

June 2004

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 75

SYLLABUS/COMPONENT: 9709/03, 8719/03

**MATHEMATICS AND HIGHER MATHEMATICS
Paper 3 (Pure 3)**



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- 1 Show correct sketch for $0 \leq x < \frac{1}{2}\pi$ B1
Show correct sketch for $\frac{1}{2}\pi < x < \frac{3}{2}\pi$ or $\frac{3}{2}\pi < x \leq 2\pi$ B1
Show completely correct sketch B1 **3**
[SR: for a graph with $y = 0$ when $x = 0, \pi, 2\pi$ but otherwise of correct shape, award B1.]
- 2 *EITHER:* State or imply non-modular inequality $(2x+1)^2 < x^2$ or corresponding quadratic equation or pair of linear equations $(2x+1) = \pm x$ B1
Expand and make a reasonable solution attempt at a 3-term quadratic, or solve two linear equations M1
Obtain critical values $x = -1$ and $x = -\frac{1}{3}$ only A1
State answer $-1 < x < -\frac{1}{3}$ A1
OR: Obtain the critical value $x = -1$ from a graphical method, or by inspection, or by solving a linear inequality or equation B1
Obtain the critical value $x = -\frac{1}{3}$ (deduct B1 from B3 if extra values are obtained) B2
State answer $-1 < x < -\frac{1}{3}$ B1 **4**
[Condone $\leq$ for $<$; accept -0.33 for $-\frac{1}{3}$.]
- 3 *EITHER:* State $6y \frac{dy}{dx}$ as the derivative of $3y^2$ B1
State $\pm 4x \frac{dy}{dx} \pm 4y$ as the derivative of $-4xy$ B1
Equate attempted derivative of LHS to zero and solve for $\frac{dy}{dx}$ M1
Obtain answer 2 A1
[The M1 is conditional on at least one of the B marks being obtained. Allow any combination of signs for the second B1.]
OR: Obtain a correct expression for y in terms of x B1
Differentiate using chain rule M1
Obtain derivative in any correct form A1
Substitute $x = 2$ and obtain answer 2 only A1 **4**
[The M1 is conditional on a reasonable attempt at solving the quadratic in y being made.]

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- 4 (i) State or imply $2^{-x} = \frac{1}{y}$ B1
Obtain 3-term quadratic e.g. $y^2 - y - 1 = 0$ B1 2
- (ii) Solve a 3-term quadratic, obtaining 1 or 2 roots M1
Obtain answer $y = (1 + \sqrt{5})/2$, or equivalent A1
Carry out correct method for solving an equation of the form $2^x = a$, where $a > 0$, reaching a ratio of logarithms M1
Obtain answer $x = 0.694$ only A1 4
- 5 (i) Make relevant use of formula for $\sin 2\theta$ or $\cos 2\theta$ M1
Make relevant use of formula for $\cos 4\theta$ M1
Complete proof of the given result A1 3
- (ii) Integrate and obtain $\frac{1}{8}(\theta - \frac{1}{4}\sin 4\theta)$ or equivalent B1
Use limits correctly with an integral of the form $a\theta + b\sin 4\theta$, where $ab \neq 0$ M1
Obtain answer $\frac{1}{8}(\frac{1}{3}\pi + \frac{\sqrt{3}}{8})$, or exact equivalent A1 3
- 6 Separate variables and attempt to integrate M1
Obtain terms $\frac{1}{3}\ln(y^3 + 1)$ and x , or equivalent A1 + A1
Evaluate a constant or use limits $x = 0$, $y = 1$ with a solution containing terms $k\ln(y^3 + 1)$ and x , or equivalent M1
Obtain any correct form of solution e.g. $\frac{1}{3}\ln(y^3 + 1) = x + \frac{1}{3}\ln 2$ A1✓
Rearrange and obtain $y = (2e^{3x} - 1)^{\frac{1}{3}}$, or equivalent A1 6
[f.t. is on $k \neq 0$.]
- 7 (i) Evaluate cubic when $x = -1$ and $x = 0$ M1
Justify given statement correctly A1 2
[If calculations are not given but justification uses correct statements about signs, award B1.]
- (ii) State $x = \frac{2x^3 - 1}{3x^2 + 1}$, or equivalent B1
Rearrange this in the form $x^3 + x + 1 = 0$ (or vice versa) B1 2

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- (iii) Use the iterative formula correctly at least once M1
Obtain final answer -0.68 A1
Show sufficient iterations to justify its accuracy to 2d.p., or show there is a sign change in the interval $(-0.685, -0.675)$ A1 **3**

- 8 (i) *EITHER*: Solve the quadratic and use $\sqrt{-1} = i$ M1
Obtain roots $\frac{1}{2} + i\frac{\sqrt{3}}{2}$ and $\frac{1}{2} - i\frac{\sqrt{3}}{2}$ or equivalent A1
OR: Substitute $x + iy$ and solve for x or y M1
Obtain correct roots A1 **2**

- (ii) State that the modulus of each root is equal to 1 B1✓
State that the arguments are $\frac{1}{3}\pi$ and $-\frac{1}{3}\pi$ respectively B1✓ + B1✓ **3**
[Accept degrees and $\frac{5}{3}\pi$ instead of $-\frac{1}{3}\pi$. Accept a modulus in the form $\sqrt{\frac{p}{q}}$ or $\sqrt{n}$, where p, q, n are integers. An answer which only gives roots in modulus-argument form earns B1 for both the implied moduli and B1 for both the implied arguments.]

- (iii) *EITHER*: Verify $z^3 = -1$ for each root B1 + B1
OR: State $z^3 + 1 = (z + 1)(z^2 - z + 1)$ B1
Justify the given statement B1
OR: Obtain $z^3 = z^2 - z$ B1
Justify the given statement B1 **2**

- 9 (i) State or imply $f(x) \equiv \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x+1}$ B1
EITHER: Use any relevant method to obtain a constant M1
Obtain one of the values: $A = -1, B = 4$ and $C = -2$ A1
Obtain the remaining two values A1
OR: Obtain one value by inspection B1
State a second value B1
State the third value B1 **4**
[Apply the same scheme to the form $\frac{A}{x-2} + \frac{Bx+C}{x^2-1}$ which has $A = 4, B = -3$ and $C = 1$.]

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- (ii) Use correct method to obtain the first two terms of the expansion of $(x-1)^{-1}$ or $(x-2)^{-1}$
or $(x+1)^{-1}$ M1

Obtain any correct unsimplified expansion of the partial fractions up to the terms in x^3

(deduct A1 for each incorrect expansion)

A1✓ + A1✓ + A1✓

Obtain the given answer correctly

A1 5

[Binomial coefficients involving -1 , e.g. $\binom{-1}{1}$, are not sufficient for the M1 mark. The f.t. is on A, B, C.]

[Apply a similar scheme to the alternative form of fractions in (i), awarding M1*A1✓A1✓ for the expansions, M1(dep*) for multiplying by $Bx + C$, and A1 for obtaining the given answer correctly.]

[In the case of an attempt to expand $(x^2 + 7x - 6)(x-1)^{-1}(x-2)^{-1}(x+1)^{-1}$, give M1A1A1A1 for the expansions and A1 for multiplying out and obtaining the given answer correctly.]

[Allow attempts to multiply out $(x-1)(x-2)(x+1)(-3+2x-\frac{3}{2}x^2+\frac{11}{4}x^3)$, giving B1 for reduction to a product of two expressions correct up to their terms in x^3 , M1 for attempting to multiply out at least as far as terms in x^2 , A1 for a correct expansion up to terms in x^3 , and A1 for correctly obtaining the answer $x^2 + 7x - 6$ and also showing there is no term in x^3 .]

[Allow the use of Maclaurin, giving M1A1✓ for $f(0) = -3$ and $f'(0) = 2$, A1✓ for $f''(0) = -3$, A1✓ for $f'''(0) = \frac{33}{2}$, and A1 for obtaining the given answer correctly (f.t. is on A, B, C if used).]

- 10 (i) State x-coordinate of A is 1

B1 1

- (ii) Use product or quotient rule

M1

Obtain derivative in any correct form e.g. $-\frac{2\ln x}{x^3} + \frac{1}{x} \cdot \frac{1}{x^2}$

A1

Equate derivative to zero and solve for $\ln x$

M1

Obtain $x = e^{\frac{1}{2}}$ or equivalent (accept 1.65)

A1

Obtain $y = \frac{1}{2e}$ or exact equivalent not involving $\ln$

A1 5

[SR: if the quotient rule is misused, with a 'reversed' numerator or x^2 instead of x^4 in the denominator, award M0A0 but allow the following M1A1A1.]

- (iii) Attempt integration by parts, going the correct way

M1

Obtain $-\frac{\ln x}{x} + \int \frac{1}{x} \cdot \frac{1}{x} dx$ or equivalent

A1

Obtain indefinite integral $-\frac{\ln x}{x} - \frac{1}{x}$

A1

Use x-coordinate of A and e as limits, having integrated twice

M1

Obtain exact answer $1 - \frac{2}{e}$, or equivalent

A1 5

[If $u = \ln x$ is used, apply an analogous scheme to the result of the substitution.]

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11 (i) EITHER: Obtain a vector in the plane e.g. $\overrightarrow{PQ} = -3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ B1

Use scalar product to obtain a relevant equation in a, b, c e.g. $-3a + 4b + c = 0$ or

$6a - 2b + c = 0$ or $3a + 2b + 2c = 0$ M1

State two correct equations in a, b, c A1

Solve simultaneous equations to obtain one ratio e.g. $a : b$ M1

Obtain $a : b : c = 2 : 3 : -6$ or equivalent A1

Obtain equation $2x + 3y - 6z = 8$ or equivalent A1

[The second M1 is also given if say c is given an arbitrary value and a or b is found.

The following A1 is then given for finding the correct values of a and b .]

OR: Substitute for P, Q, R in equation of plane and state 3 equations in a, b, c, d B1

Eliminate one unknown, e.g. d , entirely M1

Obtain 2 equations in 3 unknowns A1

Solve to obtain one ratio e.g. $a : b$ M1

Obtain $a : b : c = 2 : 3 : -6$ or equivalent A1

Obtain equation $2x + 3y - 6z = 8$ or equivalent A1

[The first M1 is also given if say d is given an arbitrary value and two equations in

two unknowns, e.g. a and b , are obtained. The following A1 is for two correct

equations. Solving to obtain one unknown earns the second M1 and the following

A1 is for finding the correct values of a and b .]

OR: Obtain a vector in the plane e.g. $\overrightarrow{QR} = 6\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ B1

Find a second vector in the plane and form correctly a 2-parameter equation for the plane M1

Obtain equation in any correct form e.g. $\mathbf{r} = \lambda(-3\mathbf{i} + 4\mathbf{j} + \mathbf{k}) + \mu(6\mathbf{i} - 2\mathbf{j} + \mathbf{k}) + \mathbf{i} - \mathbf{k}$ A1

State 3 equations in x, y, z, λ , and μ A1

Eliminate λ and μ M1

Obtain equation $2x + 3y - 6z = 8$ or equivalent A1

OR: Obtain a vector in the plane e.g. $\overrightarrow{PR} = 3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ B1

Obtain a second vector in the plane and calculate the vector product of the two vectors, e.g. $(-3\mathbf{i} + 4\mathbf{j} + \mathbf{k}) \times (3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ M1

Obtain 2 correct components of the product A1

Obtain correct product e.g. $6\mathbf{i} + 9\mathbf{j} - 18\mathbf{k}$ or equivalent A1

Substitute in $2x + 3y - 6z = d$ and find d or equivalent M1

Obtain equation $2x + 3y - 6z = 8$ or equivalent A1

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- (ii) *EITHER*: State equation of SN is $\mathbf{r} = 3\mathbf{i} + 5\mathbf{j} - 6\mathbf{k} + \lambda(2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k})$ or equivalent B1✓
Express x, y, z in terms of λ e.g. $(3 + 2\lambda, 5 + 3\lambda, -6 - 6\lambda)$ B1✓
Substitute in the equation of the plane and solve for λ M1
Obtain $\overrightarrow{ON} = \mathbf{i} + 2\mathbf{j}$, or equivalent A1
Carry out method for finding SN M1
Show that $SN = 7$ correctly A1

- OR*: Letting $\overrightarrow{ON} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$, obtain two equations in x, y, z by equating scalar
product of $\overrightarrow{NS}$ with two of $\overrightarrow{PQ}, \overrightarrow{QR}, \overrightarrow{RP}$ to zero B1✓+ B1✓
Using the plane equation as third equation, solve for x, y , and z M1
Obtain $\overrightarrow{ON} = \mathbf{i} + 2\mathbf{j}$, or equivalent A1
Carry out method for finding SN M1
Show that $SN = 7$ correctly A1

- OR*: Use Cartesian formula or scalar product of $\overrightarrow{PS}$ with a normal vector to find SN M1
Obtain $SN = 7$ A1
State a unit normal $\hat{\mathbf{n}}$ to the plane B1✓
Use $\overrightarrow{ON} = \overrightarrow{OS} \pm 7\hat{\mathbf{n}}$ M1
Obtain an unsimplified expression e.g. $3\mathbf{i} + 5\mathbf{j} - 6\mathbf{k} \pm 7(\frac{2}{7}\mathbf{i} + \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k})$ A1✓
Obtain $\overrightarrow{ON} = \mathbf{i} + 2\mathbf{j}$, or equivalent, only A1

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