

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

November 2003

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 75

SYLLABUS/COMPONENT: 9709/03, 8719/03

MATHEMATICS
Mathematics and Higher Mathematics : Paper 3



Page 1	Mark Scheme	Syllabus	Paper
	A AND AS LEVEL – NOVEMBER 2003	9709/8719	3

- 1 *EITHER:* State or imply non-modular inequality $-5 < 2^x - 8 < 5$, or $(2^x - 8)^2 < 5^2$ or corresponding pair of linear equations or quadratic equation B1
 Use correct method for solving an equation of the form $2^x = a$ M1
 Obtain critical values 1.58 and 3.70, or exact equivalents A1
 State correct answer $1.58 < x < 3.70$ A1

- OR:* Use correct method for solving an equation of the form $2^x = a$ M1
 Obtain one critical value (probably 3.70), or exact equivalent A1
 Obtain the other critical value, or exact equivalent A1
 State correct answer $1.58 < x < 3.70$ A1

[4]

[Allow 1.59 and 3.7. Condone $\leq$ for $<$. Allow final answers given separately. Exact equivalents must be in terms of $\ln$ or logarithms to base 10.]

[SR: Solutions given as logarithms to base 2 can only earn M1 and B1 of the first scheme.]

- 2 *EITHER:* Obtain correct unsimplified version of the x^2 or x^4 term of the expansion of $(1 + \frac{1}{2}x^2)^{-2}$ or $(2 + x^2)^{-2}$ M1
 State correct first term $\frac{1}{4}$ B1
 Obtain next two terms $-\frac{1}{4}x^2 + \frac{3}{16}x^4$ A1+A1

[The M mark is not earned by versions with unexpanded binomial coefficients such as $\binom{-2}{1}$.]

[SR: Answers given as $\frac{1}{4}(1 - x^2 + \frac{3}{4}x^4)$ earn M1B1A1.]

[SR: Solutions involving $k(1 + \frac{1}{2}x^2)^{-2}$, where $k = 2, 4$ or $\frac{1}{2}$ can earn M1 and A1 for a correct simplified term in x^2 or x^4 .]

- OR:* Differentiate expression and evaluate $f(0)$ and $f'(0)$, where $f'(x) = kx(2 + x^2)^{-3}$ M1
 State correct first term $\frac{1}{4}$ B1
 Obtain next two terms $-\frac{1}{4}x^2 + \frac{3}{16}x^4$ A1+A1

[Allow exact decimal equivalents as coefficients.]

[4]

- 3 Use correct $\cos 2A$ formula, or equivalent pair of correct formulas, to obtain an equation in $\cos \theta$ M1
 Obtain 3-term quadratic $6\cos^2 \theta + \cos \theta - 5 = 0$, or equivalent A1
 Attempt to solve quadratic and reach $\theta = \cos^{-1}(a)$ M1
 Obtain answer 33.6° (or 33.5°) or 0.586 (or 0.585) radians A1
 Obtain answer 180° or π (or 3.14) radians and no others in range A1

[The answer $\theta = 180^\circ$ found by inspection can earn B1.]

[Ignore answers outside the given range.]

[5]

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4(i) EITHER Obtain terms $\frac{1}{2\sqrt{x}}$ and $\frac{1}{2\sqrt{y}} \frac{dy}{dx}$, or equivalent B1+B1

Obtain answer in any correct form, e.g. $\frac{dy}{dx} = -\sqrt{\frac{y}{x}}$ B1

OR: Using chain or product rule, differentiate $(\sqrt{a} - \sqrt{x})^2$ M1

Obtain derivative in any correct form A1

Express $\frac{dy}{dx}$ in terms of x and y only in any correct form A1

OR: Expand $(\sqrt{a} - \sqrt{x})^2$, differentiate and obtain term $-2 \cdot \frac{\sqrt{a}}{2\sqrt{x}}$, or equivalent B1

Obtain term 1 by differentiating an expansion of the form $a + x \pm 2\sqrt{a}\sqrt{x}$ B1

Express $\frac{dy}{dx}$ in terms of x and y only in any correct form B1

[3]

(ii) State or imply coordinates of P are $(\frac{1}{4}a, \frac{1}{4}a)$ B1

Form equation of the tangent at P M1

Obtain 3 term answer $x + y = \frac{1}{2}a$ correctly, or equivalent A1

[3]

5 (i) Make recognizable sketch of $y = \sec x$ or $y = 3 - x^2$, for $0 < x < \frac{1}{2}\pi$ B1

Sketch the other graph correctly and justify the given statement B1

[2]

[Award B1 for a sketch with positive y -intercept and correct concavity. A correct sketch of $y = \cos x$ can only earn B1 in the presence of $1/(3 - x^2)$. Allow a correct single graph and its intersection with $y = 0$ to earn full marks.]

(ii) State or imply equation $\alpha = \cos^{-1}(1/(3 - \alpha^2))$ or $\cos \alpha = 1/(3 - \alpha^2)$ B1

Rearrange this in the form given in part (i) i.e. $\sec \alpha = 3 - \alpha^2$ B1

[2]

[Or work *vice versa*.]

(iii) Use the iterative formula with $0 \leq x_1 \leq \sqrt{2}$ M1

Obtain final answer 1.03 A1

Show sufficient iterations to justify its accuracy to 2d.p. or show there is a sign change in the interval (1.025, 1.035) A1

[3]

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- 6 (i) Use product or quotient rule to find derivative M1
Obtain derivative in any correct form A1
Equate derivative to zero and solve a linear equation in x M1
Obtain answer $3\frac{1}{2}$ only A1

[4]

- (ii) State first step of the form $\pm \frac{1}{2}(3-x)e^{-2x} \pm \frac{1}{2} \int e^{-2x} dx$, with or without 3 M1
State correct first step e.g. $-\frac{1}{2}(3-x)e^{-2x} - \frac{1}{2} \int e^{-2x} dx$, or equivalent, with or without 3 A1
Complete the integration correctly obtaining $-\frac{1}{2}(3-x)e^{-2x} + \frac{1}{4}e^{-2x}$, or equivalent A1
Substitute limits $x = 0$ and $x = 3$ correctly in the complete integral M1
Obtain answer $\frac{1}{4}(5 + e^{-6})$, or exact equivalent (allow e^0 in place of 1) A1

[5]

- 7 (i) EITHER: Attempt multiplication of numerator and denominator by $3 + 2i$, or equivalent M1
Simplify denominator to 13 or numerator to $13 + 26i$ A1
Obtain answer $u = 1 + 2i$ A1

- OR: Using correct processes, find the modulus and argument of u M1
Obtain modulus $\sqrt{5}$ (or 2.24) or argument $\tan^{-1} 2$ (or 63.4° or 1.11 radians) A1
Obtain answer $u = 1 + 2i$ A1

[3]

- (ii) Show the point U on an Argand diagram in a relatively correct position B1√
Show a circle with centre U B1√
Show a circle with radius consistent with 2 B1√

[3]

[f.t. on the value of u .]

- (iii) State or imply relevance of the appropriate tangent from O to the circle B1√
Carry out a complete strategy for finding $\max \arg z$ M1
Obtain final answer 126.9° (2.21 radians) A1

[3]

[Drawing the appropriate tangent is sufficient for B1√.]

[A final answer obtained by measurement earns M1 only.]

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- 8 (i) *EITHER*: Divide by denominator and obtain a quadratic remainder M1
Obtain $A = 1$ A1
Use any relevant method to obtain B, C or D M1
Obtain one correct answer A1
Obtain $B = -1, C = 2, D = 0$ A1

- OR*: Reduce *RHS* to a single fraction and identify numerator with that of $f(x)$ M1
Obtain $A = 1$ A1
Use any relevant method to obtain B, C or D M1
Obtain one correct answer A1
Obtain $B = -1, C = 2, D = 0$ A1

[5]

- (ii) Integrate and obtain terms $x - \ln(x - 1)$, or equivalent B1√
Obtain third term $\ln(x^2 + 1)$, or equivalent B1√
Substitute correct limits correctly in the complete integral M1
Obtain given answer following full and exact working A1

[4]

[If $B = 0$ the first B1√ is not available.]

[SR: If A is omitted in part (i), treat as if $A = 0$. Thus only M1M1 and B1√B1√M1 are available.]

- 9 (i) Separate variables and attempt to integrate $\frac{1}{\sqrt{(P - A)}}$ M1
Obtain term $2\sqrt{(P - A)}$ A1
Obtain term $-kt$ A1

[3]

- (ii) Use limits $P = 5A, t = 0$ and attempt to find constant c M1
Obtain $c = 4\sqrt{A}$, or equivalent A1
Use limits $P = 2A, t = 2$ and attempt to find k M1
Obtain given answer $k = \sqrt{A}$ correctly A1

[4]

- (iii) Substitute $P = A$ and attempt to calculate t M1
Obtain answer $t = 4$ A1

[2]

- (iv) Using answers to part (ii), attempt to rearrange solution to give P in terms of A and t M1
Obtain $P = \frac{1}{4}A(4 + (4 - t)^2)$, or equivalent, having squared $\sqrt{A}$ A1

[2]

[For the M1, $\sqrt{(P - A)}$ must be treated correctly.]

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- 10 (i)** Express general point of l or m in component form e.g. $(1 + 2s, s, -2 + 3s)$ or $(6 + t, -5 - 2t, 4 + t)$ B1
Equate at least two corresponding pairs of components and attempt to solve for s or t M1
Obtain $s = 1$ or $t = -3$ A1
Verify that all three component equations are satisfied A1
Obtain position vector $3\mathbf{i} + \mathbf{j} + \mathbf{k}$ of intersection point, or equivalent A1
[5]
- (ii) EITHER:** Use scalar product to obtain $2a + b + 3c = 0$ and $a - 2b + c = 0$ B1
Solve and find one ratio e.g. $a : b$ M1
State one correct ratio A1
Obtain answer $a : b : c = 7 : 1 : -5$, or equivalent A1
Substitute coordinates of a relevant point and values of a, b and c in general equation of plane and calculate d M1
Obtain answer $7x + y - 5z = 17$, or equivalent A1
- OR:** Using two points on l and one on m (or *vice versa*) state three simultaneous equations in a, b, c and d e.g. $3a + b + c = d$, $a - 2c = d$ and $6a - 5b + 4c = d$ B1√
Solve and find one ratio e.g. $a : b$ M1
State one correct ratio A1
Obtain a ratio of three unknowns e.g. $a : b : c = 7 : 1 : -5$, or equivalent A1
Use coordinates of a relevant point and found ratio to find fourth unknown e.g. d M1
Obtain answer $7x + y - 5z = 17$, or equivalent A1
- OR:** Form a correct 2-parameter equation for the plane, e.g. $\mathbf{r} = \mathbf{i} - 2\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) + \mu(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$ B1√
State 3 equations in x, y, z, λ and μ M1
State 3 correct equations A1√
Eliminate λ and μ M1
Obtain equation in any correct unsimplified form A1
Obtain $7x + y - 5z = 17$, or equivalent A1
- OR:** Attempt to calculate vector product of vectors parallel to l and m M1
Obtain two correct components of the product A1
Obtain correct product, e.g. $7\mathbf{i} + \mathbf{j} - 5\mathbf{z}$ A1
State that the plane has equation of the form $7x + y - 5z = d$ A1√
Substitute coordinates of a relevant point and calculate d M1
Obtain answer $7x + y - 5z = 17$, or equivalent A1
[6]
- [The follow through is on $3\mathbf{i} + \mathbf{j} + \mathbf{k}$ only.]