June 2004

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/07, 8719/07

MATHEMATICS AND HIGHER MATHEMATICS
Paper 7 (Probability and Statistics 2)

UNIVERSITY of CAMBRIDGE
International Examinations

1 (i) H_0: $\mu = 15$ or $p = 0.25$ H_1; $\mu > 15$ or $p > 0.25$	B1	**1**	For H_0 and H_1 correct
(ii) Test statistic $z = \pm \frac{21.5 - 15}{\sqrt{60 \times 0.25 \times 0.75}} = 1.938$ OR test statistic $z = \pm \frac{22/60 - 0.5/60 - 15/60}{\sqrt{\frac{0.25 \times 0.75}{60}}} = 1.938$	M1 A1		For attempt at standardising with or without cc, must have $\sqrt{}$ something with 60 in on the denom For 1.94 (1.938)
CV $z = 1.645$ In CR Claim justified	M1 A1ft	 **4**	For comparing with 1.645 or 1.96 if 2-tailed, signs consistent, or comparing areas to 5% For correct answer(ft only for correct one-tail test)
2 (i) Mean = 3.5 + 2.9 + 3.1 = 9.5 Var = $0.3^2 + 0.25^2 + 0.35^2$ (=0.275) St dev = 0.524	B1 M1 A1	 **3**	9.5 as final answer For summing three squared deviations For correct answer
(ii) $z = \frac{9 - 9.5}{\sqrt{\frac{their\ \text{var}}{4}}} = -1.907$ or $z = \frac{36-38}{\sqrt{(4 \times their\ \text{var})}} = -1.907$ $\Phi(1.907) = 0.9717$ = 0.972	M1 M1 A1	 **3**	For standardising, no cc For $\sqrt{\frac{their\ \text{var}}{4}}$ or √(4 x *their* var) in denom - no 'mixed' methods. For correct answer
3 (i) E(2*X*-3*Y*) = 2E(*X*) –3E(*Y*) = 16 – 18 = - 2	M1 A1	 **2**	For multiplying by 2 and 3 resp and subt For correct answer
(ii) Var (2*X*-3*Y*) = 4Var (*X*) +9Var (*Y*) = 19.2 + 54 = 73.2	B1 M1 M1 A1	 **4**	For use of var (*Y*) = 6 For squaring 3 and 2 For adding variances (and nothing else) For correct final answer
4 (i) $\bar{x} = 375.3$ $\sigma^2_{n-1} = 8.29$	B1 M1 A1	 **3**	For correct mean (3.s.f) For legit method involving *n*-1, can be implied For correct answer
(ii) p = 0.19 or equiv. $0.19 \pm 2.055 \times \sqrt{\frac{0.19 \times 0.81}{200}}$ 0.133 < p < 0.247	B1 M1 B1 A1	 **4**	For correct p For correct form $p \pm z \times \sqrt{\frac{pq}{n}}$ either/both sides For z = 2.054 or 2.055 For correct answer

5 (i) $\frac{c-54}{3.1/\sqrt{10}} = -1.282$ $c = 54 - 1.282 \times \frac{3.1}{\sqrt{10}} = 52.74$	B1 M1 A1 A1 **4**	For + or – 1.282 seen For equality/inequality with their z ($\pm$) (must have used tables), no $\sqrt{10}$ needed (c can be numerical) For correct expression (c can be numerical, but signs must be consistent) For correct GIVEN answer. No errors seen.
(ii) P($\bar{x}$ > 52.74) = 1 - $\Phi\left(\frac{52.74-51.5}{3.1/\sqrt{10}}\right)$ = $1 - \Phi(1.265) = 1 - 0.8971$ = 0.103 or 0.102	B1 M1 A1 A1 **4**	For identifying the outcome for a type II error For standardising , no $\sqrt{10}$ needed For $\pm$ 1.265 (accept 1.26-1.27) For correct answer
6 (i) P(5) = $e^{-6} \times \frac{6^5}{5!}$ = 0.161	M1 A1 **2**	For an attempted Poisson P(5) calculation, any mean For correct answer
(ii) P($X \geq 2$) = 1 –{P(0) + P(1)} = 1 - $e^{-1.6}(1+1.6)$ = 0.475	B1 M1 A1 **3**	For μ = 1.6, evaluated in a Poisson prob For 1 – P(0) – P(1) or 1 – P(0) – P(1) – P(2) For correct answer
(iii) P(1 then 4 $\mid$ 5) = $\frac{\left(e^{-3} \times 3\right) \times \left(e^{-3} \times \frac{3^4}{4!}\right)}{e^{-6} \times \frac{6^5}{5!}}$ = 0.156 or 5/32	M1 M1 A1 **3**	For multiplying P(1) by P(4) any (consistent) mean For dividing by P(5) any mean For correct answer
7 (i) $c\int_0^5 t(25-t^2)\,dt = 1$ $c\left[\frac{25t^2}{2} - \frac{t^4}{4}\right]_0^5 = 1$ $c\left[\frac{625}{2} - \frac{625}{4}\right] = 1 \Rightarrow c = \frac{4}{625}$	M1 A1 A1 **3**	For equating to 1 and a sensible attempt to integrate For correct integration and correct limits For given answer correctly obtained
(ii) $\int_2^4 ct(25-t^2)\,dt = \left[\frac{25ct^2}{2} - \frac{ct^4}{4}\right]_2^4 = c[136] - c[46]$ = $\frac{72}{125}$ (0.576)	M1* M1*dep A1 **3**	For attempting to integrate f(t) between 2 and 4 (or attempt 2 and 4) For subtracting their value when t = 2 from their value when t = 4 For correct answer
(iii) $\int_0^5 ct^2(25-t^2)\,dt = \left[\frac{4}{625} \times \frac{25t^3}{3} - \frac{4}{625} \times \frac{t^5}{5}\right]_0^5$ = $\frac{8}{3}$	M1* A1 M1*dep A1 **4**	For attempting to integrate $tf(t)$, no limits needed For correct integrand can have c (or their c) For subtracting their value when t=0 from their value when t=5 For correct answer