

June 2003

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/07, 8719/07

**MATHEMATICS AND HIGHER MATHEMATICS
Paper 7 (Probability and Statistics 2)**



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1 (i) 2.5 1.25 (ii) 5 5	B1 B1 2 B1ft B1ft 2	For correct mean. For correct variance For correct mean. For correct variance
2 $H_0 : p = 0.6$ $H_1 : p > 0.6$ $P(X \geq 10) = {}_{12}C_{10}0.6^{10}0.4^2 + {}_{12}C_{11}0.6^{11}0.4^1 + 0.6^{12}$ $= 0.0834$ Reject H_0 , i.e. accept claim at 10% level S.R. Use of Normal scores 4/5 max $z = \frac{9.5 - 7.2}{\sqrt{2.88}}$ (or equiv. Using $N(0.6, 0.24/12)$) $= 1.3552$ $Pr(>9.5) = 1 - 0.9123 = 0.0877$ Reject H_0 , i.e. accept claim at 10% level	B1 M1* M1*dep A1 B1ft 5 B1 M1 A1 B1ft	For correct H_0 and H_1 For one Bin term ($n = 12, p = 0.6$) For attempt $X = 10, 11, 12$ or equiv. For correct answer (or correct individual terms and dig showing 0.1) For correct conclusion For correct H_0 and H_1 Use of $N(7.2, 2.88)$ or $N(0.6, 0.24/12)$ and standardising with or without cc For correct answer or 1.3552 and 1.282 seen For correct conclusion
3 (i) $31 \pm 2.326 \times \frac{3}{\sqrt{20}}$ $= (29.4, 32.6)$ (ii) 30% is inside interval Accept claim (at 2% level)	B1 M1 B1 A1 4 ftB1* ftB1*dep 2	For correct mean Calculation of correct form $\bar{x} \pm z \times \frac{s}{\sqrt{n}}$ (must have $\sqrt{n}$ in denominator) $z = 2.326$ Correct answer S.R. Solutions not using (i) score B1ft only for correct working and conclusion
4 (i) $P(X > 1.5) = \left[x - \frac{x^2}{4} \right]_{1.5}^2$ or $1 - \left[x - \frac{x^2}{4} \right]_{.0}^{1.5}$ $= 0.0625$	M1 A1 2	For substituting 2 and 1.5 in their $\int f(x)dx$ (or area method $\frac{1}{2}$ their base x their height) For correct answer

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(ii) $E(X) = \int_0^2 \left(x - \frac{1}{2}x^2\right) dx = \left[\frac{x^2}{2} - \frac{x^3}{6}\right]_0^2 = 2/3$	M1 A1	2	For evaluating their $\int xf'(x)dx$ For correct answer
(iii) $m - \frac{m^2}{4} = 0.5$ $m = 0.586 (2 - \sqrt{2})$	M1 A1	3	For equating their $\int f(x)dx$ to 0.5 For solving the related quadratic For correct answer
5 (i) $P(X < 1.7) = \Phi\left(\frac{1.7 - 2.1}{0.9/\sqrt{20}}\right) = 1 - \Phi(1.9876) = 0.0234$	B1 M1 A1 A1	4	For identifying prob Type I error For standardising For correct standardising and correct area For correct final answer
(ii) $P(\text{Type II error}) = P(X > 1.7) = 1 - \Phi\left(\frac{1.7 - 1.5}{0.9/\sqrt{20}}\right) = 1 - \Phi(0.9938) = 0.160$	B1 M1 A1 A1	4	For identifying prob for Type II error For standardising using 1.5 and their 1.7 For correct standardising and correct area For correct final answer
6 (i) $\lambda = 1.25$ $P(X < 4) = e^{-1.25} \left(1 + 1.25 + \frac{1.25^2}{2} + \frac{1.25^3}{6}\right) = 0.962$	M1 M1 A1	3	For attempting to find new λ and using it For summing $P(0, 1, 2, 3)$ or $P(0, 1, 2, 3, 4)$ using a Poisson expression For correct answer
(ii) $X \sim N(182.5, 182.5)$ $P(> 200 \text{ breakdowns}) = 1 - \Phi\left(\frac{200.5 - 182.5}{\sqrt{182.5}}\right) = 1 - \Phi(1.332) = 0.0915 (0.0914)$	B1 M1 A1ft A1	4	For correct mean and variance For standardising process with or without continuity correction For correct standardising and correct tail For correct answer
(iii) $\lambda = 5$ for phone calls $\lambda = 6.25$ for total $P(X = 4) = e^{-6.25} \left(\frac{6.25^4}{4!}\right) = 0.123$	B1 M1 A1	3	For summing their two λ s and using a Poisson expression OR alt. method using sep. distributions 5 terms req. For correct answer

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7 (i) 20 of $A \sim A^*$ $\sim N(401, 20 \times 0.15^2)$ $\sim N(401, 0.45)$ 20 of $B \sim B^* \sim N(401, 1.458)$ $A^* - B^* \sim N(0, 1.908)$ $P(A^* - B^* > 2)$ $= 1 - \Phi\left(\frac{2-0}{\sqrt{1.908}}\right)$ $= 1 - \Phi(1.4479)$ $= 0.0738$ <u>OR</u> $\bar{A} \sim N(20.05, 0.15^2/20),$ $\bar{B} \sim N(20.05, 0.27^2/20)$ $\bar{A} - \bar{B} \sim N(0, 0.00477)$ $P(\bar{A} - \bar{B} > 0.1)$ $= 1 - \Phi\left(\frac{0.1-0}{\sqrt{0.00477}}\right)$ $= 0.0738$ (ii) $1.96 = \frac{20.07 - 20.05}{(0.15/\sqrt{n})}$ $n = 216$	B1		For correct mean for either
	B1		For variance 20×0.15^2 or 20×0.27^2
	M1		For adding their two variances
	M1		For consideration of their $A^* - B^* > 2$
	M1		For standardising and finding correct area
	A1	6	For correct answer
	B1		For correct mean for either
	B1		For variance $0.15^2/20$ or $0.27^2/20$
	M1		For adding their variances
	M1		For consideration of their $\bar{A} - \bar{B} > 0.1$
	M1		For standardising and finding correct area
	A1	6	For correct answer
	M1		For an equation of correct form on RHS involving $\sqrt{n}$
	B1		For 1.96 used
	M1		For solving an equation of correct form (any z)
	A1	4	For correct answer