

# MATHEMATICS

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| <p><b>Paper 9709/11</b><br/><b>Paper 11</b></p> |
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## Key messages

This paper provided several opportunities to use 'completion of the square' to reach a conclusion from a quadratic expression. Candidates would be well advised to ensure they are proficient in applying this technique when the coefficient of  $x^2$  is negative or fractional.

In questions where a numerical answer is required from use of calculus such as **Question 2** and **10(iii)** it is essential that working is shown to justify correct solutions.

## General comments

Again it needs to be noted that although no GCSE topics are directly examined basic algebraic, geometrical and trigonometric techniques are required to answer some A-level questions.

## Comments on specific questions

### Question 1

- (i) The use of the general term of this type of expansion provided the quickest route to the required coefficients but with so few terms complete expansions leading to correct coefficients were also seen regularly. Those who bracketed the  $-2x$  term were a lot less likely to make sign errors.
- (ii) Candidates who scored full marks in **part (i)** generally went on to gain full marks in this part with most candidates being able to see the connection between the two parts. Some of those who did not make the connection managed to expand three sets of brackets to find the required terms.

Answers: (i)  $1 - 10x + 40x^2$ , (ii)  $a = 3$

### Question 2

The need to find the  $\frac{dy}{dx}$  at  $x = 1$  was seen by many candidates some of whom went on to use the chain rule correctly to find  $\frac{dy}{dt}$ . Correct notation was a feature of the better answers.

Answer:  $-0.06$

### Question 3

The use of integration was appreciated by those able to start this question and a lot of correct integrations were seen with some candidates being prepared to differentiate their result to check its accuracy. Those candidates who included a constant of integration were usually able to obtain the equation of the curve and the required point.

Answer:  $(\frac{1}{2}, 0)$

#### Question 4

- (i) This was often completed correctly with the majority of candidates expanding the left hand side of the identity and applying the trigonometric identity  $\cos^2\theta + \sin^2\theta = 1$  twice to reach the right hand side of the identity. Some candidates chose to apply the same identity twice to the right hand side and successfully factorised this expression to equal the left hand side. A few found the quickest route applying the trigonometric identity in the second bracket on the left hand side and expanding the brackets to give the right hand side.
- (ii) Most who completed **part (i)** saw the connection to **part (ii)** and obtained an expression for  $\tan^3\theta$ . The given range encouraged candidates to look for a second solution and to work in degrees. Most followed the guidance in presenting answers to one decimal place.

Answer: (ii)  $51.6^\circ$  and  $231.6^\circ$

#### Question 5

- (i) The coordinate geometry required to solve this part was generally well understood and many completely correct solutions were seen.
- (ii) A variety of methods were employed to solve this part. The most straightforward involved using the equations from **part (i)** to find the point of intersection of the lines and doubling the coordinates of that point. Other successful methods involved trigonometry and the properties of the kite. Some of these became quite complex and candidates needed to clearly show their working. For full marks the answers had to be exact.

Answers: (i)  $AC; y = \frac{-x}{2} + 4$ ,  $OB; y = 2x$  (ii) (3.2, 6.4)

#### Question 6

- (i) A well answered question with successful candidates realising that the required area was the result of the difference between the area of triangle AOT and sector AOB. The area of the sector was seen correctly more often than the trigonometry required to find AT or OT for the area of the triangle.
- (ii) Of the three parts of the perimeter the arc length was the one found most successfully. Those who were able to use basic trigonometry with the angle expressed in radians generally went on to find the other two lengths and the complete perimeter

Answers: (i)  $\frac{1}{2}r^2(\tan\theta - \theta)$  or equivalent (ii) 16.6

#### Question 7

- (i) This part was completed correctly by the majority of candidates
- (ii) Whilst many candidates were able to find the mid-point vector some incorrectly thought  $\overrightarrow{OM}$  was  $\frac{1}{2}\overrightarrow{AC}$ . The method for finding a unit vector was widely known.
- (iii) Those who had obtained the correct answer in part (i) often went on to score full marks in this part. The required scalar product formula was well understood and equally well applied.

Answers: (i)  $2i + 4j - 4k$  (ii)  $\frac{1}{\sqrt{5}(2i - j)}$  (iii)  $79.0^\circ$

### Question 8

- (a) The sum to infinity and  $n^{\text{th}}$  term formulae were often used correctly to form a quadratic equation in either the common ratio or the first term leading to the two possible answers. As the question asked for 'possible values,' valid methods had to lead to two solutions.
- (b)(i) The candidates who recognised the general term as that of an arithmetic progression usually went on to identify the common difference and use an appropriate  $S_n$  formula. The correct identification of the first term proved difficult for all but the higher scoring candidates.
- (ii) Candidates who had found an  $S_n$  formula in **part (b)(i)** usually showed the algebraic skills required to use it to find value for  $p$  and  $q$ .

Answers: (a) 18 and 36 (b)(i)  $\frac{n}{2}(2p + nq + q)$  or equivalent (b)(ii)  $p = 5, q = 2$

### Question 9

- (i) Finding the points of intersection by solving the two equations simultaneously was well understood and carried out correctly by many candidates. It should be noted that to fully describe a point both the  $x$  and  $y$  coordinates are required.
- (ii) Most successful answers came from using the  $x$  values from **part (i)**. Those who referred to their original quadratic from equating  $f(x)$  and  $g(x)$  or tested values around  $-3$  and  $4$  usually found the correct set.
- Some only gave one boundary.
- (iii) The formation of composite functions was well understood and many correct expressions for  $fg(x)$  were seen. Some of these went on, either by use of calculus or completing the square, to reach the maximum of the function and, hence, its range. The use of calculus, although more rarely seen, usually proved more successful than completing the square on this type of quadratic.
- (iv) Correct answers were seen from candidates who appreciated that  $x = k$  was the line of symmetry of  $h(x)$  or that  $x = k$  at the turning point. Calculus, completing the square and simple sketches provided successful methods.

Answers: (i)  $(4.0)$  and  $(-3, -3.5)$  (ii)  $x < -3, x > 4$  (iii)  $fg(x) \leq \frac{1}{4}$  (iv)  $k = 1$

### Question 10

- (i) Many completely correct solutions were seen. Finding the gradient and showing it could not equal zero was sufficient to gain full marks. Most chose to use the discriminant but those who could complete the square and interpret their result were equally successful.
- (ii) For many candidates this proved to be the most difficult part of a question. Those who identified  $m$  as their derivative from **part (i)** produced the best attempts by using calculus or by completing the square to find and identify the stationary value. Of those who found the correct  $x$  value few actually quoted the corresponding value of  $m$ . Although the use of the second derivative of  $m$  was the quickest route to identifying the nature of the stationary value other carefully explained methods were acceptable.
- (iii) A straightforward part-question well answered by those candidates who attempted it.

Answers: (ii)  $m = \frac{11}{3}$ , minimum (iii) 270

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| <p><b>Paper 9709/12</b><br/><b>Paper 12</b></p> |
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## Key messages

When definite integrals are evaluated, it is important that both of the limits can be clearly seen to have been substituted into the integral. For example  $\left(\frac{6^3}{12} - 6(6) - \frac{36}{6}\right) - \left(\frac{2^3}{12} - 6(2) - \frac{36}{2}\right)$ .

## General comments

The paper seemed to be well received by the candidates and many good and excellent scripts were seen. The paper seemed to work well with a number of questions being reasonably straightforward, particularly near the beginning of the paper, giving all candidates the opportunity to show what they had learned and understood, but also some questions which provided more of a challenge, even for candidates of good ability. The vast majority of candidates appeared to have sufficient time to complete the paper. The standard of presentation was generally good with candidates setting their work out in a clear readable fashion.

## Comments on specific questions

### Question 1

The question proved to be a very accessible start to the paper with a great many candidates demonstrating a good knowledge of the binomial expansion. They were very often able to write down the relevant terms (usually from a full expansion), add them together, form the required equation and solve it to find the required value of  $a$ . A number of candidates correctly wrote down the term including  $\left(\frac{x}{2}\right)^2$  but then forgot to square the 2. A small number tried to multiply the terms together rather than adding them and some weaker ones still included  $x^2$  in part but not all of the equation needed to find  $a$ .

Answer:  $a = 3$

### Question 2

In both parts of this question, the vast majority of candidates attempted to use the discriminant. Unfortunately, in part (i) many thought that it should either  $= 0$ , be  $> 0$ , or  $\geq 0$  and therefore received no credit. In part (ii), however, many correct solutions were obtained by making it equal to 0. Other valid methods were sometimes attempted such as finding the minimum  $y$  value either by differentiation or completing the square in part (i) or equating the gradients in part (ii).

Answers: (i)  $k > 9$ , (ii) 11

### Question 3

Most candidates recognised this as a geometric series question but a good number of weaker ones thought that it was an arithmetic one. Many were able to correctly interpret the given scenario and recognise that the common ratio would be 1.02 but some used 0.02. Some weaker candidates were confused as to when to use the formula for the  $n^{\text{th}}$  term and when to use the sum of  $n$  terms. Most did use them correctly, although some misunderstood the required number of terms.

Answers: (i) 9 950 (ii) 107 000



#### Question 4

In part (i) many candidates were able to use the information given to form two simultaneous equations and solve them correctly. Part (ii) proved much more challenging with a good number taking the phrase ‘no solution’ to mean that the discriminant had to be used rather than using the values obtained in part (i) to consider when the trigonometric equation would have no solution. Those who considered the graph of the function and its maximum and minimum values were usually successful.

Answers: (i)  $a = 7, b = -4$ , (ii)  $k < 3, K > 11$

#### Question 5

Many fully correct solutions to this vector question were seen. The overwhelming majority of candidates seemed very confident in using the scalar product in part (ii), although a good number were unable to express correctly the required two vectors in part (i). Some did not use their answers from part (i) in part (ii) but instead re-calculated them or used them in reverse. It should be noted that those candidates who did not use the scalar product in part (ii) received no credit.

Answers: (i)  $6i - 4k, 6i - 5j - 4k$ , (ii)  $34.7^\circ$

#### Question 6

In part (i) many candidates recognised that the shape  $OATB$  was two triangles put together, used one of these triangles to find the length of  $AT$  or  $BT$  as  $r \tan \theta$ , formed the required equation and were able to show the given result. Weaker candidates sometimes used the formula for the area of a segment and made no progress. The formulae for the area of the sector and the arc length were well known but there was confusion in both parts over when to use  $2\theta$ . Some candidates who were unable to show the result in part (i) did manage to find the required areas in part (ii) although premature approximation was sometimes an issue.

Answers: (ii) 87.8

#### Question 7

Many correct solutions to **Question 7** were seen with full marks often being scored by good candidates. Less able candidates were sometimes unable to see the connections between the different parts. In part (i), all but the weakest candidates were able to obtain an expression in the requested form but sign errors were common often leading to incorrect values for  $a$  and  $b$ . Part (ii) followed directly from part (i) but many candidates re-started and correctly found the stationary value of  $x$  using differentiation but not always the corresponding  $y$  value. Again part (iii) followed directly from the first two parts but candidates sometimes did not see the connection. In part (iv), only those who started from the completed square form were usually successful. A significant number of candidates did not make clear that the whole of  $\frac{25-x}{2}$  was square

rooted and answers which looked more like  $\frac{\sqrt{25-x}}{2}$  were common.

Answers: (i)  $25 - 2(x + 3)^2$ , (ii)  $(-3, 25)$ , (iii)  $-3$ , (iv)  $\sqrt{\frac{25-x}{2}} - 3$

#### Question 8

This multi-stage problem was frequently very well done with full marks often awarded. Many candidates found the gradient of  $AB$  in terms of  $h$  and equated this to  $\frac{2}{3}$ . This led to a straightforward equation, which could be solved to find the value of  $h$ . Using this value and the midpoint of the line  $AB$  led to the value of  $k$ .

Other candidates found the equation of  $AB$  again using  $\frac{2}{3}$  and one of the points on the line. Substituting another point led to an equation in terms of  $h$  only. Some candidates thought that the gradient of the bisector was 3 or  $-3$  but they did often know the required method from that point onwards. Some weaker ones incorrectly substituted the coordinates of  $A$  or  $B$  into the equation of the bisector or correctly used the midpoint but made no further progress.

Answer:  $h = 2, k = 36$

### Question 9

This question, particularly part (i), was very well done by many candidates. The vast majority recognised the need to integrate  $\frac{dy}{dx}$  in order to find the equation of the curve with very few forgetting about the  $+c$ . A few weaker candidates used the equation of a straight line or tried to split up the square rooted expression. Some forgot to divide by 4 having otherwise integrated correctly. In part (ii) most interpreted the given information correctly although a good number multiplied when they should have been dividing. In part (iii) many candidates differentiated correctly although some forgot to multiply by 4 and others substituted the value  $x = 2$  into their expression, rather than showing that it was always constant. Many weaker candidates missed out both of the last two parts.

Answers: (i)  $y = \frac{(4x+1)^{\frac{3}{2}}}{6} + \frac{1}{2}$ , (ii) 0.02

### Question 10

Part (i) of this question was very well done by the vast majority of candidates but a significant number struggled to draw the graphs requested in part (ii), and part (iii) proved too difficult except for the most able. In part (i) most realised that the easiest approach was to try and use  $\frac{\sin x}{\cos x} = \tan x$  although sign errors were quite common. Some candidates rearranged the equation, then correctly squared both sides, then used  $\sin^2 x + \cos^2 x = 1$  but this usually led to extra invalid solutions. Weaker ones sometimes obtained 0 when  $\cos x$  was divided by itself or thought that  $\frac{\cos x}{\sin x} = \tan x$ . In part (ii) many very good sketches were seen and candidates usually followed the instruction to put them on the same diagram. Weaker candidates often resorted to plotting points, which frequently led to unsatisfactory curves or straight lines. Some also did not sketch their curves for the whole of the specified domain or missed out this part completely. This was true even more so in part (iii) where all but the most able candidates struggled. Those who followed the guidance given and used the first two parts of the question were far more successful than those who tried to solve the inequality from scratch. Some candidates realised that the points of intersection were significant but gave the range when the function would be  $< 0$  rather than  $> 0$ .

Answers: (i)  $146.3^\circ, 326.3^\circ$ , (iii)  $x < 146.3^\circ, x > 326.3^\circ$

### Question 11

There was a mixed response to this final question. Many fully correct solutions were seen to both parts but a good number also omitted both or at least part (ii). In part (i) all but the weaker candidates were able to solve the two equations and obtain the  $x$  coordinates of points  $P$  and  $Q$ . A good number then stopped after confirming that the  $y$  coordinate of both points was 4. Most realised though the need to differentiate, find the equations of the tangents at the two points and confirm that they met at the point  $(3,3)$  and therefore on the line  $y=x$ . In part (ii) the formula for the volume of revolution about the  $x$ -axis was well known but many candidates had trouble squaring  $\left(\frac{x}{2} + \frac{6}{x}\right)$  correctly. The line  $y = 4$  was an extra layer of complexity that many could not deal with correctly. Those who attempted both volumes separately and then subtracted were generally more successful than those who attempted to do both parts together. The integral of  $4^2 - \left(\frac{x}{2} + \frac{6}{x}\right)^2$

was often seen incorrectly written as the integral of  $(4 - (\frac{x}{2} + \frac{6}{x}))^2$ . Although it occurred rarely, it is important to re-emphasise that candidates who showed no working but obtained the correct answer by using the integral function on their calculator, received no credit.

Answers: (i) (3,3), (ii)  $\frac{32}{3}\pi$

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| <p><b>Paper 9709/13</b><br/><b>Paper 13</b></p> |
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## Key message

Despite the generally high standard of mathematics exhibited by candidates in this paper it is surprising to report the high incidence of a very basic mistake. This occurred in **Question 11(ii)** which required candidates to square  $(x + 1)^2 + (x + 1)^{-1}$ . It was very common indeed to see high scoring candidates simply square the two terms individually without including the product of the two terms. Not an insignificant proportion of these candidates managed, somehow, to arrive at the correct answer. It should be noted that an answer unsupported by correct working will not gain credit. It is also necessary to show some working for the substitution of the limits.

## General comments

The paper was generally well received by candidates and many very good scripts were seen. Almost all candidates seemed to have sufficient time to finish the paper. It is again pleasing to report that candidates have become more and more aware of the amount of space allowed for their working and in the great majority of cases were able to fit their working into the given space without the need to resort to supplementary sheets. One issue is the accuracy of numerical answers, often mentioned in previous reports, which remains a source of many lost marks. Unless specified otherwise, apart from angles given in degrees which need to be given correct to 1 decimal place, answers should be given correct to 3 significant figures. Candidates often fail to appreciate, however, that in order to achieve this it is usually necessary to carry *more* than 3 significant figures in their working. This is particularly necessary in **Question 5**.

## Comments on specific questions

### **Question 1**

This question was done very well indeed with most candidates scoring full marks.

*Answer:*  $3(x - 2)^2 - 5$ .

### **Question 2**

This question attracted a high proportion of correct answers – although a variety of different methods was seen. The most efficient approach was to realise that in order to achieve a final term in  $\frac{1}{x}$ , terms in  $x^2$  and

$\left(-\frac{2}{x}\right)^3$  were needed, together with a binomial coefficient of  ${}^5C_3$ . Other more pedestrian methods were seen

including highly structured approaches to determine which power of  $r$  to consider, and also writing out the complete expansion before choosing the required term. Pascal's triangle was also seen occasionally. The most common error was to not include the minus sign, or the 2, in the term to be cubed.

*Answer:*  $-80$ .

### Question 3

This question was done reasonably well – although a significant number of candidates, after scoring the first 2 marks for using the formulae for  $S_{100}$  and  $S_{\infty}$ , instead of dividing, equated the two sums with confusion resulting. (Note that this was one occasion when a different level of accuracy (2 significant figures) was requested.)

Answer: 63%.

### Question 4

This question was generally well done. The vast majority of candidates knew exactly what they had to do but it was particularly pleasing to note the high level of accuracy in performing the integration. There was, however, a very common error made,  $(-1)^{\frac{2}{3}}$  being evaluated as  $-1$  instead of  $+1$ , which lost the last mark.

Answer:  $-\frac{1}{2}$ .

### Question 5

A few candidates, seeing 5 and a right angle, assumed it was a 5, 12, 13 triangle and other candidates were not secure in their basic trigonometry when finding either  $AB$  or  $OB$ . Apart from this, very efficient methods from most candidates were seen using the expected route. However, as described above in **General comments**, many candidates lost the last mark by not carrying sufficient accuracy in their working. The use of radians when finding lengths of arcs and areas of sectors is one prescribed section of the syllabus. It is therefore surprising to find some candidates converting to, and using, degrees in these circumstances. Although candidates could gain full credit for using this method, this method has the disadvantage of being less accurate than using 1.2 radians (which is *exact*). Some candidates were unable to obtain the correct answer (to 3 significant figures) by converting to degrees.

Answer: 17.2.

### Question 6

In part (i), almost all candidates wrote down correctly the initial expression for the gradient of  $AB$ . But it is still disappointing to see a number of candidates not using brackets in these situations and inevitably mistakes were made when simplifying the numerator and denominator which, of course, meant that the gradient did not reduce to  $\frac{1}{2}$  (independent of  $k$ ). There were also a few attempts at cancelling individual terms in the numerator and denominator of an algebraic fraction. Similar (bracket) errors were also seen in part (ii) in finding the mid-point of  $AB$  which meant that simplification of the equation of the perpendicular bisector was severely limited. Nevertheless, there were many successful attempts at this question.

Answers: (i)  $\frac{1}{2}$ ; (ii)  $y + 2x = 6$ .

### Question 7

A variety of different routes were used by candidates in part (a)(i) but broadly the routes depended either in the use of the identities  $\tan\theta = \frac{\sin\theta}{\cos\theta}$  followed by  $\sin^2\theta + \cos^2\theta = 1$  or in the use of  $1 + \tan^2\theta = \sec^2\theta$  followed by  $\sin^2\theta + \cos^2\theta = 1$ . The use of  $1 + \tan^2\theta = \sec^2\theta$  is permitted even though it does not appear until the syllabus for Pure Mathematics 2. Candidates who multiplied throughout by  $\cos^2\theta$  at an early stage were generally able to progress more elegantly to the final answer. In part (a)(ii) candidates could choose to solve the equation in the form presented in the question or to use the result of part (a)(i). Candidates were usually successful whichever method they chose. Parts (b)(i) and (b)(ii) were usually successful, although marks were often lost due to the answers (being in radians) not being given correct to 3 significant figures.

Answers: (a)(i)  $2\sin^2\theta - 1$ ; (a)(ii)  $-52.2^\circ$ ; (b)(i) 1.11; (b)(ii)  $-0.894$  or  $-0.895$  or  $-0.896$ .

### Question 8

In part (i) almost all candidates were able to differentiate correctly and a high proportion went on to equate to  $-3$  and proceed to a correct answer. Some candidates, however, equated the derivative to zero while others equated to  $2 - 3x$  or to  $\frac{1}{3}$ . In part (ii) most candidates either set their derivative equal to zero or to  $> 0$  and either factorised obtaining the values 2 and 4, or, less frequently, applied a completing the square approach, obtaining  $(x - 3)^2 - 1 > 0$ . The first approach required candidates to choose one or other of the critical values and a significant number incorrectly chose 2. In the second approach several candidates concluded that  $x$ , and therefore  $k$ , should be 3. Despite these errors this question proved to be a very good source of marks for many candidates.

Answers: (i)  $y - 6 = -3(x - 3)$ ; (ii) Smallest value of  $k$  is 4.

### Question 9

Rather surprisingly, part (i) was done relatively badly. There was a considerable number of candidates who were unable to construct a convincing explanation of why **OE** = **1.6i + 1.2j**, the given answer, and there was also a significant proportion of no-responses. The approach most commonly used was to show that **OB** = 10 and therefore that **OE** =  $\frac{1}{5}$  **OB**. Another successful approach was to use trigonometry. In Part (ii) many candidates used an incorrect strategy for finding **OD** and **BD** and as a consequence were probably able to score only 2 marks out of 7 for this part. Those who found **OD** and **BD** successfully were usually able to go on and score all 7 marks.

Answer: (ii)  $64.8^\circ$ .

### Question 10

Most candidates scored the 1 mark on offer for part (i). In part (ii), finding the inverse function was usually done very efficiently with the only occasional blemish being the retention of the  $\pm$  sign. Candidates had more difficulty with finding the domain and sometimes this was not attempted and at other times was found incorrectly. Part (iii) was somewhat unusual and needed considerable care in writing down the first line of the solution, from which it was very common for a 2 to be omitted. Candidates who wrote the first line correctly still needed to take care in finding the solution in the correct form and also had to reject a negative solution in order for  $x$  to satisfy the initial condition of being greater than  $c$  where  $c = 4$ .

Answers: (i) Smallest value of  $c$  is 2; (ii)  $f^{-1}(x) = \sqrt{x - 2} + 2$ ; (iii)  $x = 2 + \sqrt{7}$ .

### Question 11

There were many good attempts in part (i) and stronger candidates coped well. The question produced a range of scores from 0 to 5 with some weaker candidates not attempting it. Although the differentiation was often done correctly, a convincing explanation that  $2(x + 1)^3 = 1$  eluded a considerable number of candidates

– particularly those who opted to express  $2(x + 1)$  as  $2x + 2$ . Those who found  $\frac{dy}{dx}$  successfully usually went

on to arrive at a correct expression for  $\frac{d^2y}{dx^2}$ . There were, however, quite a few who mistakenly saw  $\frac{d^2y}{dx^2}$  as the derivative of  $2(x + 1)^3 = 1$ . A significant number of candidates who had scored the first 3 marks then used

a non-exact approach (i.e. a decimal value for  $x$ ) to evaluate  $\frac{d^2y}{dx^2}$  and lost the last 2 marks. Part (ii) produced

a mixture of responses. Finding  $y^2$  was the key to this question and caused most problems (a few thought they only had to integrate  $y$ ). As reported in the **Key message** above, many just squared the first and last term, losing the middle term and 3 marks immediately. Some tried to integrate whilst  $y^2$  was still unprocessed in brackets. Others tried to create a common denominator only to find this expression could not be integrated. Some candidates used a substitution for their integration and achieved a wholly correct result including a change of limits. A number of otherwise good solutions were spoilt by the omission of  $\pi$  in the final answer.

Answers: (i)  $\frac{d^2y}{dx^2} = 6$ ; (ii)  $9.7\pi$ .

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| <p><b>Paper 9709/21</b><br/><b>Paper 21</b></p> |
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## Key messages

It is essential that candidates set out their work in a clear and logical way, showing each step of their solution. This ensures that the Examiner is able to determine exactly what marks are available. It is also important that the instructions on the front of the paper are read as this will reinforce the accuracy to which candidates are expected to give their answers.

## General comments

It was evident that many candidates had not prepared or been prepared sufficiently well for this examination. As a consequence, many candidates were unable to attempt every part of each question. This having been stated, there were also candidates who had clearly prepared well and produced scripts of a high standard, showing a good understanding of the syllabus aims and objectives. It was pleasing to note that the additional page provided in the examination booklet for the continuation of questions was used appropriately.

It is important that, when a question specifies no use of calculator, that candidates show each step of their work and give their answers in an exact form.

## Comments on specific questions

### Question 1

Many candidates failed to score marks as they did not recognise the given equation as a quadratic equation in  $e^x$ . Of those that did, most factorised correctly and were able to reach the solutions  $x = \ln \frac{1}{3}$  and  $x = \ln 27$ . Far fewer candidates were able to rearrange these solutions into the form  $k \ln 3$  as required.

*Answer:*  $-\ln 3, 3 \ln 3$

### Question 2

Many candidates failed to score marks in this question too. This was because whichever approach was used, they were often unable to substitute the correct numerical values into equations that they had formed.

Very few were able to obtain the correct relationship  $\ln y = \ln A + (\ln B) \ln x$  to start with. Of those that did, many then substituted incorrectly e.g.  $\ln 4.908 = \ln A + \ln 2.2 \ln B$  and similar variations. Other common incorrect statements were  $4.908 = AB^{\ln 2.2}$ ,  $4.908 = AB^{2.2}$  and  $\ln 4.908 = AB^{\ln 2.2}$ .

Many candidates were able to find the gradient of the graph but were unsure what to equate it to. Many found the equation of the straight line in the form  $y = mx + c$  but were again unable to equate the gradient and the intercept to the relevant terms involving  $A$  and  $B$ .

Some candidates made use of the correct equations  $e^{4.908} = AB^{2.2}$  and  $e^{11.008} = AB^{5.9}$ , and, providing they were able to solve these equations correctly, usually obtained the correct value of  $A$  and of  $B$ .

*Answer:*  $A = 3.6, B = 5.2$

### Question 3

It was essential that the integrand was written as  $4e^{2x} + 4e^{-x}$  before attempting integration, unless a lengthier method of integration by parts was used. This latter method still involved having to re-write an integrand. Provided this was done, most candidates were able to integrate correctly and make the appropriate substitution of the limits to obtain a correct exact answer. As the question specified not using a calculator, those candidates that gave their final answer in decimal form, without showing a previous simplified exact form, were unable to gain the final accuracy mark.

Those candidates that did not re-write the integrand usually failed to score any marks.

Answer:  $2e^4 - 4e^{-2} + 2$

### Question 4

- (i) Most candidates were able to gain at least the first 2 marks in this question provided they recognised that they had to either differentiate a quotient or differentiate an associated product. There were occasional slips with terms in the incorrect order or incorrect differentiation of  $5\ln x$ .

For the last accuracy mark, candidates were expected to identify the  $x$ -coordinate of P as having a value of 1, make the substitution into the derivative obtained and evaluate. Unfortunately, many candidates appear to associate finding a derivative with equating it to zero and solving, which was a common occurrence in this question.

- (ii) It was expected in this part of the question that candidates equate their numerator of their derivative (or equivalent if using a product) to zero and re-arrange to obtain the given form. Many candidates attempted this with varying levels of success, but it was evident that many candidates had not met questions of this type before.
- (iii) It was evident that some candidates had either not covered this syllabus topic or were not able to recall it due to the number of 'no responses' given for this part of the question. For those candidates that did attempt this part, many correct solutions were seen, but marks were often lost due to either not using the required level of accuracy or not doing enough iterations. It was essential that candidates choose an appropriate starting point, as a value of  $0 < x < 1$  does not lead to a converging sequence of numbers.

Answer: (i)  $\frac{5}{3}$  (iii) 3.181

### Question 5

- (i) Most candidates realised that they had to differentiate each of the given parametric equations with respect to  $\theta$  and use them appropriately to obtain the gradient function. Accuracy marks were usually lost due to incorrect differentiation of  $2\cos 2\theta$  and calculators being in degree mode rather than radian mode for the substitution of  $\theta = 1$  radian.
- (ii) Only the better prepared candidates realised that the gradient of the tangent parallel to the  $y$ -axis is undefined and hence the denominator of the gradient function obtained in part (i) had to be equated to zero. For those candidates that did, most were able to gain full marks.

Answer: (i) 1.25 (ii)  $\frac{3}{8}$

### Question 6

Many candidates were able to gain a significant number of marks in this question. It is clearly an area of the syllabus with which they are confident.

- (i) Most candidate were able to obtain the correct value of the constant  $a$  and go on to use one of a variety of methods to obtain a quadratic factor and hence factorise the given function completely. Errors were usually of the form of arithmetic slips – a candidate should recognise that if they are not obtaining a remainder of zero when attempting to find the quadratic factor, that an error has occurred somewhere and that they should go back and check their arithmetic. Likewise, this should be done if a required constant in a question of this type appears to be an awkward fraction. Some candidates failed to gain the last accuracy mark as they did not give their answer as 3 linear factors.
- (ii) Provided candidates recognised the importance of substituting  $2x$  correctly, most were able to gain at least one mark and often the first accuracy mark if attempting to obtain a cubic equation, which was the most common approach. To obtain the marks for the solution of this cubic equation, it was essential that candidates show all their full working clearly as the use of a calculator in this part of the question was not allowed.

The method of solution keeping everything in terms of linear factors was less common. Those candidates that used this method often cancelled the common factors but failed to include the solutions  $x = -1$  and  $x = -2$  as part of their solution.

Answer: (i)  $a = 7$ ,  $(x+1)(x+2)(x+4)$  (ii)  $x = -1$ ,  $x = -2$ ,  $x = \frac{8}{5}$

### Question 7

- (i) It was evident that some candidates had either not covered this syllabus topic or were not able to recall it due to the number of 'no responses' given for this part of the question. Most candidates were able to find the correct value of  $R$  and were able to attempt to find  $\alpha$  using a trigonometric approach. Errors were usually sign errors in calculation of  $\alpha$  or working in degrees rather than radians.
- (ii) It was essential that the given equation was re-arranged to  $5\cos\theta - 2\sin\theta = 4$  and then make use of the answer to part (i) as requested. There were fewer correct solutions than hoped for. Many candidates erroneously attempted to solve  $5\cos\theta - 2\sin\theta = 2$ , but they were given credit in the form of method marks if they used correct approach to the solution of this equation using their answer to part (i).
- (iii) Very few correct solutions were seen. Very few candidates associated the integrand with the form  $5\cos\theta - 2\sin\theta$  they had been using in the first 2 parts of the question and replaced  $\theta$  by  $\frac{x}{2}$ . The integrand could then be simplified to  $\frac{1}{29}\sec^2\left(\frac{x}{2} + 0.3805\right)$ , which is in a form that can be integrated. Many candidates produced solutions that were totally incorrect.

Answer: (i)  $\sqrt{29}\cos(\theta + 0.3805)$  (ii) 0.353, 5.17 (iii)  $\frac{2}{29}\tan\left(\frac{x}{2} + 0.3805\right) + c$

# MATHEMATICS

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Paper 9709/22  
Paper 22

## Key messages

It is essential that candidates set out their work in a clear and logical way, showing each step of their solution. This ensures that the Examiner is able to determine exactly what marks are available. It is also important that the instructions on the front of the paper are read as this will reinforce the accuracy to which candidates are expected to give their answers.

## General comments

It was evident that most candidates had been prepared sufficiently well for this examination. However, some candidates were unable to attempt every part of each question. There were excellent candidates who had clearly prepared well and produced scripts of a high standard, showing a good understanding of the syllabus aims and objectives. It was pleasing to note that the page in the examination booklet for the use of continuation of questions was made use of appropriately when necessary.

It is important that, when a question specifies no use of a calculator, that candidates show each step of their work and give their answers in an exact form.

## Comments on specific questions

### Question 1

This was a good starter question for most candidates. The most popular method of solution was to square each side of the inequality and, by factorisation or otherwise, obtain the critical values. Some candidates, having obtained the critical values, then omitted the actual solution. There were some errors in factorisation, but in general, there has been a marked improvement in the solution of inequalities, with the solutions of the type  $x < \frac{7}{2}$ ,  $x < -\frac{3}{4}$  being less common than in the past.

Answer:  $-\frac{3}{4} < x < \frac{7}{2}$

### Question 2

- (i) Most candidates knew the form of differentiation of logarithmic functions. The main error was to give an answer of  $\frac{3}{2x+9}$  for  $\frac{d}{dx}(\ln(2x+9))$ , rather than the correct  $\frac{2}{2x+9}$ . The solution of this type of function equated to zero sometimes taxed the algebraic skills of some candidates.
- (ii) This part of the question was rarely done well. The candidates were asked to determine the nature of the stationary point. It was expected that some kind of reasoning in the form of use of the second derivative or inspection of the gradient either side of the stationary point or similar method be employed. No marks were available for a correct statement with no supporting evidence. For those candidates that chose to use the second derivative, it was essential that they showed each step of their working, including the substitution of their x-value followed by the correct numerical value of the derivative to gain both marks available. Similar detail was also expected for the alternative methods.

Answer: (i)  $x = 9$ , (ii) Minimum

### Question 3

- (i) Algebraic long division was the most popular method used by candidates, usually with great success. Some candidates were confused by the fact that there was no term in  $x$  in the divisor which then sometimes caused problems in the process of the actual division. Some candidates chose to form an identity and compare coefficients of terms. Both methods usually produced full marks for the majority of candidates. It was essential that the steps in the algebraic long division were shown clearly so that it could be seen that the remainder was equal to 1.
- (ii) Most candidates made a correct connection of the equation in this part of the question with the equation in part (i). There were occasional errors with the inclusion of a term of 1 which showed a lack of understanding of the relationship with the remainder of 1 in part (i). Most candidates chose to use the discriminant to show that  $x^2 - 2x + 2 = 0$  has no real roots. An application of the quadratic formula to show that this equation has no real roots was also acceptable. It was also essential to mention that  $x^2 + 6 = 0$  also has no real roots in order to gain full marks.

Answer: (i)  $x^2 - 2x + 2$

### Question 4

- (i) It was important that candidates dealt with the term  $2\ln(2x)$ , writing it as either  $\ln 4x^2$  or  $\ln(2x)^2$  as a first step. If this was not done then it was almost impossible for candidates to gain any of the subsequent marks. For those that did, an application of the subtraction rule to the left hand side of the equation and the power rule to  $4\ln 2$  usually lead to a correct quadratic equation which could be easily factorised. Many candidates however, lost the final accuracy mark as they included  $x = -2$  in their solution. Candidates should always check that their solutions are valid, especially in an equation involving logarithms.
- (ii) It was very uncommon to see this part of the question related to the first part of the question in spite of the word 'Hence'. This means that the result from part (i) can be used to solve part (ii). The mark allocation of 2 marks should also be a hint that not too much work is needed to solve the given equation. Many candidates started again and often made errors when dealing with powers of 2 in terms of  $u$ . There were candidates who did make the link and gain the marks available by setting the value of  $x$  obtained in part (i) equal to  $2^u$  or equivalent. It was at this point that many candidates who had included  $x = -2$  in their answer to part (i) realised that it would not produce a solution in part (ii). This should have alerted them to the fact that  $x = -2$  could not be a solution to (i) and they could have then changed their answer, thus gaining an extra mark.

Answer: (i)  $x = 6$  (ii) 2.585

### Question 5

If a candidate did not attempt to use implicit differentiation of a product then the only mark that could be gained was for the value of  $y$  when  $x = 0$ . It seemed evident that some candidates had not been taught to differentiate implicitly. Fortunately, most candidates made an attempt to differentiate  $y^3 \sin 2x$  implicitly with respect to  $x$ . There were the occasional slips with the numerical coefficients of some of the terms and occasional arithmetic slips in the evaluation of the gradient at the point  $(0, 2)$ .

Answer:  $y = -4x + 2$

### Question 6

- (i) It was essential that the integrand was expanded out before integration was attempted. If not, then it was impossible to gain any marks as it was impossible to reach any of the steps necessary to obtain the given answer. Integration of a correct integrand together with the correct application of the limits and equating to 15, yielded the equation  $a + e^a + 4e^{\frac{a}{2}} = 15$ . Many candidates were able to reach this result. It was necessary for candidates to show that  $e^a + 4e^{\frac{a}{2}} = e^2 \left( e^{\frac{a}{2}} + 4 \right)$  in the process of simplification to obtain the given answer, but very often this essential step was omitted.
- (ii) There were not many completely correct methods used by candidates. Many used a correct approach, usually the substitution of both  $x = 1.5$  and  $x = 1.6$  into  $a - 2\ln \left( \frac{15 - a}{4 + e^{\frac{a}{2}}} \right)$  or equivalent, obtaining a numerical result, which was usually correct. It was essential that a correct conclusion followed this work. Many candidates did not give a conclusion. It was expected that mention be made of a change of sign between the two numerical values, which implied  $1.5 < \alpha < 1.6$ .
- (iii) It was evident that some candidates had either not been taught iterative methods or were unable to recall them as there were quite a few blank spaces instead of work. For those candidates that were able to perform iterations, very often there were not enough iterations to justify an answer to 3 significant figures. Many candidates gave an answer of 1.55 even though their iterations implied an answer of 1.56. Accuracy is very important in questions of this type and it is important that candidates check that they have given their final answer to the accuracy required. Many candidates lost the last accuracy mark as their final answer was not to 3 significant figures.

Answer: (iii) 1.56

### Question 7

- (i) Many candidates did not achieve in this part of the question. Many attempted to make use of the identity  $\operatorname{cosec}^2 2x = 1 + \cot^2 2x$  which was not the way forward. Candidates should always look at the right hand side of the given identity to see what form they should try to make use of. In this case, there were no double angles, so the use of the double angle formulae involving  $\sin 2x$  and  $\cos 2x$  needed to be made use of together with the fact that  $\operatorname{cosec} 2x = \frac{1}{\sin 2x}$ . For those candidates that did make use of a correct approach, errors involving  $\sin^2 2x = 2 \sin^2 x \cos^2 x$  meant that it was impossible to obtain the required result. When this happens, it is better not to 'contrive' to obtain the given answer as often possible method marks are lost.
- (ii) Usually done very well, with most candidates making use of the result from part (i) to obtain a quadratic equation in  $\tan x$ . Most were able to obtain the correct solution for  $\tan x = 5$  but the solution for  $\tan x = -4$  was more problematic. For those candidates that chose to work in degrees, only the method marks were available unless the final answers were in radians. Candidates should be guided by the range given in the question. If  $\pi$  is mentioned then the implication is that a radian answer is required.
- (iii) Correct answers to this part of the question were very rare. Again, candidates should be guided by the mark allocation as to how much work is needed. The link to part (i) was very rarely made and as a result, many candidates produced lots of incorrect work. Of those few candidates that did make the connection with part (i), use of  $4y + 2$  rather than the correct  $2y + 1$  was more common. In questions of this type, candidates should always check for a link with previous parts. It was necessary to identify the integrand as  $\sec^2 (ay + b)$  for any progress to be made.

Answer: (ii) 1.37, 1.82 (iii)  $\frac{1}{2} \tan(2y + 1) + c$

# MATHEMATICS

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Paper 9709/23  
Paper 23

## Key messages

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It is important that, when a question specifies no use of a calculator, that candidates show each step of their work and give their answers in an exact form.

## Comments on specific questions

### Question 1

This was a good starter question for most candidates. The most popular method of solution was to square each side of the inequality and, by factorisation or otherwise, obtain the critical values. Some candidates, having obtained the critical values, then omitted the actual solution. There were some errors in factorisation, but in general, there has been a marked improvement in the solution of inequalities, with the solutions of the type  $x < \frac{7}{2}$ ,  $x < -\frac{3}{4}$  being less common than in the past.

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### Question 2

- (i) Most candidates knew the form of differentiation of logarithmic functions. The main error was to give an answer of  $\frac{3}{2x+9}$  for  $\frac{d}{dx}(\ln(2x+9))$ , rather than the correct  $\frac{2}{2x+9}$ . The solution of this type of function equated to zero sometimes taxed the algebraic skills of some candidates.
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Answer: (iii) 1.56

### Question 7

- (i) Many candidates did not achieve in this part of the question. Many attempted to make use of the identity  $\operatorname{cosec}^2 2x = 1 + \cot^2 2x$  which was not the way forward. Candidates should always look at the right hand side of the given identity to see what form they should try to make use of. In this case, there were no double angles, so the use of the double angle formulae involving  $\sin 2x$  and  $\cos 2x$  needed to be made use of together with the fact that  $\operatorname{cosec} 2x = \frac{1}{\sin 2x}$ . For those candidates that did make use of a correct approach, errors involving  $\sin^2 2x = 2 \sin^2 x \cos^2 x$  meant that it was impossible to obtain the required result. When this happens, it is better not to 'contrive' to obtain the given answer as often possible method marks are lost.
- (ii) Usually done very well, with most candidates making use of the result from part (i) to obtain a quadratic equation in  $\tan x$ . Most were able to obtain the correct solution for  $\tan x = 5$  but the solution for  $\tan x = -4$  was more problematic. For those candidates that chose to work in degrees, only the method marks were available unless the final answers were in radians. Candidates should be guided by the range given in the question. If  $\pi$  is mentioned then the implication is that a radian answer is required.
- (iii) Correct answers to this part of the question were very rare. Again, candidates should be guided by the mark allocation as to how much work is needed. The link to part (i) was very rarely made and as a result, many candidates produced lots of incorrect work. Of those few candidates that did make the connection with part (i), use of  $4y + 2$  rather than the correct  $2y + 1$  was more common. In questions of this type, candidates should always check for a link with previous parts. It was necessary to identify the integrand as  $\sec^2 (ay + b)$  for any progress to be made.

Answer: (ii) 1.37, 1.82 (iii)  $\frac{1}{2} \tan(2y + 1) + c$

# MATHEMATICS

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| <p><b>Paper 9709/31</b><br/><b>Paper 31</b></p> |
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## Key messages for candidates:

- Set work out clearly.
- When working towards a given answer, it is essential to include enough detailed steps so that there are no gaps in the solution.
- If a question asks you to “show all necessary working” then it is not appropriate to use short cuts available on a calculator.
- Do not overwrite a pencil solution in ink, or overwrite an erased solution – the result is often totally illegible when scanned.
- Study all topics on the specification to give yourself the maximum chance of success.

In the following comments on individual questions it should be understood that the given answers are not always the only acceptable form of the final answer.

## General comments

The responses of the candidates to this paper were very varied. There were several very good responses, but also a minority of candidates who offered no responses to most or all of the questions. The stronger candidates had plenty of time to offer responses to all ten questions.

The best work was clearly set out with a narrative explaining what the candidate was doing. In contrast, some candidates did not use mathematical notation correctly, most notably the absence of brackets in expressions.

Some candidates are not showing sufficient working to support their answers. This is particularly true when a question has a given answer, but also when the question requests “all necessary working”.

The questions that the candidates found most difficult were **Question 5(a)** (integration by substitution), **Question 5(b)** (integration of  $\cos^2 \theta$ ), **Question 7(iv)** (proof of an equilateral triangle), **Question 9(i)** (partial fractions), and **Question 10** (vectors). Many candidates did not seem to have a confident working knowledge of complex numbers or of vectors.

## **Question 1**

The majority of candidates who understood how to apply the laws of logarithms to transform the given equation usually arrived at the correct equation,  $3x^4 = 16$ , and from there a correct solution. The solution  $x = -1.52$  is not possible because of the term  $\ln x$  in the original equation.

Many candidates made no progress with the question at all because they started with the incorrect statement  $\ln(x^4 - 4) = \ln x^4 - \ln 4$ .

*Answer:* 1.52

## Question 2

- (i) The majority of candidates started by using the correct expansions of  $\cos(A - B)$  and  $\sin(A - B)$ . Many then went on to substitute the exact values and rearrange the equation to obtain an exact value for  $\tan x$ . The most common errors were to use  $\tan x = \frac{\cos x}{\sin x}$ , or a completely false start with expansions such as  $\sin(x - 60^\circ) = \sin x - \sin 60^\circ$ .
- (ii) Many candidates scored at least one mark here because they were able to use the periodicity of  $\tan x$  to obtain a second solution from their first solution.

Answer: (i)  $\frac{3\sqrt{2} + \sqrt{3}}{1 - 3\sqrt{2}}$  (ii)  $118.5^\circ, 298.5^\circ$

## Question 3

A few candidates used the product rule for differentiation, but the majority used the quotient rule. The differentiation was usually correct in form, but there were often errors in the coefficients, particularly with  $\sec^2 \frac{x}{2}$  in place of  $\frac{1}{2} \sec^2 \frac{x}{2}$ . Some candidates stopped at this point, apparently unable to find a way

forward. Many did go on to form an equation in  $\tan \frac{x}{2}$  or  $\sin x$ . A minority of candidates did reach the two correct solutions, although answers were not always given to the required accuracy. Although the question clearly expects an answer in radians, a few candidates gave answers in degrees.

Answer: 0.340 2.802

## Question 4

Those candidates who completed the division correctly or who used an unknown quadratic factor  $x^2 + Bx + C$  usually reached the correct factor and the correct values for  $a$  and  $b$ . A few candidates who obtained the correct quadratic factor then used remainder theorem with the factors  $(x + 1)$  and  $(x + 2)$  to deduce the values of  $a$  and  $b$ . Many candidates who attempted the division made arithmetic slips in the course of their working.

Some candidates attempted to use the complex factors of  $x^2 - x + 1$  but none of them got as far as considering the real and imaginary parts of the resulting equation.

Answer:  $a = 1, b = 2$

## Question 5

- (i) Few candidates were successful in demonstrating the given result. Many did not consider  $\frac{dx}{d\theta}$  at all, and the square root in the integral was frequently overlooked. A minority of candidates showed fully correct working, including an explanation for swapping the limits to deal with the minus sign.
- (ii) The minority of candidates who knew that they needed to use the double angle formula to express  $2\cos^2 \theta$  as  $1 + \cos 2\theta$  usually reached the correct answer. A large number of incorrect alternatives were seen.

Answer: (ii)  $\frac{\pi}{6}$

### Question 6

- (i) Those candidates who recognised this as a variable separable equation usually made good progress, and often reached a correct relation between  $x$ ,  $t$  and  $k$ . After a correct start, the most common error was to confuse  $k$  with a constant of integration. Most incorrect approaches involved the substitution of  $x = 100$  before any attempt to deal with the differential equation.
- (ii) Candidates with a relation between  $x$ ,  $t$  and  $k$  did adopt the correct approach of using  $t = 25$ ,  $x = 80$  to find an expression for  $k$  and then using their  $k$  with  $x = 40$  to find  $t$ .

Answer: (i)  $\ln x = -\frac{2}{3}kt\sqrt{t} + \ln 100$  (ii) 64.1

### Question 7

- (i) This question required candidates to show all necessary working, so simply writing down the two roots was not acceptable. Candidates needed to recognise this as a quadratic equation in  $z$ , and to use an appropriate method to solve it. Many candidates did get as far as a correct expression involving  $\sqrt{-8}$ , but not all of them simplified this correctly.  $8i$  was a common error, and several candidates did not split 8 as  $4 \times 2$ .
- (ii) There were many blank responses to this question implying candidates were not familiar with drawing an argand diagram.
- (iii) The work here needed to follow correctly from the candidate's answer to part (i), and this was often the case. However, some candidates were clearly working back from the result of part (iv). The work required was straight forward trigonometry; candidates working with their answers from (i) usually scored at least two marks.
- (iv) It was not always evident that candidates understood what was needed to demonstrate that the triangle is equilateral. With a correct answer to part (iii) the simplest approach was to show that their two roots had equal moduli, hence achieving an isosceles triangle with angle  $60^\circ$ . This approach was not seen, most candidates found the lengths of all three sides of the triangle.

Answer: (i)  $z = -\sqrt{6} \pm \sqrt{2}i$  (iii)  $60^\circ$

### Question 8

- (i) Many candidates demonstrated a correct method for integration by parts. The most common error was in division by  $\frac{1}{2}$ . Most candidates used the limits correctly, although a minority did not appear to consider the lower limit. The final step, to get to the given equation proved to be quite a challenge for many candidates. A large number of incorrect algebraic statements were seen.
- (ii) A large number of candidates considered the value of  $2\ln(a+2) - a$  at the given points and reached a correct solution. Candidates who considered the value of  $2\ln(a+2)$  did not always go on to make the correct comparison with the value of  $a$ .
- (iii) Candidates who started with a value of  $a_0$  close to 3 had more work to do than candidates who started at the other end of the interval. This iteration converges quite slowly, so some candidates drew the false conclusion  $a = 3.35$ , when a couple more iterations would have shown that this was not correct.

Answer: (iii) 3.36

### Question 9

- (i) Only a small minority of candidates scored full marks for this part of the question. Most candidates chose the form of partial fractions based on the denominator of the given fraction, and did not consider the numerator, so they did not recognise this as a top heavy fraction, leaving themselves a maximum score of 1 mark since any values obtained from an incorrect form are entirely fortuitous.
- (ii) Candidates who understood how to apply the binomial expansion were able to score marks here if they used their partial fractions correctly. The basic process was clearly understood by several candidates, but there were many slips in the algebra and arithmetic. Many candidates attempted the transformations  $(x-1)^{-1} = (-1)^{-1}(1-x)^{-1}$  and  $(3x+2)^{-1} = 2^{-1}\left(1+\frac{3x}{2}\right)^{-1}$  but only a minority achieved the accuracy to complete the expansions correctly. With no constant term in the partial fractions, a maximum of three marks was available.

Answer: (i)  $4 + \frac{3}{x-1} - \frac{1}{3x+2}$  (ii)  $\frac{1}{2} - \frac{9}{4}x - \frac{33}{8}x^2$

### Question 10

Many candidates offered no response to the vector question, and those who did try it often had more success with the second part.

- (i) Those candidates who used the first method on the mark scheme to find an expression for  $\overrightarrow{PQ}$  and then find  $Q$  if  $\overrightarrow{PQ}$  is perpendicular to the line, usually completed this part of the question correctly. The other popular approach was to follow the third alternative in the mark scheme; find  $\overrightarrow{AP}$  and consider the length of the projection of  $\overrightarrow{AP}$  on the line. This led to several partial solutions, with candidates unsure about how to reach the final answer from their initial working.
- (ii) The most popular approach here was to use the vector product of two vectors in the plane. Those candidates who followed this route usually completed it correctly (without making a sign error in the vector product). The other common approach was to assume a normal vector  $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$  and form equations in  $a$ ,  $b$  and  $c$ . For both of these approaches, the most common error was to use the direction of the line, rather than the position vector of a point, to evaluate  $d$ .

Answer: (i) 1.22, (ii)  $4x + y - 2z = 8$

# MATHEMATICS

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|                                                 |
|-------------------------------------------------|
| <p><b>Paper 9709/32</b><br/><b>Paper 32</b></p> |
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## Key messages for candidates:

- Presentation and correct use of notation (in particular the correct use of brackets) is important and can help to avoid errors.
- Work in pencil that is then over-written in pen can result in a response that is completely illegible when scanned. A similar problem occurs if work is erased and overwritten. Strike through the first attempt and start afresh.
- If a question asks for a given result, or asks for “all necessary working” to be shown, then those candidates who take a short cut using functions on their calculator will not earn full marks. Similarly, if a question clearly indicates that a calculator should not be used, then an alternative method should be employed.
- Be prepared to answer questions on the whole of the syllabus.
- Check your work carefully. Many marks are lost through arithmetic slips and steps that defy the basic rules of algebra.

In the comments on individual questions that follow it should be noted that the answers given are not necessarily the only acceptable forms.

## General

This paper proved to be accessible to the majority of candidates; most candidates offered solutions to all ten questions, and some of the work seen was of a very high standard. There was a minority of candidates who were inadequately prepared for the paper, offering no attempt in several topics.

The best work was clearly set out, with an indication of what the candidate was attempting to do at each stage. At the other extreme, some work did not appear to follow any logical path and contained many arithmetic and algebraic errors.

The first two questions (indices and modulus, and trigonometry) should have been straight forward, but processing errors meant that relatively few candidates reached the correct answers. It was pleasing to note that the responses to **Question 5** (implicit differentiation followed by use of the gradient) showed a marked improvement on a similar question in last year’s paper. The questions that caused most difficulty were **Question 7(ii)** (angle on an Argand diagram) and **Question 10(iii)** (equation of a plane).

## **Question 1**

Most candidates did start by attempting to achieve a non-modular equation or pair of equations. Those candidates who opted for two separate equations usually succeeded in reaching at least one correct value for  $2^x$ . Candidates who obtained a value for  $2^x$  usually went on to find the corresponding value of  $x$ . Some candidates went directly from a value of  $2^x$  to a value of  $x$  with no indication of method, despite the question asking for “all necessary working”. The most common errors centred in attempts to square the original equation. Some candidates squared only one side of the equation, or forgot to square the 3. There were many incorrect statements such as  $3 \times 2^x = 6^x$ , from which candidates could not recover. The algebra can be simplified by using a substitution such as  $u = 2^x$ , but those candidates who opted for  $x = 2^x$  should be advised to think more carefully about how they name variables. Some candidates obtained both possible values for  $x$  but then rejected the negative answer.

*Answer:*  $-0.415, 0.585$

## Question 2

Many candidates started well by rewriting the given equation in terms of either  $\tan \theta$  or  $\sin \theta$  and  $\cos \theta$ . A few quoted the formula for  $\cot(A+B)$  in terms of  $\cot A$  and  $\cot B$ . Only a minority of candidates were successful in following through with accurate algebra to obtain the correct final answers. Those candidates who substituted 1 for  $\tan 45^\circ$  at an early stage were more likely to maintain accuracy than those who did not.

Several candidates who obtained  $\tan^2 \theta = \frac{1}{3}$  lost the final mark because they did not consider both possible square roots.

Most candidates did recognise the function  $\cot \theta$  but in their attempts to obtain an equation in terms of more familiar functions, a significant minority opted for  $\tan \theta + \tan(\theta + 45^\circ) = \frac{1}{2}$ . Another common error was to

claim that  $\cot(\theta + 45^\circ) = \frac{\cos(\theta + 45^\circ)}{\sin x}$ .

Answer:  $30^\circ$ ,  $150^\circ$

## Question 3

- (i) Many candidates were unsure about what was required here. Some tried to overcomplicate matters with the use of chain rule and differential equations, and some thought it had something to do with finding the area of the triangle. A straight forward approach starting by showing the lengths of  $PN$  and  $TN$  on the diagram and then finding the gradient of  $PT$  was sufficient.
- (ii) Those candidates who separated the variables correctly often made good progress with this part of the questions. A surprising number of candidates claimed to be substituting  $(4,3)$  but actually used  $(3,4)$ . Several candidates reached a correct expression for  $\ln y$  but did not go on to find  $y$  in terms of  $x$ . Some candidates made errors at the final stage, often using the incorrect form  $e^{a+b} = e^a + e^b$ . Any correct form of the final answer was accepted, so it was not necessary to simplify.

Answer: (ii)  $y = 3e^{\frac{1}{2}x-2}$

## Question 4

- (i) The majority of candidates started by working on the left hand side of the identity, and applied the double angle formulae correctly. There were some sign errors, and some inappropriate cancelling of terms, but many candidates did eventually find a way to simplify the fraction to demonstrate that it was identical to the right hand side. Some solutions took longer than necessary because candidates did not immediately recognise  $1 - \cos^2 \theta$  as a difference of two squares. Those candidates who expressed the denominator in terms of  $\sin \theta$  denominator and reached the expression  $\frac{1 - \cos x}{\sin x}$  were not always able to convert this to the required form.
- (ii) Many candidates completed the integral correctly. The most common error was to omit the "-" in the integration, but the limits were usually applied correctly. Some candidates did not immediately recognise the integral, but still reached the correct answer by applying the substitution  $u = 1 + \cos x$  and using the limits in the correct order.

Answer: (ii)  $\ln \frac{3}{2}$

### Question 5

- (i) All but the weakest candidates recognised the need to use implicit differentiation. A small number of candidates gained an additional  $\frac{dy}{dx}$  by writing this at the beginning of their first line of work, and a few candidates did not obtain zero when differentiating the constant term. There were some slips in the algebra but most candidates who scored the first two marks went on to achieve the required result.
- (ii) The response to this question was better than the response to similar questions in the past, but candidates were often let down by poor algebra. Most candidates who equated  $\frac{dy}{dx}$  to  $-1$  went on to reach  $y = -2x$ , but some overlooked the possibility that  $y = 0$ . A disappointing feature of the work this time was the number of candidates who reached  $x^3 = 1$  and concluded that  $x = \pm 1$ . Even when candidates had correctly obtained  $x = 1$ , they did not always use  $y = -2x$  to obtain the  $y$ -coordinate.

Answer: (ii)  $(\sqrt[3]{3}, 0)$ ,  $(1, -2)$

### Question 6

- (i) Several candidates did not appear to be aware of the formula  $\frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$  for the area of a segment of a circle – some found the area of the isosceles triangle  $ABC$  by splitting it into two right angled triangles, and then needed to use the double angle formula. The most common error was use of the incorrect radius for the semi-circles. There was occasional fudging of the areas of the semicircles, with belated amendments by a factor of 2 in order to produce the required answer.
- (ii) Many candidates evaluated  $f(\theta) = \theta - \frac{\pi}{2} - \sin\theta$  (or its negative) at 2.2 and 2.4 and most of these drew attention to the change of sign. Those who simply evaluated  $g(\theta) = \frac{\pi}{2} + \sin\theta$  usually failed to complete the argument with appropriate comparison of  $\theta$  and  $g(\theta)$ .
- (iii) Almost all candidates were able to iterate successfully at least once, many reaching 2.31, although not all gave enough values to 4 d.p. to justify this. A small number of candidates worked in degrees, leading to an incorrect answer of 1.60.

Answer: (iii) 2.31

### Question 7

- (i) This was well answered by candidates. A few candidates failed to simplify  $-6\sqrt{3}i + \sqrt{3}i$  leaving it as  $(-6\sqrt{3} + \sqrt{3})i$  and similarly in calculations for  $\frac{u}{v}$ . A small number of candidates used their answer to  $uv$  in the numerator of  $\frac{u}{v}$ , despite having indicated the intention to multiply by the complex conjugate. A few candidates did not read the question carefully enough and only evaluated one complex number, either  $uv$  or  $\frac{u}{v}$ . A small number of candidates simply wrote down the answers for both  $uv$  and  $\frac{u}{v}$  with no working shown, and scored no marks.

- (ii) Most candidates gained the mark for plotting  $u$  and  $v$  in an Argand diagram, though some plotted  $uv$  and  $\frac{u}{v}$ . Very few candidates provided a correct demonstration that  $\angle AOB = \frac{2}{3}\pi$ . It was common to see an answer of the form  $\angle AOB = \pi - \tan^{-1} \frac{2}{\sqrt{3}} - \tan^{-1} \frac{1}{3\sqrt{3}} = \frac{2}{3}\pi$  with no supporting working. A minority of candidates used the cosine rule in triangle AOB to good effect, and a small number used  $\arg\left(\frac{u}{v}\right)$ , but these solutions often involved the incorrect statement  $-\frac{\pi}{3} = \frac{2\pi}{3}$ .

Answer: (i)  $-11 - 5\sqrt{3}i$ ,  $-1 + \sqrt{3}i$

### Question 8

- (i) Most candidates applied the product rule correctly and went on to find the value of  $x$  at  $M$ . In the differentiation there were some errors in differentiating  $e^{-\frac{1}{3}x}$ , with 3 often appearing in place of  $\frac{1}{3}$ . Many candidates made the working unnecessarily complicated by not discounting the possibility of  $e^{-\frac{1}{3}x} = 0$  at an early stage. There were several errors in simplifying  $1 - \frac{x+1}{3}$ , almost all involving signs.
- (ii) The integration was often completed successfully. Most candidates attempted to use integration by parts “the right way round”, but there were some errors in dividing by  $-\frac{1}{3}$ . Although the question refers to the shaded region, and this is clearly indicated on the diagram, several candidates used limits 0 and 1 or 0 and 2 to complete the integration.

Answer: (i)  $x = 2$ , (ii)  $9e^{\frac{1}{3}} - 12$

### Question 9

- (i) Candidates were well prepared for this question, and most errors were arithmetic slips. Most candidates obtained  $A = -3$  first by multiplying up and substituting  $x = 3$ , then substituted further values or compared coefficients to deduce the values of  $B$  and  $C$ . Some compared coefficients of  $1, x, x^2$  at the start, and solved the system of three simultaneous equations. A small number started by obtaining  $A = -3$  then added  $\frac{3}{3-x}$  to the original expression and simplified, to obtain the remaining terms.
- (ii) Although most candidates were clearly familiar with the process of using the binomial expansion, there were often errors at the stage of using  $(3-x)^{-1} = 3^{-1}\left(1 - \frac{x}{3}\right)^{-1}$  and  $(2+x^2)^{-1} = 2^{-1}\left(1 + \frac{x^2}{2}\right)^{-1}$ . Arithmetic errors often resulted in candidates scoring only the two method marks.

Answer: (i)  $\frac{-3}{3-x} + \frac{x+2}{2+x^2}$  (ii)  $\frac{x}{6} - \frac{11x^2}{18} - \frac{31x^3}{108}$

### Question 10

- (i) The majority of candidates used a correct method to show that the two lines do not intersect. Many made arithmetic errors, but they usually scored at least two marks. Only a minority of candidates completed the question correctly by also showing that the two lines are not parallel. Some candidates who considered the possibility of the lines being parallel stated that the direction vectors were not equal, which is not the same thing.

- (ii) The majority of candidates used a correct method to find a vector perpendicular to both  $l$  and  $m$ , and many gave a correct answer. Some candidates who used the vector product made a sign error in the  $j$  term, and all methods were subject to arithmetic slips.
- (iii) Only a minority of candidates adopted a correct strategy for finding the equation of  $p$  – many offered no response at all. Most successful candidates adopted the first method on the mark scheme, starting by finding the midpoint of a line segment joining a point on  $l$  to a point on  $m$ . Those candidates who attempted to use the formula for the distance of a point from a plane often misquoted the formula.

Answer: (ii)  $5i - 3j + k$  (iii)  $5x - 3y + z = 7$

# MATHEMATICS

Paper 9709/33  
Paper 33

## Key messages

- In questions where it states 'showing all necessary working' or 'without using a calculator' candidates will receive no marks if they fail to obey this instruction.
- When answers are given in the question, for example **Question 4**, **Question 5(i)**, **Question 7(ii)** and **Question 8(i)**, it is essential to include enough detailed steps so that there are no gaps in the solution.

## General comments

The standard of work on this paper varied considerably and resulted in a spread of marks from zero to full marks. All questions were accessible to well-prepared candidates. The questions or parts of questions that were generally done well were **Question 1** (Binomial expansion), **Question 2** (disguised quadratic equation and logarithms), **Question 3** (integration by parts), **Question 4(i)** (stationary point), **(iii)** (iterative formula), **Question 5(ii)** (solution of trigonometric equation), **Question 6(i)** (partial fractions), **Question 8(i)** (implicit differentiation), **Question 9(a)** (solution of complex number equation) and **Question 10(i)** (intersection of lines). Those that were done least well were **Question 4(ii)** (sketching graphs to show roots), **Question 7(ii)** (evaluation of integral), **Question 8(ii)** (solving for particular value of gradient), **Question 9(b)** (Argand diagram), **Question 10(ii)** (using angle between two vectors to establish a quadratic equation in terms of a parameter).

It was pleasing to see that candidates are aware of the need to show sufficient working in their solutions. Previous reports mentioned this in the context of solving a quadratic equation and substituting limits into an integral. Where answers are given after the Comments on specific questions, it should be understood that the form given is not necessarily the only 'correct answer'.

## Comments on specific questions

### Question 1

Many candidates started the paper confidently by gaining full marks on this question. However, there were a considerable number of candidates for whom it proved to be difficult or impossible. The solution requires the careful extraction of a constant, followed by some index work in order to reach  $A(1+y)^\alpha$ , finishing with the use of the binomial expansion. The extraction of the constant should be undertaken whilst the expression is in its given form, and at the same time the negative sign should be incorporated into the  $\left(\frac{3x}{4}\right)$  term to

produce  $\frac{4}{\left(\sqrt{(4)(1+y)^{\frac{1}{2}}}\right)}$ , where  $y = \frac{-3x}{4}$ . Then moving  $(1+y)^{\frac{1}{2}}$  into the numerator by an index sign change

results in  $\left(\frac{4}{\sqrt{(4)(1+y)^{-\frac{1}{2}}}}\right)$ . The binomial expansion completes the solution. Unfortunately A often had the

incorrect value of  $\frac{1}{2}$ , 4, 8 or 16, likewise  $\alpha$  was  $\frac{1}{2}$  or  $-1$  and  $y$  was  $\frac{+3x}{4}$  or just  $-3x$ .

Answer:  $2 + \frac{3}{4}x + \frac{27}{64}x^2$

## Question 2

This question requested that all necessary working was shown, hence solving the given equation on a calculator to produce 0.638 was not acceptable since no working was shown. Unfortunately too few candidates realised that this equation was a disguised quadratic equation in  $5^x$ . Once this was spotted then this equation needed solving. This should have been undertaken following a clear statement of the quadratic formula. Whilst it is acceptable to solve such an equation on the calculator any error in that process means, as stressed in earlier reports, the method mark cannot be awarded, unlike the quadratic formula approach. Once this solution for  $5^x$  had been found it was necessary to show the working for the final answer, either from  $x = \ln(\text{solution of quadratic equation}) / \ln 5$  or  $\ln_5(\text{solution of quadratic equation})$ . There should only be one answer.

Answer:  $x = 0.638$

## Question 3

Many fully correct solutions were seen. However too many candidates either omitted the coefficient  $\left(\frac{1}{3}\right)$  or introduced a negative sign when integrating. Some appeared to find the derivative of  $\cos x$  instead of its integral. When substituting limits candidates would be well advised to just substitute their limits in one step and do the actual evaluation in the following step. Trying to combine the two steps required the evaluation of  $\sin\left(\frac{\pi}{2}\right)$ ,  $\cos\left(\frac{\pi}{2}\right)$  and  $\cos(0)$ . If any errors were introduced, it was often not possible to decipher whether the limits had actually been substituted correctly. If this was the situation then the method mark was not awarded.

Answer:  $\frac{(\pi - 2)}{18}$

## Question 4

- (i) This was another question that usually resulted in candidates scoring full marks. Very few candidates made the mistake of substituting  $x = p$  before differentiating. Candidates who used the quotient formula with the signs in the numerator reversed could still obtain the given answer but scored no marks as they used an incorrect method.
- (ii) In this question some very weak mathematics was observed. Both graphs caused serious difficulty. As  $\ln x$  only has a domain  $x > 0$  both graphs could be restricted to this domain. However, it was essential that the graphs did cover the whole of this region, namely  $\ln x$  for  $x > 0$  and not just  $x \geq 1$ . In addition this graph should have had the asymptote  $x = 0$ , passed through  $(1, 0)$  and had a decreasing gradient. The other graph  $1 + \frac{3}{x}$  should also have had the asymptote  $x = 0$ , but in addition the asymptote  $y = 1$ . Most had this graph approaching the  $x$ -axis, and even if they didn't it was usually clearly below the point  $(0, 1)$  marked on the  $y$ -axis. Furthermore it was necessary to extend the separation of the graphs well past their point of intersection. This would indicate that there isn't another root. Finally the root needs to be clearly indicated by marking the intersection point or making some comment about only one intersection point, hence one root.
- (iii) Nearly every candidate scored full marks, working to 4 d.p. and giving the correct answer to 2 d.p. However, a very small number of candidates could do no more than substitute  $p$  into the formula, failing to realise that they had to take a numerical value to commence their iterative procedure.

Answer: (iii) 4.97

### Question 5

- (i) Candidates found this question extremely difficult. Most could expand the expression in brackets, as the question suggested, the binomial expansion proving to be an efficient method. Many candidates then failed to realise that Pythagoras was required, as this bracket had the value of 1. Candidates often had the expansion equal to zero and introduced the 1 into their answer with no justification to try to reach the given expression. Because of the given answer, a logical argument was needed with no gaps in the working. For example, candidates should not be jumping from  $3\cos^2 x \cdot \sin^2 x$  to  $\left(\frac{3}{4}\right)(\sin^2 2x)$ . Since this expression was in the given answer, such a large gap in working loses the method mark and hence the accuracy mark. A complete answer would include either defining  $\sin 2x$  or introducing the working  $3\cos^2 x \cdot \sin^2 x = \left(\frac{3}{4}\right)(2\cos x \cdot \sin x)^2 = \left(\frac{3}{4}\right)(\sin^2 2x)$ .
- (ii) Candidates usually produced accurate working but a significant number did not manage to obtain all four answers because they did not extend the interval to take account of the  $2x$ . There were a few candidates who either failed to square root  $4/9$  or who had  $x$  instead of  $2x$ . As stated in the rubric for this paper, values of angles need always to be given to 1 d.p.

Answer: (ii)  $20.9^\circ$ ,  $69.1^\circ$ ,  $110.9^\circ$ ,  $159.1^\circ$

### Question 6

- (i) This was another question where candidates usually performed well. Although a few candidates chose completely inappropriate partial fractions, such as  $\frac{1}{y}$ , or  $\frac{1}{(4-y)}$  and  $\frac{1}{(4+y)}$ . Unfortunately several candidates confused themselves by believing the factors of  $(4-y^2)$  to be  $(y-2)$  and  $(y+2)$ . However some amended the sign and were able to recover the correct answer.
- (ii) Nearly all candidates separated the variables correctly and integrated to gain the necessary  $\ln$  terms, although too many omitted to deal with the negative sign in their integration of  $\frac{1}{(2-y)}$ . This meant that they were only able to gain the method mark for determining the value of the constant. Many candidates failed to realise how to extract an expression for  $y$  in terms of  $x$  from their correct  $\ln\left[\frac{(2+y)}{(2-y)}\right]$  or  $\frac{(2+y)}{(2-y)}$  expressions.

Answers: (i)  $\left(\frac{1}{4}\right)\left[\frac{1}{(2+y)} + \frac{1}{(2-y)}\right]$  (ii)  $y = \frac{2(3x^4 - 1)}{(3x^4 + 1)}$

### Question 7

- (i) This question clearly stated that a calculator was not permitted, something that had clearly been ignored once  $\alpha = 1.107$  appeared. Had candidates followed the instructions and expanded  $R \cos(\theta - \alpha)$  correctly, expressions for  $R \cos \alpha$  and  $R \sin \alpha$  were easily available, then the use of Pythagoras and some simple trig produced the correct exact expressions for  $R$  and  $\tan \alpha$ . Some candidates finished with  $\tan \alpha = \frac{1}{2}$ , perhaps because they were expanding  $R \cos(\theta - \alpha)$  incorrectly or calculating  $\tan$  as  $\cos/\sin$ .
- (ii) Most candidates used the result from (i), however too few spotted that this needed to be converted into  $\sec^2(\theta - \alpha)$  in order to integrate to  $\tan(\theta - \alpha)$ . At this stage, if  $\alpha$  was shown as 1.107 instead of  $\tan^{-1} 2$ , candidates could make no more progress without further violation of the rubric. Once the limits were substituted and  $\tan^{-1} 2$  was included, use of the  $\tan(A - B)$  formula was needed.

Answer: (i)  $R = \sqrt{5}$ ,  $\tan \alpha = 2$

### Question 8

- (i) Since the answer was given in the question, all steps in the solution needed to be clearly seen. That involved including brackets and showing clearly any cancelling. Often there was no indication that the right hand side had been differentiated to zero. When candidates failed to show such detail, this often resulted in the method mark and hence the accuracy mark being withheld.
- (ii) This question, along with **Question 5(i)** and **Question 10(ii)**, proved very challenging for the majority of the candidates. Here the problem lay in not understanding what was meant by 'parallel to the y-axis'. Instead of realising that  $\frac{dy}{dx}$  had to be infinite, the majority of candidates set  $\frac{dy}{dx}$  to zero or even 1. Both of these resulted in complex equations to solve for which candidates received no credit. The candidates who successfully set the denominator to zero often omitted the pair of coordinates resulting from the solution  $y = 0$ .

Answer: (ii)  $(a, 0)$ ,  $(-a, 2a)$

### Question 9

- (a) Some candidates tried to solve immediately for  $z$ , and whilst it is possible with some clever mathematics, few candidates could make any progress. The more sensible approach, and the one adopted by successful candidates, was to set  $z = x + iy$  and equate real and imaginary parts to obtain two linear equations in  $x$  and  $y$ . The usual problem was obtaining the correct real term from  $-iz^*$ , as it involved the evaluation of  $(-i)$  squared.
- (b) The main difficulty in the Argand diagram sketch was understanding that  $|z| \leq 3$  represents a circle of radius 3 and centre  $(0, 0)$ . Too often the circle had the incorrect centre and there was no indication of what its radius was. Likewise the line  $y = 2$  was rarely labelled. In fact several candidates had their line clearly shown at  $x = 2$ . For some reason the trigonometry to calculate the greatest value of  $\arg$  proved difficult. In fact with the right-angled triangle with sides 2 and 3, this was a trivial matter of subtracting  $\sin^{-1}$  of one angle from  $\pi$  or adding  $\cos^{-1}$  of the other angle to  $\frac{\pi}{2}$ .

Answers: (a)  $1 + 2i$  (b) 2.41

### Question 10

- (i) This was another question that candidates found relatively easy, especially if they noticed that the  $j$  component of the equated vectors produced one of the parameter values without the need to solve simultaneous equations. However, a considerable number of candidates scored zero since they equated the line  $l$  with the direction vector **AB** instead of with the line through points  $A$  and  $B$ . Most candidates adopted the method of showing that one of the three components was not satisfied by their two parameters. To follow this approach it is necessary to determine the actual values of the components, hence  $A \neq B$ , and not leave their answer as  $a + b \neq c + d$ . A few candidates found a second, different, value for one of the parameters to show inconsistency.
- (ii) All candidates found this question difficult. A sketch would have been useful, as it would have stopped candidates working with vectors **PA** or **BA**. It would also have prevented them from using vector **BP** instead of **AP** or **AB**. In addition far too many candidates had a problem in determining the actual vector **AP**, by subtracting the vector **OA** from the line  $l$  given in (ii). Those candidates that did overcome this problem often had arithmetical errors in their expression. The methods for the magnitudes and scalar product were usually correct, and it was pleasing to see the detailed working which has often been absent in the past. However due to arithmetical or algebraic errors, only a few candidates achieved the given quadratic equation in  $t$ . Despite these errors, many candidates salvaged a method mark at the end by solving the given equation and substituting their correct solutions into vector **OP**. Only the very able candidates realised the importance of starting with the correct vectors **AP** and **AB** and the need to square their expression to reach the answer given. The squaring process resulted in two final answers for **OP**, but one is spurious since it corresponds to an angle  $PAB$  of  $60^\circ$  rather than  $120^\circ$ . This can easily be seen as substituting

$t = \frac{-2}{3}$  into the expression for the scalar product leads to a positive value rather than a negative value.

*Answer:* (ii)  $2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$

# MATHEMATICS

Paper 9709/41  
Paper 41

## Key messages

- Non-exact numerical answers are required correct to three significant figures or angles correct to one decimal place as stated on the question paper and cases where this was not adhered to were seen in **Question 2, Question 3, Question 4, Question 6** and **Question 7**. Candidates would be advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.
- When answering questions involving an inclined plane, a force diagram could help candidates to include all relevant terms when forming a Newton's Law equation or a work/energy equation. This was particularly noticeable here in **Question 3, Question 6** and in both parts of **Question 7**.
- In questions such as **Question 4** in this paper, where displacement is given as a cubic function of time, then calculus must be used and it is not possible to apply the equations of constant acceleration.

## General comments

There were some excellent candidates who produced very good answers on this paper. Overall a wide range of performance was seen but the paper was generally well answered.

Some candidates lost marks due to not giving answers to 3 significant figures as requested and also due to prematurely approximating within their calculations leading to the final answer. This was often seen in **Questions 2, 3, 4, 6** and **7**. In **Question 6** the sine of an angle was given in the question. In such questions it was not necessary to determine the actual angle to one decimal place as this often leads to premature approximation and frequently also to a loss of accuracy marks.

One of the rubrics on this paper is to take  $g = 10$  and it has been noted that virtually all candidates are now following this instruction. In fact in some cases it is impossible to achieve a correct given answer, such as in this paper in **Question 3**, unless this value is used.

## Comments on specific questions

### Question 1

There are two possible approaches to this question. By realising that the final position of the particle is 5 metres below the start point, the total time for the journey can be determined from a single application of the constant acceleration equation  $s = ut + \frac{1}{2}at^2$  by setting  $s = -5$  with  $u = 24$  and  $a = -g = -10$ . Alternatively using the equation  $v = u + at$  with  $u = 24$ ,  $v = 0$  and  $a = -g = -10$ , the time,  $T_1$ , taken to reach the highest point can be found. In order to find the time,  $T_2$ , taken to fall to the ground from the highest point it is necessary to find the height at which the particle comes to rest. Once this is found the constant acceleration equations can be used to find the required time to fall from rest at the highest point. The total time from projection until  $P$  reaches the ground is  $t = T_1 + T_2$ . An error that was often seen was for candidates to find the time to reach the highest point and then simply double this value. However, this only finds the time taken to reach the original start position and not the time to the ground

*Answer:* Time from projection until  $P$  reaches the ground is 5 seconds

## Question 2

Although it is possible to solve this problem using either Lami's theorem, where the required angles appear directly in the Lami equations, or by considering the triangle of forces, most candidates chose to consider this problem by resolving forces. It is necessary to introduce an acute angle  $\alpha$ , say, between the 10 N force and the direction directly opposite to the 8 N force. Alternatively the acute angle,  $\beta$ , say, between the 10 N force and the direction directly opposite to the 6 N force could be used. Once either of these angles are found then the problem is solved by using the given diagram. Most candidates performed well on this question and a large number of correct answers were seen. Some candidates stopped once they had found either  $\alpha$  or  $\beta$  without completing the problem

**Answers:** The angle between the 6 N force and 10 N force is  $126.9^\circ$  (to 1 decimal places)  
The angle between the 8 N force and 10 N force is  $143.1^\circ$  (to 1 decimal places)

## Question 3

- (i) In this question the answer is given and so care must be taken to ensure that all working is shown. In addition, the given answer will not be achieved unless the value of  $g = 10$  is used. It is given that particle  $P$  is in equilibrium and so the most straightforward method is to use the fact that the forces acting on  $P$  parallel to the direction of the plane are in balance, namely the component of the weight down the plane balances the component of the 100 N force up the plane. In order to find the normal reaction between the plane and  $P$ , it is necessary to state the balance of forces perpendicular to the plane involving the component of the weight, the component of the 100 N force and the normal reaction. Some candidates wrongly assumed that the reaction was merely the component of the weight. As suggested in the key points above, it is most useful here to draw a good and complete force diagram for the situation. Many students correctly solved this problem although several used the wrong components when resolving forces.
- (ii) In this situation, motion takes place and since the acceleration is required then it is necessary to use Newton's second law applied to the particle in the direction parallel to the plane. There are two forces involved, the component of the weight down the plane and the component of the 100 N force up the plane. It may not be clear to candidates whether the acceleration will be up or down the plane but the sign of the acceleration found will determine this. Most candidates scored well on this question.

**Answers:** (i)  $\theta = 66.4$  (given) The normal reaction  $R = 161$  N (to 3 significant figures)  
(ii) The acceleration of  $P$  is  $5.83 \text{ ms}^{-2}$

## Question 4

- (i) Here the displacement is given as a cubic function of time. This means that it cannot be assumed that the motion is one of constant acceleration. Some candidates made this mistake when approaching this problem. In this part the velocity is required and it is necessary to use calculus and the fact that  $v = \frac{ds}{dt}$  and so the velocity is found merely by differentiating the given expression for  $s$ . Most candidates who used calculus achieved the correct answer here.
- (ii) Most candidates realised that it was necessary to solve the equation  $v = 0$  to find the values of  $t$  for which  $P$  is at instantaneous rest. Since the expression found for  $v$  in part (i) is a quadratic function it is necessary to solve a quadratic equation. Most candidates successfully factorised this quadratic and some used the formula method to find the required values. The majority of candidates found the values correctly. Some candidates only expressed their answer in decimals rather than using fractions and often the  $\frac{2}{3}$  answer was stated wrongly to 3sf. Some candidates wrongly tried to continue using constant acceleration equations.

- (iii) There are two possible approaches to this question. Using calculus, the minimum value of velocity can be found by differentiation of the expression for  $v$  and setting this to zero. This will give the value of  $t$  at which the minimum occurs and most candidates took this approach. Alternatively the expression for  $v$  can be written using completing the square and the problem can be solved this way. Some candidates realised that as the expression for  $v$  is a quadratic and the two zeros are known, then the minimum value will occur midway between these two values of  $t$ . Some candidates tried to evaluate  $v$  at a number of different  $t$  values in order to find the minimum but this rarely achieved the correct result

Answers: (i) The velocity is given by  $v = 3t^2 - 8t + 4$  (ii) The two values of  $t$  at which  $P$  is at instantaneous rest are  $t = 2$  and  $t = \frac{2}{3}$  (iii) The minimum velocity of  $P$  occurs when  $t = \frac{4}{3}$  and is  $v = -\frac{4}{3} \text{ ms}^{-1}$

### Question 5

- (i) From the information given in the question there are two possible approaches. A  $v$ - $t$  diagram for the motion could be set up and this would give an excellent visual approach. Alternatively the constant acceleration formulae could be used for each part of the motion. This was the approach that most candidates took. Using constant acceleration formulae the distance is found in two parts. The first 6 seconds involves constant (uniform) acceleration and so the distance can be found using  $s = \frac{1}{2}(u + v) \times t$  with  $u = 0$ ,  $v = 12$  and  $t = 6$ . The second part has zero acceleration and so the distance travelled is  $s = ut$  since  $a = 0$ , and using  $u = 12$  and  $t = 10$  the distance from  $t = 6$  to  $t = 16$  is found. Adding these two parts gives the required distance travelled in the first 16 seconds. If the  $v$ - $t$  approach is taken, the distance travelled can be found with one evaluation using the area of a trapezium as  $s = \frac{1}{2}(16 + 10) \times 12$ . Most candidates scored well on this part of the question. However, the sketch of the displacement-time graph caused problems for many candidates. Most thought that the displacement-time graph consisted of three straight lines. In fact when acceleration or deceleration takes place the displacement-time graph is a curve.
- (ii) In this part of the question again a  $v$ - $t$  graph could be used but most candidates used the constant acceleration formula  $s = \frac{1}{2}(12 + V) \times 4$  for the motion over the final 4 seconds using their own value of  $s$  as  $200 - 156$  with 156 replaced by their answer in part (i). Several wrongly thought that  $V$  could be found by dividing the remaining distance by 4, assuming no deceleration.

Answers: (i) The distance travelled in the first 16 seconds is 156 m  
The displacement time graph involves three curves which join smoothly.  
A curve, concave upwards from (0,0) to (6,36).  
A straight line from (6,36) to (16,156).  
A curve, concave downwards from (16,156) to (20,200)  
(ii) The value of  $V$  is 10

### Question 6

- (i) In this question it was necessary to realise that since the car was moving at constant speed, the net force acting on the car is zero. This means that the driving force must exactly balance the resistive force of 850 N. Some candidates wrongly thought that the driving force was related to the weight of the car. Use of the equation  $P = Fv$  enabled the power (the rate at which the engine of the car is working) to be determined. The question also asked for this to be given in kW and some candidates failed to adjust the units of their answer.
- (ii) Once again the car is moving at constant speed but in this case up a hill and with a different power. The balance of forces still exists but here the driving force must balance the resistance force and the component of the weight. Hence a three term force equation must be set up in terms of the unknown speed,  $v$ , of the car. The driving force can be expressed as  $P/v$  which in this case is  $63000/v$  and the equation can be solved for  $v$ . Some errors that were seen were when candidates

either forgot to include the resistance force or forgot to include the weight component. Many candidates found the correct result

- (iii) In this part of the question, the car is travelling down the hill with a different power and a different unknown non-constant resistance. Candidates must be careful here not to assume constant resistance and also to note that the question asks for use of energy methods. There are four distinct terms which are needed in order to set up a work-energy equation. The increase in kinetic energy can be found from the information given. Also the loss of potential energy of the car can be found. The work done by the engine of the car requires use of the formula Work Done = Power  $\times$  Time. The final term in the work-energy equation is the work done against the resistance force which is to be determined. The relationship between these terms is that WD by the engine + PE loss = KE gain + WD against resistance. On substitution of three of these terms, the work done against resistance can be found. Candidates found this to be one of the more difficult questions on the paper. Many found some of the terms in the work energy equation while others often found the three required terms but used the wrong signs in the equation itself. A number of candidates did in fact assume that resistance is constant.

Answers: (i) The rate at which the engine of the car is working is 30.6 kW (ii) The speed of the car is  $30 \text{ ms}^{-1}$   
(iii) The work done against the resistance is 270 000 J

### Question 7

- (i) This question involves connected particles, each on an inclined plane and as there is no friction involved, one method is to apply Newton's second law parallel to the plane in each case. For both particles there are two forces acting, the component of the weight and the tension. The tension is the same throughout the string but some wrongly used different tensions acting on each particle. When the two equations of motion are stated, they produce two equations each involving the tension  $T$  and the acceleration  $a$ . These equations can now be solved simultaneously to find the value of  $a$ , which is needed in order to find the required value of  $v$ . Alternatively, since only  $a$  was needed, the equation for the system can be found and many candidates took this approach. The main cause of error was candidates using the wrong component of weight parallel to the plane such as  $0.8g \cos 45$  for particle  $A$  and  $1.2g \cos 30$  for particle  $B$  or even not using a component but using  $0.8g$  parallel the plane for  $A$  and  $1.2g$  for  $B$ . Once the acceleration is found then use of  $v^2 = u^2 + 2as$  with this value of  $a$ ,  $u = 0$  and  $s = 0.4$  enabled the final speed  $v$  to be found. An alternative approach to this problem is to use energy methods applied to the whole system. Some candidates attempted this successfully. Some tried to solve the problem by considering only one particle but if this is attempted then the work done by the tension must also be included.
- (ii) In this part it is first necessary to resolve forces perpendicular to each plane in order to determine the normal reaction in each case so that the friction forces can be found using  $F = \mu R$ . The two normal reactions are different but some wrongly took them to be the same as one another and also this lead to the friction forces being wrongly taken to be the same on each particle. The forces acting parallel to the plane for both  $A$  and  $B$  are in equilibrium and in both cases these forces consist of the weight component, the tension in the string and the friction term. The two equations will involve the unknown tension  $T$  and the coefficient of friction  $\mu$ . Solving these simultaneously and eliminating  $T$  gives the required value of  $\mu$ . One error which was often seen was that candidates wrongly assumed that the tension in this case was the same as that found in 7(i). Similar errors with components were seen here as were highlighted in 7(i).

Answers: (i) The speed of  $A$  after each particle has travelled a distance 0.4 m is  $0.370 \text{ ms}^{-1}$  (to 3 sf)  
(ii) The value of  $\mu$  for which the system is in limiting equilibrium is 0.0214 (to 3 sf)

# MATHEMATICS

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Paper 9709/42  
Paper 42

## Key messages

- Non-exact numerical answers are required correct to three significant figures as stated on the question paper and cases where this was not adhered to were seen in **Question 2**, **Question 3**, **Question 5** and **Question 7**. Candidates would be advised to carry out all working to at least 4 significant figures if a final answer is required to 3 significant figures.
- When answering questions involving an inclined plane, a force diagram could help candidates to include all relevant terms when forming a Newton's Law equation or a work/energy equation. This was particularly noticeable here in **Question 5** and both parts of **Question 7**.
- In questions such as **Question 6** in this paper, where acceleration is given as a linear function of time, then calculus must be used and it is not possible to apply the equations of constant acceleration.

## General comments

The paper was generally well done by many candidates although as usual a wide range of marks was seen.

The presentation of the work was good in most cases and as the papers are now scanned, it is important to write answers clearly using black pen.

In **Question 5** and **Question 7**, angles of  $30^\circ$  and  $60^\circ$  were given and these can often lead to exact answers.

The examination allowed candidates at all levels to show their knowledge of the subject, whilst differentiating well between even the stronger candidates. **Question 3** was found to be the easiest question whilst both parts of **Question 7** proved to be the most challenging.

One of the rubrics on this paper is to take  $g = 10$  and it has been noted that virtually all candidates are now following this instruction. In fact in some cases it is impossible to achieve a correct given answer unless this value is used.

## Comments on specific questions

### **Question 1**

Because this question gives information about the total work done by the man, it cannot be assumed that the driving force supplied by the man is constant. This means that this problem must be solved using energy principles. The increase in kinetic energy can be evaluated using the formula  $KE = \frac{1}{2}mv^2$  and the work done against the constant resistance force  $P$  as the man runs 60m can be written as  $WD = 60P$ . The work-energy equation can then be written as Total WD by the man = Increase in KE + WD against resistance. When this equation is solved the value of  $P$  can be found. Several candidates did in fact assume that the driving force supplied by the man was constant and in doing so lost marks.

Answer:  $P = 10.5$

## Question 2

Here the question asks for the power of the engine of the train. This can be determined by use of the formula  $P = Fv$  where  $F$  is the driving force acting on the train and  $v$  is the speed of the train. Since it is given in the question that  $v = 15$ , it is necessary to determine the driving force. This is achieved by applying Newton's second law to the motion of the train. There are three forces acting on the train, the unknown driving force, the given resistance force of 18000 N and the component of the weight acting down the plane. When equating this combination of forces to mass  $\times$  acceleration this leads to a 4 term equation from which the driving force can be found. Once this driving force,  $DF$ , is found, use of the formula  $P = DF \times 15$  gives the required power. Some errors which were seen were not including the weight component in the equation, using the wrong component or using mass rather than weight. Many candidates mistakenly used 0.2 for the acceleration instead of  $-0.2$  since the deceleration was given. However, many candidates produced a well laid out correct answer.

**Answer:** The power of the engine of the train is 2 060 000 W = 2060 kW = 2.06 MW (to 3 significant figures)

## Question 3

Most candidates made a good attempt at this question. The majority of candidates resolved forces vertically and horizontally. When resolving horizontally this produced a two term equation from which the value of  $\theta$  could be determined. Resolving vertically introduced an equation for  $P$  which also included  $\theta$ . Using the value found for  $\theta$  enabled  $P$  to be found. Almost all candidates took this approach and performed extremely well on this question. It is also possible to use either Lami's equation or the triangle of forces method to solve this problem but very few candidates attempted these methods.

**Answers:**  $\theta = 41.4$  (to 1 decimal place)  $P = 3.92$  (to 3 significant figures)

## Question 4

- (i) This question includes a given answer and so candidates need to be careful to show all of their working. It is given that the particle  $P$  moves with constant acceleration, so use of any of the constant acceleration formulae is allowed. There are many different approaches to this problem depending on which equations are used and also which distances are considered. One possible approach is to use  $u$  as the speed at  $A$  and  $a$  as the acceleration. Applying the constant acceleration equation  $s = ut + \frac{1}{2}at^2$  to  $AB$  gives  $100 = 4u + 8a$  and applying the same equation to  $AC$  gives  $248 = 8u + 32a$ . These equations can then be solved for  $u$  and  $a$  which are the values required in the question. When solving these equations some candidates relied on their calculator to solve these simultaneous equations but since one answer is given, more detail of the solution should be shown. Some candidates tried to use these equations applied to  $AB$  and  $BC$  but wrongly assumed that  $u$  was the initial velocity in both equations. Another possible method which could be used is to apply  $s = vt - \frac{1}{2}at^2$  to  $AB$  and  $s = ut + \frac{1}{2}at^2$  to  $BC$ . In this case the final speed  $v$  for  $AB$  is the same as the initial speed  $u$  for  $BC$  since in both cases it is the speed of  $P$  at  $B$  and this leads to equations  $100 = 4v - 8a$  and  $148 = 4v + 8a$  and the given value of  $a$  can be shown very easily. Many other variations on these methods were seen. A common error that was seen was to write  $\frac{100}{4} = 25$  as the speed on  $AB$  and also  $\frac{148}{4} = 37$  as the speed on  $BC$  when in fact the speed varies throughout as the particle travels from  $A$  to  $D$ . These methods did not score any marks. Some candidates merely used the given value of  $a = 3$  to find the speed at  $A$  and this alone earned very few marks.
- (ii) Again there are several methods available to find the distance  $CD$ . One method is to use the given information about the speed at  $D$  along with the value found for the speed at  $A$ . By applying the constant acceleration equation  $v^2 = u^2 + 2as$  to the distance  $AD$  using  $u = 19$ ,  $v = 61$ ,  $a = 3$  then  $s$ , the distance  $AD$  can be found. Once  $AD$  is found the required distance can be found using  $CD = AD - 100 - 148$ . Many candidates correctly used this method to find  $CD$  but those who wrongly found the speed at  $C$  as  $\frac{148}{4}$  from part (i) could only score some of the method marks.

- Answers: (i) The acceleration of  $P$  is  $3 \text{ ms}^{-2}$  (given) and the speed of  $P$  at  $A$  is  $19 \text{ ms}^{-1}$   
(ii) The distance  $CD = 312 \text{ m}$

### Question 5

In this problem it is necessary to consider two extreme cases of when the particle is about to slip up the plane (this will lead to the greatest value of  $P$ ) and when the particle is about to slip down the plane (this will lead to the least value of  $P$ ). There is also another variation in this question. It is not stated in the question whether the force  $P$  is applied up or down the plane. Consider first the case when  $P$  acts up the plane in both cases. In order to use the relationship  $F = \mu R$  needed to find the friction force,  $F$ , the normal reaction,  $R$ , must be found by resolving perpendicular to the plane. This gives a simple equation  $R = 20g \cos 60 = 100$ . In the case of the particle being on the point of slipping down the plane (friction will act up the plane) three forces are in equilibrium and this can be stated as  $P(\text{least}) = 20g \sin 60 - F$ . When the particle is about to slip up the plane (friction will act down the plane) the statement of equilibrium is  $P(\text{greatest}) = 20g \sin 60 + F$ . The given relationship that the greatest value of  $P$  is twice the least value can be used to find the value of  $\mu$ . However it is possible that friction could be large enough that for the case of slipping down the plane,  $P$  may also act down the plane. For slipping up the plane clearly  $P$  must also act up the plane. In this case the equations take the form  $P(\text{least}) = F - 20g \sin 60$  and  $P(\text{greatest}) = F + 20g \sin 60$ . Again use of the given relationship  $P(\text{greatest}) = 2P(\text{least})$  will lead to a value for  $\mu$ . Full marks were available to candidates who chose either of these methods. Most candidates scored well on this question. Some errors made by candidates were to use mass rather than weight in their equations, not including  $F$  in one of the cases and some used  $P(\text{least}) = 2P(\text{greatest})$  which leads to a negative value for  $\mu$ .

Answers: When  $P$  acts up the plane in both cases  $\mu = \frac{\sqrt{3}}{3} = 0.577$

When  $P$  acts down the plane for the least value and up the plane for the greatest  $\mu = 3\sqrt{3} = 5.20$

### Question 6

- (i) In this question the acceleration,  $a$ , is given as a linear function of  $t$  and so calculus must be used and not the constant acceleration equations. It is given that the particle is at instantaneous rest at  $t = 20$  and it is required to find the value of  $t$  when this happens again. The velocity must be found by integrating the expression for acceleration. It is vital that a constant of integration is included. The constant of acceleration can be found by using the condition that  $v = 0$  at  $t = 20$ . This gives a 3 term quadratic expression for  $v$ . The equation  $v = 0$  can be solved, giving one solution as  $t = 20$  and the other will be the required value of  $t$ . An error which was seen was not to include the constant of integration and this meant that it was not possible to use the given information that  $v = 0$  at  $t = 20$  and also lead to an incorrect solution, usually  $t = 50$ . In some cases candidates wrongly attempted to use the constant acceleration equations and this method did not score any marks.
- (ii) In this part of the question the distance travelled between the times of instantaneous rest is required. This involves integrating the expression for  $v$  and using limits of  $t = 20$  and  $t = 30$ . Most candidates integrated correctly and used limits. Some candidates wrongly thought that they could now use the constant acceleration formulae to find the required distance. Candidates who found an incorrect answer to part (i) were still able to score all of the method marks in this part.

Answer: (i) The particle is again at instantaneous rest at  $t = 30$   
(ii) The distance travelled between the times of instantaneous rest is  $20 \text{ m}$

### Question 7

- (i) Most candidates attempted to solve this problem by using Newton's second law applied either to each particle or to the system of two particles in order to find the tension  $T$  and the acceleration  $a$ . When applying Newton's law parallel to the horizontal plane to particle  $A$ , the only force acting is the tension  $T$  in the string. When Newton's law is applied to particle  $B$  in the direction parallel to the slope the forces acting are the weight component and the tension  $T$ . The equations take the form  $T = 1.6a$  and  $2.4g \sin 30 - T = 2.4a$ . These simultaneous equations can be solved to find the required values of  $T$  and  $a$ . An error that was often seen was that the weight was also included when considering the motion of particle  $A$  even though this is in a direction perpendicular to the motion, and some candidates wrongly used  $2.4g$  as the force acting along the plane. Many candidates

scored well on this question. It is possible to use energy methods in this question but very few candidates followed that approach.

- (ii) In this case friction acts on the horizontal surface only. It is necessary to first determine the friction force. The normal reaction,  $R$ , is given by  $R = 1.6g$  and the friction force,  $F$ , is found from  $F = \mu R = 0.2 \times 1.6g = 3.2$  in this case. The problem is broken into two stages. In stage 1 particle  $B$  moves towards the barrier and it is necessary to find the speed of the particles as  $B$  hits the barrier and stops. This will be the initial speed of particle  $A$  in stage 2 as particle  $A$  is brought to rest by the friction force. The approach taken by most candidates was again to use Newton's second law applied either to each particle or to the system of two particles in order to find the new acceleration. Newton's second law equations applied to particles  $A$  and  $B$  take the form  $T - 3.2 = 1.6a$  and  $2.4g \sin 30 - T = 2.4a$ . Again these can be solved to give the new value of  $a$  as  $2.2$ . Using this value of  $a$  in the equation  $v^2 = u^2 + 2as$  with  $u = 0$ ,  $a = 2.2$  and  $s = 1$  will give the final speed of the particles at the end of stage 1. In stage 2 particle  $A$  continues to move parallel to the horizontal plane and the only force now acting in this direction is the friction force  $F = -3.2$  (negative since it acts in the direction opposite to the motion of  $A$ ) and the equation of motion for  $A$  is  $-3.2 = 1.6a$  and hence  $a = -2$ . Particle  $A$  now slows down subject to this deceleration and the distance travelled is once again found by using the equation  $v^2 = u^2 + 2as$  with  $v = 0$ ,  $a = -2$  and the value of  $u$  is the speed found at the end of stage 1. This gives the distance  $s$  travelled in stage 2. The total distance travelled by particle  $A$  is then  $s + 1$ . Again it is possible to approach this problem using energy methods but most candidates used the Newton's law approach. Some common errors that were seen were to assume that the tension in the string and the acceleration here were the same as in part (i), that friction applied to both particles and to use mass instead of weight in the equations of motion.

Answers: (i)  $T = 4.8 \text{ N}$  acceleration of  $A = 3 \text{ ms}^{-2}$  (ii) The total distance travelled by  $A$  is  $2.1 \text{ m}$

# MATHEMATICS

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Paper 9709/43  
Paper 43

## Key messages

Candidates should maintain sufficient accuracy in their working in order to achieve the required accuracy in their final answers.

When forming an equation of motion or an energy equation, candidates are reminded to check that all relevant forces or all energies have been considered e.g. **Question 4** and **Question 6** parts (ii) and (iii).

The instructions on the examination paper show the requirement that  $g = 10$  be used for the acceleration due to gravity. A small number of scripts were seen using  $g = 9.8$  or  $g = 9.81$ .

## General comments

Much of the work seen was of a very high standard with clearly presented, accurate solutions supported by appropriate explanation and diagrams. The easiest questions were found to be **Question 1** and **Question 5**, whilst **Question 4** and **Question 6** parts (ii) and (iii) were found to be more challenging.

## Comments on specific questions

### Question 1

- (i) Nearly every response showed the correct acceleration. This was usually found either by calculating the gradient or by applying  $v = u + at$ .
- (ii) The majority of candidates attempted to form and solve an equation in e.g.  $T$ , with  $T$  representing either the time travelled at constant speed or the time travelled until deceleration. The most concise solutions recognised distance as the area of the trapezium rather than separating the journey into three time intervals. The most common error was to give an answer of 570 seconds (including the 40 seconds of acceleration). A few candidates oversimplified the situation by considering an isosceles trapezium with equal times for acceleration and deceleration. Some others read 'length' of time and calculated the distance travelled at constant speed.
- (iii) This was straightforward for the majority of candidates who calculated the area of the appropriate triangle. A number of candidates took the decelerating time to be 70 seconds ( $600 - 530$ ) without allowing for the 40 seconds of acceleration. A few candidates calculated the deceleration rather than the distance travelled whilst decelerating.

Answers: (i)  $0.4 \text{ ms}^{-2}$  (ii) 530 s (iii) 240 m

## Question 2

This was a straightforward question for many candidates who applied constant acceleration formulae with  $a = g$  or  $a = -g$  for the two stages of motion. A few candidates answered the question as if the ball was projected **upwards** instead of downwards with a consequent increase in the time taken to reach the ground. A few mistakenly used ' $u = 0$ ', overlooking the projection speed ( $5 \text{ ms}^{-1}$ ). Errors for the second stage of motion included leaving the speed as  $V \text{ ms}^{-1}$  on rebound, or combining  $V$  and  $\frac{V}{2}$  instead of  $\frac{V}{2}$  and 0 to find the rebound time. Although the times for each stage could be found directly using  $v = u + at$ , it was common to see the solution of a quadratic equation in  $t$  from the use of  $s = ut + \frac{1}{2} at^2$ . For the rebound time, this two stage method involved calculating the maximum height first. This sometimes led to errors in accuracy from using an approximation for the height reached. Candidates who chose to find  $V$  alternatively by an energy method sometimes omitted to consider the initial non-zero kinetic energy.

*Answer:* 13, 1.45 s

## Question 3

Most candidates were able to resolve the forces accurately, with only the occasional sin/cos error or missing component. It was common to see the magnitude and direction of the resultant force rather than the additional force needed for equilibrium. The main difficulty for candidates was in stating the direction of the additional force precisely enough. Some used a diagram to support their description.

*Answer:* 27.6 N, 27.9° below 'negative x-axis'

## Question 4

This question was challenging for many candidates, mainly because of the requirement to 'use an energy method'. Those who considered the kinetic and potential energies of both particles and formed a single equation frequently presented a concise and accurate solution. The many candidates who considered each particle separately often omitted to account for the tension in the string. Despite the connected particles, many candidates appeared to believe that the speeds of the two particles were different. Some candidates ignored the energy requirement and solved the problem by forming equations of motion for the two particles in order to calculate the acceleration, and then applied  $v^2 = u^2 + 2as$ . The usual errors in forming energy equations were missing terms, sign errors or a potential energy duplicated in the equation as 'work done'.

*Answer:*  $2.16 \text{ ms}^{-1}$

## Question 5

Candidates frequently gained full marks for this question, showing an understanding that the frictional force could act up or down the plane. Since 0.394 was a given answer, candidates were expected to show sufficient working rather than an algebraic formula without any substitution. Some candidates considered  $P$  acting down the plane leading to  $P = -0.394$  without interpreting the negative result or stating the magnitude, whilst others stated erroneously  $P = 0.35 \times 3g \cos 20^\circ - 3g \sin 20^\circ = 0.394$ , ignoring the negative result. Other responses solved for  $P$  in just one situation without identifying it as the least or greatest value.

*Answer:* 20.1

## Question 6

This question was found to be harder, with some misunderstandings evident in all three parts. Nevertheless, there were still many fully correct solutions seen.

- (i) The majority of candidates used  $P = Fv$  with the driving force equal to the resistance as expected. The power was frequently calculated successfully with the occasional answer given in watts instead of kilowatts. For some, the resisting force was left as  $40v$  rather than  $40 \times 56$ .

- (ii) Whilst most candidates attempted to apply  $F = ma$ , some omitted to include either the resistance  $\left(\frac{125440}{32} = 1400a \rightarrow a = 2.8 \text{ ms}^{-2}\right)$  or the driving force  $(40 \times 32 = 1400a \rightarrow a = 0.914 \text{ ms}^{-2})$ . Others appeared to be unaware of the variable driving force  $(40 \times 56 - 40 \times 32 = 1400a \rightarrow a = 0.686 \text{ ms}^{-2})$  or the variable resistance  $\left(\frac{125440}{32} - 40 \times 56 = 400a \rightarrow a = 1.2 \text{ ms}^{-2}\right)$ .
- (iii) As in part (ii), some candidates oversimplified the situation by resolving parallel to the incline to form an equation with two instead of three terms. A few overlooked the 'constant speed' and included ' $ma$ ', using the acceleration found in the previous part of the question. Candidates who believed that the driving force acted **up** the hill, solved  $1400g \sin \theta = 40 \times 50 + \frac{60000}{12}$ , ( $\theta = 13.2$ ), instead of solving  $1400g \sin \theta = 40 \times 50 - \frac{60000}{12}$ .

Answers: (i) 125 kW (ii)  $1.89 \text{ ms}^{-2}$  (iii) 3.3

### Question 7

This question differentiated between candidates who usually attempted all parts with varying degrees of success, particularly in parts (ii) and (iii).

- (i) The most common approach was to find  $t$  by solving  $\frac{dv}{dt} = 0$  and then calculating  $v(t)$ . A common misconception was that  $v(2)$  was the maximum velocity.
- (ii) This part was well answered by those who used  $a = \frac{dv}{dt}$  in both cases for  $t = 2$  and were thus able to conclude 'no instantaneous change'. Some candidates calculated  $v(2) = 8$  from each equation and concluded mistakenly that the same speed implied no instantaneous change in acceleration.
- (iii) The sketch produced a large variety of curve/straight line combinations which were not necessarily in agreement with the answers to the other parts of the question. Whilst the graph for  $2 \leq t \leq 4$  was usually linear as expected, the section for  $0 \leq t \leq 2$  was quite commonly misrepresented either with one or more straight lines or with an increasing curve from  $(0,0)$  to  $(0,8)$ . Candidates need to be aware that a sketch does not require the scaling of axes and point plotting which in this case sometimes resulted in a trapezium. It was expected that the key values 1.5, 8 and 9 would be seen.
- (iv) Despite the variety of graphs in part (iii), most candidates used some integration to find the distance travelled. Many fully correct solutions were seen even following linear graphs. Some candidates did relate the distance to the area under their incorrect graph, calculating for example the area of an isosceles triangle or a trapezium. Those who introduced a constant of integration created a more complicated solution but still frequently completed successfully.

Answers: (i)  $9 \text{ ms}^{-1}$  (ii) No instantaneous change (iv) 21.3 m

# MATHEMATICS

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Paper 9709/51  
Paper 51

## Key messages

Some candidates are still losing marks for not giving answers to 3 significant figures and also due to premature approximating. Candidates should be reminded that if an answer is required to 3 significant figures then their working should be performed to at least 4 significant figures

Candidates should always refer to the formula booklet if in doubt about a formula.

## General comments

It is pleasing to see that very few candidates now use  $g = 9.8/9.81$ . The paper does request candidates to use  $g = 10$ .

The easier questions proved to be **1**, **3** and **5(i)**.

The harder questions proved to be **5(ii)**, **6(ii)**, **7(i)** and **7(ii)**.

## Comments on specific questions

### Question 1

This question was generally well done. It simply required candidates to find the horizontal and vertical velocities after 4 seconds and then to use Pythagoras's theorem and the trigonometry of a right angled triangle.

Answers:  $23.4 \text{ m s}^{-1}$ ,  $39.8^\circ$  below the horizontal

### Question 2

This question was usually well done. Candidates needed to take moments about A.

Answers:  $0.433 \text{ m}$

### Question 3

- (i) This part of the question was well done by most of the candidates.
- (ii) Most candidates scored all 3 marks on this part of the question. A few candidates were unable to integrate  $e^{-t}$ .
- (iii) This part of the question was extremely well done.

Answers: (i)  $\frac{dv}{dt} = 2t - 5e^{-t}$  (ii)  $v = t^2 + 5e^{-t} + 5.16$  (iii)  $10.2 \text{ m s}^{-1}$

### Question 4

- (i) This part of the question was usually well done. A small number of candidates simply quoted the equation of the trajectory and made the necessary substitutions to get the required answer. This meant that they did not obey the instructions.



- (ii) This part of the question simply required candidates to substitute the values given into the equation found in part (i).
- (iii) This part of the question was usually well done.

Answers: (i)  $y = x - \frac{10x^2}{V^2}$  (ii) 31.0 (iii) 36 and 60

#### Question 5

- (i) This part of the question was well done by many candidates.
- (ii) This part of the question proved to be very difficult for many candidates. The extension had to be found at the equilibrium position which is where the acceleration is zero and the velocity is at its greatest. After this a 5 term energy equation has to be set up and solved.

Answers: (i) 0.8 (ii)  $2.78 \text{ m s}^{-1}$

#### Question 6

- (i) The majority of candidates found this part of the question fairly straightforward.
- (ii) This part of the question proved too difficult for many candidates. By taking  $e$ , as the extension, the radius of the circle would be  $0.5 + e$  and so the tension would be  $\frac{15e}{0.3}$ . Newton's Second Law can now be applied horizontally to find the extension and hence the value of HP.

Answers: (i)  $2.74 \text{ m s}^{-1}$  (ii) 0.591 m

#### Question 7

This question proved to be the hardest one on the paper.

- (i) The use of similar triangles had to be used to find the height of the conical tip. The volume removed could then be calculated by applying the volume formulae for a cone and a cylinder.
- (ii) This part of the question rarely produced a correct answer. Various volumes had to be calculated and then a moment equation about the base had to be set up. Even when a good attempt was made, too many errors occurred.

Answers: (i) 0.72 m,  $0.0352\pi \text{ m}^3$  (ii) 0.22 m

# MATHEMATICS

Paper 9709/52  
Paper 52

## Key messages

Some candidates are still losing marks for not giving answers to 3 significant figures and also due to premature approximating. Candidates should be reminded that if an answer is required to 3 significant figures then their working should be performed to at least 4 significant figures.

Candidates should always refer to the formula booklet if in doubt about a formula.

## General comments

Most candidates now use  $g = 10$  as instructed.

The easier questions proved to be **4(i)** and **7** whilst the harder questions were **3**, **4(ii)**, **5(i)**, **5(ii)** and **6(ii)**.

## Comments on Specific Questions

### Question 1

- (i) Too many candidates did not realise that  $\tan \theta = \frac{12}{20}$  where  $\theta$  was the angle of projection.
- (ii) Many candidates tried to use  $s = ut + \frac{1}{2}at^2$  with their initial velocity and so scored the M mark.

Answers: (i)  $31.0^\circ$  ,  $25.9 \text{ m s}^{-1}$  (ii)  $4.05 \text{ m}$

### Question 2

Many candidates attempted to find the distance OP when the string was in its stretched state. An attempt was made to set up a 3 term energy equation. Too often one of the terms was incorrect or an extra term was seen. The other value was rarely attempted when the string was slack.

Answers:  $1.03 \text{ m}$  and  $0.113 \text{ m}$

### Question 3

- (i) Very few candidates even attempted this part of the question and of those who did very few were able to arrive at the correct answer. Candidates needed to realise that the distance of the centre of mass of BC from the vertical through A was  $\frac{x \sin \theta}{2} - a \cos \theta$ . Moments could then be taken about the point A, resulting in the equation  $\frac{a(a \cos \theta)}{2} = x \left( \frac{x \sin \theta}{2} - a \cos \theta \right)$ . This would then produce the required answer.
- (ii) Very few candidates were able to solve the quadratic equation  $1.25 \tan \theta - 2ax - a^2 = 0$ .

Answers: (i)  $x^2 \tan \theta - 2ax - a^2 = 0$  (ii)  $3a$

#### Question 4

- (i) This part of the question was quite well done. A few candidates did not obey the instructions and simply quoted the trajectory equation and made the necessary substitutions.
- (ii) This part of the question proved too difficult for many of the candidates. The question was solved by equating 2 expressions in  $x$ . This gave the equation  $\frac{x}{\sqrt{3}} - \frac{x^2}{60} = \frac{(x+15)}{\sqrt{3}} - \frac{(x+15)^2}{60}$ . This can then be solved and the  $y$ , the height, can be found.

Answers: (i)  $y = \frac{x}{\sqrt{3}} - \frac{x^2}{60}$  (ii) 4.06 m

#### Question 5

- (i) Very few candidates were able to solve this part of the question. Too many candidates either made no attempt or tried to take moments about the base. If  $\theta$  is the angle between the base and the vertical then  $\tan \theta = \frac{(0.6-0.5)}{0.4} = \frac{1}{4}$ . Also,  $\tan \theta = \frac{x}{0.6}$ , where  $x$  is the required distance.
- (ii) The required moment equation proved too difficult for those candidates who attempted the question.

Answers: (i) 0.15 m (ii) 0.464.

#### Question 6

- (i) Some candidates did not realise that the tension in BP was zero. Quite a number thought that the 2 tensions were equal. Generally this part of the question was well done.
- (ii) This part of the question was far too difficult for many of the candidates. It was necessary to find the tension in AP when the tension in BP was 5 N. With this value of the tension it was possible to use Newton's Second Law to find the greatest speed.

Answers: (i)  $4.39 \text{ rad s}^{-1}$  (ii)  $3.04 \text{ m s}^{-1}$

#### Question 7

- (i) Most candidates were able to solve this part of the question.
- (ii) This part was generally well done. It was solved by putting the acceleration and time equal to zero.
- (iii) Most candidates realised that it was necessary to integrate twice in order to find the required height. Too often the limits were incorrectly applied. Some candidates could not integrate  $e^{-t}$ .

Answers: (i)  $\frac{dv}{dt} = 10 + 3t - 5ke^{-t}$  (ii) 2 (iii) 18.3 m

# MATHEMATICS

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Paper 9709/53  
Paper 53

## Key messages

Some candidates are still losing marks for not giving answers to 3 significant figures and also due to premature approximating. Candidates should be reminded that if an answer is required to 3 significant figures then their working should be performed to at least 4 significant figures

Candidates should always refer to the formula booklet if in doubt about a formula.

## General comments

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The harder questions proved to be **5(ii)**, **6(ii)**, **7(i)** and **7(ii)**.

## Comments on specific questions

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### Question 3

- (i) This part of the question was well done by most of the candidates.
- (ii) Most candidates scored all 3 marks on this part of the question. A few candidates were unable to integrate  $e^{-t}$ .
- (iii) This part of the question was extremely well done.

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### Question 4

- (i) This part of the question was usually well done. A small number of candidates simply quoted the equation of the trajectory and made the necessary substitutions to get the required answer. This meant that they did not obey the instructions.



- (ii) This part of the question simply required candidates to substitute the values given into the equation found in part (i).
- (iii) This part of the question was usually well done.

Answers: (i)  $y = x - \frac{10x^2}{V^2}$  (ii) 31.0 (iii) 36 and 60

#### Question 5

- (i) This part of the question was well done by many candidates.
- (ii) This part of the question proved to be very difficult for many candidates. The extension had to be found at the equilibrium position which is where the acceleration is zero and the velocity is at its greatest. After this a 5 term energy equation has to be set up and solved.

Answers: (i) 0.8 (ii)  $2.78 \text{ m s}^{-1}$

#### Question 6

- (i) The majority of candidates found this part of the question fairly straightforward.
- (ii) This part of the question proved too difficult for many candidates. By taking  $e$ , as the extension, the radius of the circle would be  $0.5 + e$  and so the tension would be  $\frac{15e}{0.3}$ . Newton's Second Law can now be applied horizontally to find the extension and hence the value of HP.

Answers: (i)  $2.74 \text{ m s}^{-1}$  (ii) 0.591 m

#### Question 7

This question proved to be the hardest one on the paper.

- (i) The use of similar triangles had to be used to find the height of the conical tip. The volume removed could then be calculated by applying the volume formulae for a cone and a cylinder.
- (ii) This part of the question rarely produced a correct answer. Various volumes had to be calculated and then a moment equation about the base had to be set up. Even when a good attempt was made, too many errors occurred.

Answers: (i) 0.72 m,  $0.0352\pi \text{ m}^3$  (ii) 0.22 m

# MATHEMATICS

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Paper 9709/61  
Paper 61

## Key messages

To do well in this paper, candidates must work with 4 significant figures or more in order to achieve the accuracy required. Candidates should also show all working, so that in the event of a mistake being made, credit can be given for method; a wrong answer with no working shown, scores no marks. Candidates should label graphs and axes including units, and choose sensible scales.

## General comments

It was pleasing to see that at least some of the candidates who took this paper had a good knowledge of the syllabus. There were still many candidates from centres who did not even attempt the examination and handed in blank sheets of paper. A large proportion of candidates who entered this exam scored less than 10%, and did not know what a normal distribution was, did not recognise the binomial situation, had no idea what permutations and combinations were about, and were unable to understand the concepts of mean and standard deviation. They did not appear to have had any practice with past papers and consequently a large number of candidates were unable to recognise the language let alone attempt the paper.

## **Question 1**

This question was meant to be a straightforward start to the paper but in fact turned out to be one of the poorest attempted questions. Candidates seemed unaware that  $\Sigma(x - 10) = \Sigma x - 12 \times 10$  where there were 12 data points. Hence they were unable to make any headway.

*Answer:* 66, 606

## **Question 2**

- (i) The difficulty level of this question was not deemed particularly challenging yet it was surprising how frequently mistakes were made. Most candidates were able to find the median and lower and upper quartiles from the raw data. However, mistakes were often made when plotting the quartiles and the maximum and minimum values. This was often due to a non-linear scale being used.

The number of candidates failing to see the need to use a ruler to draw their box plot or to include an axis label was disappointing and penalised accordingly. It was not uncommon to see whiskers wrongly drawn through the box and some candidates drew stem and leaf diagrams or scatter graphs, instead of boxplots.

Overall, it was felt that relatively 'easy' marks were lost too frequently in this question.

- (ii) Many candidates could answer this question confidently and appeared well prepared to do so. It was disappointing to see the second mark not awarded because of an inadequate concluding statement, despite correct working. On the other hand, there were confused candidates who tried to use  $1.5 \times \text{UQ}$  and  $1.5 \times \text{LQ}$  to answer the question.

*Answer:* (i) LQ 18, median 25, UQ 50

### Question 3

- (i) There were many good solutions to this question as well as some very poor ones. Most candidates knew they had to multiply two probabilities together. Many stopped there and failed to realise that there were two options, either  $P(R, B)$  or  $P(B, R)$ . A few lost marks because they did not give answers to 3 decimal places.
- (ii) This part was mainly well done by those who understood what the question was asking for. Some thought the socks were taken with replacement and gained some credit but not all the marks.
- (iii) This part was well done by those who had produced the table.

Answers: (i)  $\frac{16}{33}$  (ii) Table with 0, 1, 2 and probabilities  $\frac{14}{33}$ ,  $\frac{16}{33}$  and  $\frac{3}{33}$  respectively. (iii)  $\frac{2}{3}$

### Question 4

- (a) The majority of candidates were able to answer this question with a lot of success and 4 or 5 marks were regularly gained. There was little problem finding the correct z-values (ignoring signs) and setting up correct standardisation equations. One of the most common errors was using 0.5 and not fully appreciating that  $-0.5$  should be used in order to find the probability of a distance more than 31,000 km. In general, candidates had little difficulty dealing with simultaneous equations.
- (b) This question was handled with much less confidence. It was common to see no attempt or no decent attempt. There were candidates who were able to standardise in terms of one variable and these often went on to gain full marks. Occasionally, some marks could be awarded to the candidate who substituted an arbitrary value for  $\mu$  or  $\sigma$  and was able to find the correct probability this way.

Answers: (a)  $\sigma = 2000$ ,  $\mu = 32\,000$  (b) 0.933

### Question 5

- (i) It was pleasing that most of the candidates who had done any work for this exam, realised that here was a binomial distribution and found the correct probabilities and number i.e.  $X \sim \text{Bin}(15, 0.22)$ . A surprising number subtracted 0.22 from 1 to make 0.88 and not 0.78. This had unfortunate repercussions in part (ii) though candidates who showed their working were able to gain method marks.
- (ii) This was also well attempted by most candidates. However, a large majority (well over half) forgot the continuity correction.

Answers: (i) 0.398 (ii) 0.861

### Question 6

- (i) A number of candidates were unable to picture this situation and struggled to answer the question. Of those who did understand the scenario, many found  $P(\text{SLL})$  and  $P(\text{SRR})$  and added them together, without realising that in fact  $P(\text{LSL})$ ,  $P(\text{LLS})$ ,  $P(\text{RSR})$  and  $P(\text{RRS})$  needed to be included.
- (ii) This conditional probability question was well attempted with many perfect solutions, even from those who did not gain full marks in part (i).

Answers: (i) 0.293 (ii) 0.137

### Question 7

- (i) 9! was given as an answer too frequently where candidates failed to remove repetitions and this did not bode well for the rest of the question which was answered fairly poorly overall.
- (ii) Full marks were very rarely awarded in this question. Candidates did not appear to be well-practiced and solutions were regularly incomplete. Many candidates opted to use the 'subtraction

method' where they started with their answer to (i) and attempted to subtract all the possible scenarios where vowels could be together in some way. This was highly complicated and a correct answer was not found from this method. Therefore, candidates should be encouraged to consider the most straightforward of methods before beginning. Detailed study of previous years' mark schemes by candidates would facilitate this.

- (iii) Too often permutations were confused with combinations or  ${}^5C_3$  was multiplied by another value rather than left on its own in this question.
- (iv) Candidates found this question reasonably accessible and lots of full solutions were seen where at least part marks could be awarded. It was pleasing to see the relatively sophisticated and simple method of  ${}^7C_3$  utilised from time to time.

Answers: (i) 90720 (ii) 10800 (iii) 10 (iv) 35

# MATHEMATICS

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Paper 9709/62  
Paper 62

## Key messages

Candidates must show sufficient method to justify their conclusions. Failure to communicate intended processes may produce uncertainty about the final answer.

Candidates need to consider whether their scale will enable their graphical solutions to be plotted to an appropriate degree of accuracy.

Candidates may avoid some accuracy errors by calculating with fractions and not the decimal equivalent.

## General comments

Although the majority of candidates presented their solutions in a logical manner, there was sometimes a lack of structure which made determining the final answer difficult for examiners. A lack of structure also appeared to result in not all scenarios being identified in **Question 6**.

Candidates are reminded that to state a 3sf final answer, working values need to be stated to at least 4sf.

Many good solutions were seen for **Questions 4 and 6**. Sufficient time seems to have been available for candidates to complete all the work they were able to. Many candidates found **Question 2** difficult.

## Comments on specific questions

### **Question 1**

This question on central tendency was anticipated to be an accessible start to the paper, and was attempted by almost all candidates.

- (i) Although the majority of candidates stated the mode, a common misconception was that the greatest value was required, rather than the most frequent value. However, a number of candidates calculated the mean of the values, or stated the median here, and in (ii).
- (ii) The correct use of the formula  $\frac{n+1}{2}$  was common approach to finding the middle term. Not all candidates arranged the values into order before stating the value. Weaker candidates used the simpler, but acceptable, approach of 'counting in' to find the median. Again, a number of candidates calculated the mean.

Candidates were less successful in calculating the interquartile range, with a final answer of 2.5 being a common error. A number of candidates found the difference of the term values. A surprising number of candidates made no attempt here, which is a change from previous sessions.

Answers: (i) 38 (ii) median = 38.5, IQR = 2

## Question 2

Almost all candidates found this question the most challenging on the paper. At this level, candidates should be aware that an answer greater than 1 must be incorrect and should be encouraged to reconsider their approach.

- (i) Although almost all candidates were able to calculate accurately the probabilities of the separate conditions stated in the question, few fully correct solutions were seen. Candidates who included a tree diagram were often more successful and identified the required branches on the diagram. By using a tree diagram, candidates could have checked whether the branch outcome fulfilled the stated conditions. A number successfully attempted a Venn diagram approach, placing probabilities in the appropriate regions.
- (ii) Candidates were reminded in the question that workings were required, and many candidates failed to provide sufficient to justify their conclusion. Good solutions clearly stated  $P(M) \times P(H) = P(M \cap H)$  as the condition required for independence. The probabilities for both sides were calculated fully and the final values compared before making a statement about independence. Weaker solutions simply stated the calculations, which could only gain partial credit as no justifications were provided. A few candidates used the generic form of the condition, and unless  $A$  and  $B$  were linked to the context of the question, were not able to achieve full credit. It was encouraging that fell 'circular' arguments were seen.

Answers: (i) 0.7 (ii) Not independent

## Question 3

This was a fairly standard normal distribution question. Candidates should be aware of the critical values that are stated below the normal table, as these are expected to be used.

- (i) Many weaker candidates are able to access questions using the normal distribution, and this was seen again. A common error was not to use the z-value from the critical value. Candidates would be well advised to sketch a simple normal distribution curve to identify the required region, as many candidates used the positive z-value. A number of candidates failed to solve their correct equation accurately.
- (ii) The best solutions had a sketch which visually interpreted the question. This allowed candidates to identify the required region and recognise that a z-value was stated in the question. However, most candidates used a standardisation approach, and then arrived back at the question information. Unfortunately, many rounding errors were introduced, or  $P(z < 1.8)$  was calculated. Almost all solutions included an attempt to calculate the number of cartons required. Candidates are reminded that this is not an approximation, but an interpretation of the context.

Answers: (i) 452 (ii) 5

## Question 4

It was encouraging that clear probability distribution tables were presented in almost all solution. Although a few candidates had probabilities greater than 1 in total, a more common error was to have probabilities totalling less than 1.

- (i) Good solutions often initially constructed the probability distribution table, with the calculations for individual terms below to support the answers within the table. The best solutions would calculate  $P(0)$  and  $P(1)$  from the question data, and then use the fact that probabilities sum to 1 to calculate the final answer. Some candidates showed calculations in fractions and stated their answers as decimals with rounding such that the final probabilities were slightly inaccurate. A common error was to calculate with replacement, and although the correct logic was applied, errors occurred in the application. Many candidates did not consider that the order of selecting the animals was important, and therefore omitted a significant number of scenarios.
- (ii) Although  $E(X)$  was stated in the question, many solutions included calculating this from first principles. Most candidates used their probability distribution table correctly, but failure to show

working where (i) was incorrect resulted in no method mark being awarded. Very few candidates did not apply the variance formula correctly.

Answers: (i) (ii)  $\frac{20}{49}$

|        |               |               |               |
|--------|---------------|---------------|---------------|
| $X$    | 0             | 1             | 2             |
| $P(X)$ | $\frac{2}{7}$ | $\frac{4}{7}$ | $\frac{1}{7}$ |

### Question 5

- (i) Although almost every candidate attempted this question, arithmetical errors were seen surprisingly frequently.
- (ii) Candidates who set their work out clearly were often more successful in calculating accurately the mean from the group frequency distribution. The best solutions clearly stated the required terms on the numerator and used the information within the question for the denominator. Where candidates had an error in (i), without this working it was not possible to determine how their answer was achieved. The context of the question required any improper fraction to be converted into a mixed number. Common errors were the use of the class interval and inaccuracy in determining the midpoint of the interval.
- (iii) Good solutions included a clear calculation of the required frequency distribution, often efficiently at the frequency table. Appropriate scales were used for the histogram so that the vertical values could be plotted to the anticipated accuracy. The best solutions had all but the final bar on horizontal gridlines, with the vertical bar lines drawn carefully on vertical gridlines. It was encouraging to see more candidates remembering to include units when labelling the length of phone calls axis. Weaker solutions were often simply bar charts, and many candidates were not able to draw vertical lines consistently.

Answers: (i) 40 (ii) 6.33 (iii) Histogram constructed

### Question 6

The combination and permutation appeared to be attempted with more confidence than in previous sessions, with many candidates providing clear statements of the scenarios being considered, and correctly interpreting the conditions stated within the contexts.

- (a) (i) The best solutions identified the letters which formed the vowel block and then considered how the 5 items were arranged. Good solutions clearly identified that the effect of the repeated As needed to be removed and accurately evaluated their expression. Some candidates did multiply their final answer by 2, which would appear to be a misconception about whether the vowel block was chosen first or second.
- (ii) Good solutions often had a simple exemplar scenario to clarify thinking, with the T clearly identified in the middle. The arrangement of end letters was then considered before multiplying by the arrangements of the remaining letters. The removal of the repeated letter impact was the final stage. Many more complex approaches were attempted unsuccessfully.
- (b) Many good solutions were seen, which clearly identified the individual scenarios before calculating how many selections were available. Weaker solutions often had less systematic listing of possible scenarios, leading to the omission of one or more. Again, some candidates attempted more complex approaches, with the most common being to calculate the total number of selections possible and then removing those not required. These approaches were almost always unsuccessful. Candidates do need to ensure that they complete the simple arithmetical aspects of these problems with accuracy; as these simple errors lead to the loss of the final accuracy mark.

Answers: (a)(i) 2400 (ii) 720 (b) 366

### Question 7

This was a fairly standard question where candidates were required to determine which approximation was appropriate. This was evidenced even where candidates were unsuccessful in answer.

- (i) The majority of candidates recognised that the binomial approximation was appropriate. A good solution would identify the conditions required clearly, state the unsimplified calculations and then proceed to a final answer stated to 3 or more significant figures. A common error was to include 11 owners within the required sample. Again, where correct expressions were seen, some numerical errors were identified, and premature approximation of answers was in evidence. A limited number of students attempted to use a normal distribution, which is an invalid approach.
- (ii) With the increase of sample size a normal distribution is an appropriate approximation. Good candidates confirmed this with the appropriate test, although this was not a question requirement. Good solutions again had a simple normal curve sketch to confirm the required probability area, stated the unsimplified expressions for the mean and variance calculations, substituted accurately into the standardisation formula and used the tables appropriately. As the data was discrete, a continuity correction was required. Weaker solutions either omitted the continuity correction, or used the upper bound, so omitting the boundary value. Few candidates failed to use the standard deviation. The number of candidates who used their probability from (i) in this part was surprising, and candidates are reminded of the benefit of re-reading the question after completion to check that they have completed the task appropriately.
- (iii) Many candidates recognised that the question required using the binomial approximation to find the limiting term. Many exponential equations were seen, using a variety of stated or calculated values. The most efficient approach was using logs, although trial and improvement solutions were clearly in evidence. Where an exponential equation was presented, it was almost always successfully solved. Candidates are advised to identify the key data requirements of the context, especially as the condition was now whether the phone was from Company B or not. Candidates should be aware of the context of the question, so that a non-integer final answer will not gain credit.

Answers: (i) 0.942 (ii) 0.231 (iii) 10

# MATHEMATICS

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Paper 9709/63  
Paper 63

## Key messages

Candidates should be encouraged not to do rough working away from the main script. There are method marks awarded in most questions and, if the Examiners can see the method used, they do not have to withhold all marks for incorrect responses. Similarly, candidates should appreciate the importance of writing down the unsimplified numerical expressions that they put into their calculator. If the answer is incorrect, the candidate may still be eligible for all the method marks if the Examiner knows how the answer was achieved. This was particularly important in **Questions 4 and 6**.

A few candidates offered more than one solution to several questions. They should be aware that we will only mark one response, normally the most complete response or, if both complete, the second response.

## General comments

This paper proved accessible to most candidates with many candidates achieving full marks. The most challenging question proved to be **Question 7** on permutations and combinations and particularly **Question 7(iv)**. The graph in **Question 1(ii)** also produced some problems, where dealing with continuous data and scaling the horizontal axis caused many candidates to lose marks. Candidates should be aware that there is no credit for shading and that using blunt pencils risks the graph being judged to be inaccurate.

The questions on the Normal Distribution were generally well answered.

## Comments on specific questions

### **Question 1**

- (i) Although most candidates understood the question and identified the position of the lower quartile, many gave the class interval that contained the median.

*Answer:* 15–19

- (ii) Most candidates realised that they needed to calculate the frequency densities before drawing a histogram and only a minority used the frequencies as the heights of their bars. However, only the better candidates correctly identified the class widths as 5, 5, 5, 10, 25 and appreciated that the bar boundaries were at 9.5, 14.5, 19.5, 24.5, 34.5 and 39.5. A significant number used the class limits rather than boundaries and worked with class widths of 4, 4, 4, 9 and 24 going on to draw their bars starting at 10. However, almost all candidates appreciated that the bars of a histogram are touching. The axes were generally well labelled with almost all candidates labelling the horizontal axis with the required 'mass' and 'kg'. The scaling of the horizontal axis proved to be more challenging. The most successful candidates indicated a break in the scale between 0 and 9.5 and drew all their bars on solid lines. Of those who did not indicate a break, the better ones knew to draw their bars starting in the space between 9 and 10. We considered that the horizontal axis was incorrectly scaled if the solid lines were labelled 9.5, 14.5 etc. and the origin was labelled as zero.

Few candidates scored full marks on this question. Many would have avoided plotting errors if they had chosen a sensible scale on the vertical axis.

## Question 2

- (i) This question was well answered. Most candidates found the critical value 0.674 associated with a probability of 0.75 and knew that they had to standardise using zero. The few who used the tables the wrong way round and equated their standardising expression to a probability were not awarded any marks.

A number of candidates did not understand the question and tried to use  $p = 0.25$  in the Binomial formulae  $np$  and  $npq$  for the mean and variance.

- (ii) Most candidates knew that this question required the Binomial Distribution with  $n = 8$  and  $p = 0.25$  and many obtained the correct answer by finding the probability that 0 or 1 value was positive. Some misinterpreted 'fewer than' to include 2 as well while others subtracted the correct answer from 1. A few incorrectly attempted to answer the question by standardising.

Answers: (i) 4.45 (ii) 0.367

## Question 3

- (i) Most successful candidates knew to use  $1 - x$  as the probability of a member being female and then substituted into one of the three possible equations. Surprisingly few used the fact that the probability of being an Advanced swimmer or a Beginner is equal to 0.5, preferring to equate the two product probabilities. Of those who did not use  $1 - x$  and introduced another letter for the probability of being female, only a handful managed to eliminate the second letter once they realised that it was equal to  $4x$  and that  $x + 4x = 1$ .

Answers: (i) 0.2 (ii) 0.12

- (ii) This question was well answered. Even many of those who struggled with part (i) were able to substitute into the Conditional Probability formula and gain follow through marks.

## Question 4

- (i) Most candidates knew how to find a weighted mean and obtained the correct final answer. The most common error was to add the two mean prices and divide by two.
- (ii) This question was well answered with most strong candidates gaining full marks. Many weaker candidates gave no response. The most common errors in finding the sums of squares were to forget to square the standard deviation or to divide the mean price by the number of holidays. A significant number correctly obtained the sums of squares and then either did not know how to proceed or used an incorrect Variance formula. If the final answer was incorrect, we needed to see the unsimplified numerical expressions if we were to award the Method marks.

Answers: (i) \$1870 (ii) 488

## Question 5

- (i) Of those who knew what a probability distribution table was, most obtained full marks. Those who did not understand the question usually listed A, B and C with the associated probabilities of a head. Some only considered the possibilities of 0, 1 and 2 heads and many did not appreciate that the probability of a head on a non-biased coin is 0.5. This commonly resulted in a table with the probabilities 0.15, 0.55 and 0.3 where they had only multiplied two probabilities together. Those who presented their working clearly were more likely to gain full marks. A surprising number showed no working at all and risked losing all the marks if they made a mistake. Some of these put the correct probabilities in the wrong cells.
- (ii) If the probabilities presented in part (i) summed to 1, candidates were eligible for both the method marks in this question. Most candidates correctly substituted into the formula for the mean. However, in the variance formula, many forgot to subtract the mean squared and some squared the probabilities rather than the number of heads.

Answers: (i)

| H (no of heads) | 0                                   | 1                                  | 2                                    | 3                                  |
|-----------------|-------------------------------------|------------------------------------|--------------------------------------|------------------------------------|
| $P(H=h)$        | $0.075 \left( \frac{3}{40} \right)$ | $0.35 \left( \frac{7}{20} \right)$ | $0.425 \left( \frac{17}{40} \right)$ | $0.15 \left( \frac{3}{20} \right)$ |

(ii) 1.65; 0.678

### Question 6

- (i) Most candidates knew how to standardise and those who drew a diagram were usually successful in finding the correct area.
- (ii) Most candidates realised that they needed to calculate a new mean and variance based on their answer to part (i). However, many risked losing marks by not showing their unsimplified numerical expressions for the mean and variance. Those who gained full marks remembered the continuity correction, knew to divide by the square root of the variance and were able to identify the correct area as being greater than 0.5.

Answers: (i) 0.168 (ii) 0.898

### Question 7

- (i) Most candidates gained the method mark for  $8!$  and many of these correctly divided by  $3!$  and  $2!$  to deal with the repeats although very few explained their reasoning. A less common solution was seen where candidates had correctly considered the number of ways of choosing each of the nine letters and then divided by  $4!$  and  $2!$ .
- (ii) The most commonly seen approach was to consider the number of ways of choosing positions for the four Es in the six possible spaces around the other five letters ( ${}^6C_4$ ), multiplying by  $5!$  to deal with the number of ways of arranging the five 'non-E' letters and dividing by  $2!$  to deal with the repeated Ns. This method was generally quite successful although a significant number presented  $\frac{5!}{2!} \times {}^6P_4$  as their final answer while others just wrote  $\frac{5!}{2!}$ . Another, less common approach was to consider the number of ways when the first E is in the first, second or third position. Not many candidates arrived at the correct final answer using this approach. A third method, chosen by a significant number, rarely led to the correct final answer. They calculated the total number of ways of arranging the nine letters  $\left( \frac{9!}{4!2!} \right)$  and then attempted to subtract the number of ways where Es are touching.
- (iii) Those who chose a listing method were most successful. There are only three choices for the fifth letter – S, V or T. However, a significant number gave  ${}^4C_2 \times {}^2C_2 \times {}^3C_1 = 18$  as their answer where they have treated the Es and Ns as being different from each other.
- (iv) The strongest candidates knew to consider the three scenarios of two, three and four Es while also thinking about the number of Ns which could be two, one or zero. A listing method within the three scenarios of two, three or four Es also worked well. However, as in part (iii), many candidates treated the Es and Ns as being different from each other and the most common wrong answer was 105. They calculated the number of ways with two Es as  ${}^4C_2 \times {}^5C_3 = 60$ , with three Es as  ${}^4C_3 \times {}^5C_2 = 40$  and with four Es as  ${}^5C_1 = 5$ . Some also came up with 105 by subtracting the number of ways with zero or one E from the total using similar reasoning. Another commonly seen wrong answer was  ${}^5C_1 + {}^5C_2 + {}^5C_3 = 25$ . These candidates gained a method mark if they explained that they were considering the three scenarios of two, three or four Es.

Answers: (i) 3360 (ii) 900 (iii) 3 (iv) 18

# MATHEMATICS

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Paper 9709/71  
Paper 71

## Key messages

When a question asks for an explanation 'in the context of the question' text book definitions will not be accepted, the answer must relate to the situation given in the question.

Candidates need to ensure they use standard deviation and variance correctly within calculations and also in statistical notation. Confusion between the two can lead to a significant loss of marks over the paper as a whole.

Candidates should check how reasonable their answers are (for example probabilities should be between 0 and 1).

## General comments

In general, candidates scored well on **Questions 1** and **6(i)** and **(ii)** whilst **Questions 2** and **7** proved more demanding for some candidates. There was a complete range of scripts from the very good to the very poor; there were cases where candidates did not seem prepared for the demands of this paper.

Most candidates kept to the required level of accuracy, though, as is often the case, there were situations where candidates lost marks for giving final answers to less than 3 significant figure accuracy through premature approximation. This was particularly seen on **Question 4**.

Timing did not appear to be a problem for candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also some very good and complete answers.

## Comments on specific questions

### **Question 1**

This question was generally well attempted, though there were some candidates who confused the two formulae for the unbiased estimate of the population variance, and some candidates who found the biased variance rather than the unbiased.

Answer: 2.04   0.369

### **Question 2**

This question was not well attempted. Part **(i)** required a confidence interval for a proportion to be calculated; there seemed to be confusion here with a confidence interval for a mean. Many candidates centred their interval on 20 and were therefore only able to score for use of the correct  $z$  value. Many candidates did use the correct value for  $z$  (1.881 or 1.882), but incorrect values were seen. When asked to calculate a confidence interval it is important that the final answer is given as an interval, and not merely two separate values.

Part (ii) was poorly answered. Candidates did not realise that they should be checking to see if  $\frac{1}{6}$  was within the CI they had calculated. It is important on this type of question that the check is clearly stated; comments such as 'it' is in the CI will not be accepted.

Answers: (i) 0.125 to 0.275 (ii) No evidence of bias as  $\frac{1}{6}$  is within this range

### Question 3

This question was reasonably well attempted. Many candidates realised that a Normal approximation,  $N(153, 153)$ , was required. The most common error was to omit a continuity correction. Weaker candidates used other, invalid, distributions.

Answer: 0.138

### Question 4

Confusion between standard deviation and variance caused loss of marks for some candidates here. Many candidates knew how to calculate the mean and the variance of a large cup of coffee, but did not calculate the standard deviation as requested, merely leaving their answer as a variance. Other candidates incorrectly calculated the variance using  $1.5 \times 10.2$  (rather than  $1.5^2 \times 10.2$ ). Both these errors caused carry forward errors into part (ii), though credit was still given for a correct method used in (ii).

In general the method required for part (ii) was well understood, but standard deviation and variance mixes were again common. Premature approximation here (for example using 33.1 or 33.2 rather than 33.15) caused a lack of accuracy in the final answer for some candidates.

Answers: (i) 155.1 4.79 (ii) 0.930

### Question 5

Part (i) of this question was reasonably well attempted. Many candidates realised that the standard deviation required (the standard error) was  $\frac{3.1}{\sqrt{50}}$  and standardised correctly. Some candidates, having calculated the correct z value, went on to choose the wrong probability; the question asked for the probability that the sample mean mass was less than 14.0, therefore the required probability should be less than 0.5. Candidates would be advised to draw a quick diagram here to prevent this error.

Part (ii) was not well answered. It is important that candidates understand the underlying theories as well as being able to perform relevant calculations. Many candidates were able to identify that  $n$  was large, but unfortunately did not give the answer 'no', it was not necessary to assume that the population was normally distributed. Many candidates thought the answer was 'yes'.

Part (iii) required a significance test to be carried out. Many candidates appreciated and followed the steps required, but not always fully correctly. Errors included stating incomplete Hypotheses, comparison with an incorrect z value, an invalid comparison (for example comparing a z value with an area), or a comparison not fully shown. The comparison must be clearly seen as an inequality statement or values clearly shown on a diagram; this could be either a comparison of z values or a comparison of areas.

Answers: (i) 0.324 (ii) No because  $n$  is large  
(iii) There is evidence that the mean mass in this area is less than 14.2

### Question 6

Parts (i) and (ii) of this question were well attempted, but there were many candidates who did not know how to approach parts (iii) and (iv); these parts were often omitted completely.

In parts (i) and (ii), candidates were required to 'show that'  $k$  was 10 and 'show that'  $E(X)$  was  $10\ln 2$ , it is therefore important here for candidates to show all stages of their working, as marks can be withheld for lack of essential working. On the whole candidates gave good solutions, though sign errors were seen frequently in part (i) and weaker candidates struggled with the integration in both parts (i) and (ii).

Attempts at parts (iii) and (iv) were varied. Integration attempts often involved use of incorrect limits, and invalid use of a Normal distribution as also seen by weaker candidates.

Answers: (iii) 0.111 (iv) 7.14

### Question 7

This question was not well attempted.

In general, candidates were confident using the Poisson distribution in parts (i) and (ii), but using the correct values for  $\lambda$  was not always seen.

In part (i) whilst many candidates knew how to find  $P(X < 3)$  some worked with separate values of 0.4 (girls) and 0.6 (boys) and were unable to combine correctly to find the probability that the total number of absences was less than 3.

In part (ii) finding  $P(X > 3)$  was done well, though some candidates interpreted 'more than 3' incorrectly and omitted a term in their expression. Again, incorrect values for  $\lambda$  were seen.

Part (iii)(a) was a question that asked for an answer 'in this context', therefore a text book definition of a Type I error was not acceptable here. In order to gain the mark the answer need to relate to the given situation and not merely mention  $H_0$  and/or  $H_1$ .

Part (iii)(b) and (c) were not well answered. In part (b) whilst reasonable attempts were made in stating the hypotheses, calculating the probability of a Type I error was poorly done, with many candidates thinking the probability was 5 per cent. In part (c) calculation of  $P(X > 3)$  was required, a large number of candidates calculated  $P(X = 3)$ . It should be noted that the conclusion should be written in context and should not be a definite statement (see below).

Answers: (i) 0.920 (ii) 0.0656

(iii)(a) Incorrectly concluding that there were more absences than usual when there were not

(b)  $H_0: \lambda = 1.5$   $H_1: \lambda > 1.5$  0.0186 (c) No evidence of more than usual male absences

# MATHEMATICS

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Paper 9709/72  
Paper 72

## Key messages

When a question asks for an explanation ‘in the context of the question’ text book definitions will not be accepted, the answer must relate to the situation given in the question.

Candidates must be aware that for a final answer to be accurate to 3 significant figures, all figures used within the calculation should be accurate to at least 4 significant figures.

## General comments

In general, candidates scored well on **Questions 1, 3, 4(i) and 6(ii) and (iii)** whilst **Questions 7 and 6(i)** proved more demanding for some candidates. Candidates were generally able to demonstrate and apply their knowledge in the situations presented, though explanations showing a statistical understanding of the situation were not always well answered. There was a complete range of scripts from very good ones to poor ones.

Most candidates kept to the required level of accuracy, though, as is often the case, there were situations where candidates lost marks for giving final answers to less than three significant figure accuracy. This was particularly seen on **Question 6(iv)** where a premature approximation caused the final answer to not have the required level of accuracy (see comments below).

Timing did not appear to be a problem for candidates.

Detailed comments on individual questions follow. Whilst the comments indicate particular errors and misconceptions, it should be noted that there were also some very good and complete answers.

## Comments on specific questions

### **Question 1**

This was a particularly well attempted question. Many candidates used the correct value of  $\lambda$  and obtained a correct expression for  $P(X < 4)$ . Errors included attempts to use separate values for  $\lambda$  (0.8, 1.2 and 2.4) and either not to combine or combine incorrectly.

*Answer:* 0.359

### **Question 2**

Although this question was reasonably well attempted, there were some candidates who did not understand how to describe a distribution fully. Some candidates, calculated correct values for the mean and variance, but did not state that the distributions of A and B were Normal. Others attempted to ‘describe’ how they would do the calculations to find the parameters without actually doing them and gave non-numerical answers.

*Answers:* A: N(6,4.8) B: N(6,2.4)

### Question 3

This question was well attempted, particularly part (i), where candidates were able to successfully find the 95 per cent confidence interval required. Errors included using an incorrect value for  $z$ , and on occasions final answers were not given as an interval, but as two separate values i.e. not in the form required.

Part (ii) was not quite as well answered. Many candidates realised that the width would be less, but a reason was required. Some candidates calculated the two widths, which was acceptable, whilst others reasoned that the value of  $n$  had increased and the width was proportional to  $\frac{1}{\sqrt{n}}$  (again acceptable)

Answers: (i) 48.7 to 55.3 (ii) Narrower because  $\left(\frac{\sigma}{\sqrt{n}}\right)$  is smaller

### Question 4

Part (i) of this question was well answered but the remainder of the question proved to be more demanding. Calculating the unbiased estimates of the mean and variance was a technique that most candidates performed well; there was occasional mixing of the two formulae for the variance, but this was not as prevalent as has been noted in the past.

Part (ii) required a significance test to be carried out. Candidates mainly knew the steps required, but did not always carry these out successfully. Hypotheses were not always given or were given incorrectly. Many candidates standardised (though  $\sqrt{150}$  was omitted on occasions and some candidates incorrectly used values from part (i)), but the major loss of marks was in the comparison and conclusion. It is important that the comparison is clearly stated as an inequality or a diagram with numerical values clearly marked. The comparison could be either  $z$  values or an area comparison. The conclusion drawn should be in context and not definite.

Some candidates, in part (iii), realised that the sample was large therefore the CLT could be applied, resulting in the distribution of the sample means being approximately Normal. However, the underlying theory did not always appear well understood.

Answers: (i) 495.9 12.8 (ii) No evidence (at 2 per cent) that mean mass of the packets is less than 505  
(iii) Large sample, so sample mean approximately Normally distributed

### Question 5

This question was reasonably well attempted, though not as well as is usually the case for this type of question. In part (i) candidates needed to realise that the area under the pdf (the area of a triangle) was equal to 1. Some candidates, rather than calculate the area of the triangle, attempted integration; this was an acceptable method if the correct form of  $f(x)$  was used, but it was a more complex approach. Some unsuccessful attempts using Pythagoras and the gradient of the line were seen.

In part (ii), which wasn't as well answered, use of  $y = mx + c$  along with the value of  $b$  in terms of  $a$  led to the required expression for  $f(x)$ . As this was a 'show that' question it is important for candidates to show all steps in their working out as marks can be withheld for lack of essential working.

Part (iii) was quite well answered, even by those who were not successful on parts (i) and (ii), though errors with integration and limits were seen, as well as algebraic errors when substituting limits into their integration attempts.

Answers: (i)  $b = \frac{2}{a}$  (iii) 1.5

### Question 6

With the exception of part (i), this was a particularly well answered question.

It is important in questions like part (i) that candidates do answer 'in context'. Answers such as 'they are independent' does not relate to the question, whereas 'accidents occur independently' would be an acceptable response.

In part (ii) most candidates used the correct Poisson expression for  $P(4)$  along with the correct value for  $\lambda$  (2.5). In part (iii) many candidates used the correct Poisson expression for  $1-P(0,1,2,3)$ , though there were occasions when candidates omitted the  $P(3)$  term. The value of  $\lambda$  required here was  $\frac{25}{12}$  or 2.08333. Some candidates used 2.08; this is a premature approximation which could lead to an accuracy error in the final answer.

Part (iv) was also quite well attempted; many candidates used the correct Normal approximation  $N(21.726, 21.726)$ . Errors here included a lack of (or an incorrect) continuity correction, and use of  $N(21.7, 21.7)$ ; there were many occasions when this premature approximation led to an accuracy error in the final answer.

Answers: (i) Accidents occur independently or accidents occur randomly (ii) 0.134 (iii) 0.158 (iv) 0.952

### Question 7

This was a particularly poorly attempted question.

In part (i), many candidates omitted or wrote incorrect Hypotheses. A Binomial expression, using  $B(9, 0.1)$ , for  $P(X \geq 3)$  was required; few candidates were able to obtain this expression fully correctly. Errors included omitting terms or adding extra terms, calculating individual probabilities only and not summing them, or using an incorrect Binomial distribution. A comparison with 0.01 (or equivalent) was then required. Candidates need to be aware that this comparison must be clearly shown as an inequality statement (or on a clear diagram with values stated). Marks were lost here for invalid comparisons; for example comparison with a  $z$  value was often seen.

Part (ii) was not always fully answered by candidates. On occasions just a text book definition for a Type I error was given.

Part (iii) proved to be demanding for a large number of candidates. Finding the critical region ( $X \geq 4$ ) was rarely shown, though some candidates used it without justification. To calculate the probability required,  $B(9, 0.5)$  should have been used to calculate  $P(X < 4)$ . Many candidates used the correct Binomial distribution, but few found  $P(X < 4)$ .

Answers: (i) No evidence (at 1 per cent level) to reject  $H_0$ , claim not justified (ii)  $H_0$  was not rejected  
(iii) 0.254

# MATHEMATICS

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| <p><b>Paper 9709/73</b><br/><b>Paper 73</b></p> |
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## Key messages

Candidates should be encouraged to use their knowledge of the conditions needed for a Poisson distribution to a variety of contexts such as students leave to study engineering independently of each other. Candidates work on probability was not as good as their work on statistical distributions, and therefore they should be encouraged to put more focus on these areas of the specification.

The responses to questions on significance testing continues to improve, with most tests being carried out correctly involving a valid comparison

## General comments

Many fully correct solutions were given in all questions. The two questions which involved the application of probability **Question 4(ii)** and **7(iii)** were exceptions to this. The questions involving the combination of normal variables, and the question on continuous distribution functions were generally done well. The need to put statistical ideas into a particular context was an area of difficulty for many candidates.

## Comments on specific questions

### Question 1

- (i) Most candidates correctly found the value of the parameter for the Poisson approximation to the Binomial and then calculated  $P(X < 3)$ . Errors were rare and only a few candidates ignored the requirement to use a suitable approximation.
- (ii) The majority of candidates correctly identified that the Poisson approximation was appropriate as  $np < 5$  and  $p > 50$ . Some candidates listed only one of these conditions, or stated  $n > 15$

Answers: (i) 0.609 (ii)  $np < 5$ ,  $n$  large

### Question 2

- (i) Many candidates correctly identified 4 different numbers between 001 and 265 ignoring repeats and numbers outside the range by using blocks of 3 consecutive digits in the strings of random numbers generated for the question. Some incorrectly used the zero from the start of the numbers while others included the repeated number 165. A number of candidates included a number outside the range. Work on this topic has improved as this type of question has been set a number of times, although some candidate's responses indicated that the need for a set of 3 digit numbers was not well understood.
- (ii) Nearly all candidates correctly obtained unbiased estimates for the mean and variance of the earnings of the students in Amy's sample. There were very few candidates who did not correctly substitute into the formula for the unbiased estimator of the variance.

- (iii) Very few candidates provided a correct interpretation of the population in this context. The required answer was the earnings of **all** the students in Amy's year. Many candidates thought the population was the students, not their earnings, while a number who identified earnings either stated it was for the 25 students in the sample in part (ii) or failed to make it clear that it was the earnings of all the students in Amy's year.

Answers: (i) 213,165, (0)73, 196 (ii) 20.4,118 (iii) The weekly earnings of all students in Amy's year

### Question 3

The question tested the understanding of confidence intervals for a proportion, and the correct use of normal tables. Candidates were required to find the variance for the proportion and to then use the size of the interval to find the value of  $z$  that gives this interval. A number of candidates included the mean in their interval before realising that as the confidence interval was symmetric then the mean could be eliminated. Those who worked with the mean and equated one side of the interval to the whole interval generally scored no more marks. Correct work resulted in a  $z$  value of 1.572 and candidates were then expected to use tables to find the size of the interval, making use of the symmetry around the mean. Additionally candidates were required to give their answer as a percentage to the nearest whole number. A significant number of candidates did not do this, giving the answer 88.4%.

Answer: 88%

### Question 4

- (i) The candidates were required in this part of the question to give one condition for  $M$  (The number of male students) to have a Poisson distribution in context. Very few candidates were able to do this. Many answers lacked context, or restated the conditions for a binomial distribution to be approximated by a Poisson. Others included female students. Any of the conditions for a Poisson distribution were accepted if in context.
- (ii) This part was very well done by most candidates. They combined the two means correctly and found  $P(M+F>3)$  by doing  $1 - P(M+F<3)$ . Some candidates did not include 3 and so calculated  $P(M+F>2)$ .
- (iii) Very few candidates obtained the correct conditional probability. The problem in most cases was finding  $P(F=0) \cap P(M>3)$  which was the numerator, together with the calculation from part (ii) which was the denominator. Many candidates used only one part of the calculation for their numerator, either  $P(F=0)$  or  $P(M>3)$ . Other candidates did not recognise  $P(M>3)$  was not their previous answer as they now had to use only the mean for males.

Answers: (i) Males leaving each year to study engineering have a constant mean (ii) 0.547 (iii) 0.308

### Question 5

- (i) This was a six mark question that required the candidates to test for a change in the mean time of a journey, having stated the assumption needed to carry out the test. While many candidates correctly carried out the test, very few identified the requirement that the standard deviation should be assumed to have remained the same. The majority of candidates stated correctly the two hypotheses, with the context of the question indicating a two tailed test was appropriate. The correct test statistic was calculated by most candidates, although some candidates omitted  $\sqrt{30}$  from their calculation. The majority of candidates then correctly compared their  $z$  value with +1.96 or -1.96 depending on the sign of their test statistic, or they correctly compared their probability with 0.025. Very few candidates failed to show a valid comparison – the most common loss of marks was by drawing but not annotating a sketch of the normal distribution with the key values. Nearly all candidates correctly concluded that there was insufficient evidence of a change in journey time.
- (ii) Most candidates correctly identified that the probability of a Type I error was 5%. The most common error was candidates who used the probability of their test statistic 3.4% or 96.6%.

- (iii) The majority of candidates were able to correctly state in context that a Type II error would be to conclude that the mean was unchanged when in reality there was a change.

Answers: (i) Standard deviation unchanged. No evidence of a change in mean (ii) 5%.  
(iii) To conclude that the mean was unchanged when in reality there is a change

#### Question 6

- (i) The majority of candidates scored full marks on this question. They met the requirement to add the means and variances to find the distribution of the total time to complete the task, and then to find the probability that this total time was less than 8.5. The error most often seen was in adding the two standard deviations rather than the variances. A small number of candidates used tables incorrectly when finding the probability in the lower tail, this was often as they had incorrectly set up the standardisation.
- (ii) This part of the question was answered very well by most candidates. They correctly found the parameters for the mean and variance of the distribution required (the time for the first task – twice the time for the second task), and found the probability this was less than 0. Most standardisations were correctly done and the correct value was found from normal tables. A small number of candidates found the sum of two times for the second task rather than using  $2^2$  in their variance calculation.

Answers: (i) 0.904 (ii) 0.478

#### Question 7

- (i) Nearly all candidates provided sufficient working to verify that  $k = \frac{8}{7}$ . There were very few errors in integrating and almost every candidate who had correctly integrated and equated to 1 showed enough working to demonstrate the correct value of  $k$ .
- (ii) The majority of candidates were able to find  $E[X]$ . Some candidates did not evaluate  $xf(x)$  before integrating while a common error was to integrate  $\frac{1}{x}$  as either 1 or  $x$ . Provided the integration was performed correctly most candidates evaluated their integral correctly.
- (iii) Fully correct answers were rare in this part although most candidates scored some marks. The first task was to find the probability that one value of  $X$  was less than 1.5. This was done by integrating the function in part (i) between 1 and 1.5. The next problem was to find the probability that  $X$  was greater than 1.5. This could be done by doing  $1 -$  the number just obtained, although many integrated between 1.5 and 2. The next stage in the calculation was to find  $P(X < 1.5) \times P(X > 1.5)^2$ . A significant number of candidates used  $2 \times P(X > 1.5)$  which led to an answer greater than 1. Finally candidates needed to realise there were 3 ways of obtaining 1 value of  $X$  less than 1.5 and 2 greater than 1.5 and therefore they needed to multiply their answer by 3. Many candidates failed to realise that multiplication by 3 was needed.

Answers: (i)  $k = \frac{8}{7}$  (ii) 1.36 (iii) 0.191