2]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(d) Find $\text{Var}(X^2)$.

[2]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

Number of unripe pears per pack	0	1	2	3	4	5	≥ 6
Observed frequency	35	48	43	15	6	3	0
Expected frequency	40.874	p	35.641	12.579	2.775	0.392	q

-
- This image shows a blank sheet of primary-ruled paper. It features ten horizontal rows, each defined by three dashed lines: a top line, a middle line, and a bottom line. The rows are evenly spaced and extend across the width of the page, providing a template for handwriting practice.

-
- This image shows a full page of white paper with ten horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and extend across the entire width of the page. There is no text or other markings on the paper.

- 4 Manet has developed a new training course to help athletes improve their time taken to run 800 m. Manet claims that his course will decrease an athlete's time by more than 2 s on average. For a random sample of 10 athletes the times taken, in seconds, before and after the course are given in the following table.

Athlete	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>	<i>E</i>	<i>F</i>	<i>G</i>	<i>H</i>	<i>I</i>	<i>J</i>
Before	150	146	131	135	126	142	130	129	137	134
After	145	138	129	135	122	135	132	128	127	137

Use a t -test, at the 5% significance level, to test whether Manet's claim is justified, stating any assumption that you make. [8]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 5 Nine balls labelled 1, 2, 3, 4, 5, 6, 7, 8, 9 are placed in a bag. Kai selects three balls at random from the bag, without replacement. The random variable X is the number of balls selected by Kai that are labelled with a multiple of 3.

(a) Find the probability generating function $G_X(t)$ of X . [3]

[illegible]

The balls are replaced in the bag.

Jacob now selects two balls at random from the bag, without replacement. The random variable Y is the number of balls selected by Jacob that are labelled with an even number.

(b) Find the probability generating function $G_Y(t)$ of Y . [2]

[illegible]

- [illegible]

- [illegible]

- 6 The blood cholesterol levels, measured in suitable units, of a random sample of 11 women and a random sample of 12 men are shown below.

Women	51	55	242	167	152	256	75	137	98	238	235	
Men	311	262	170	302	175	320	220	260	72	351	86	333

Carry out a Wilcoxon rank-sum test, at the 5% significance level, to test whether, on average, there is a difference in cholesterol levels between women and men. [9]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

This image shows a full page of a worksheet designed for handwriting practice. It features 20 evenly spaced, horizontal dashed lines across the entire width of the page. The background is plain white, and there are no margins, text, or other markings present.

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.