

CAMBRIDGE
INTERNATIONAL EXAMINATIONS

November 2003

GCE A AND AS LEVEL

MARK SCHEME

MAXIMUM MARK: 50

SYLLABUS/COMPONENT: 9709/07, 8719/07

MATHEMATICS AND HIGHER MATHEMATICS
Paper 7 (Probability and Statistics 2)



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1 $\frac{1.9}{\sqrt{n}} \times 1.96 < 1$ $n > 13.9$ (13.87) $n = 14$	M1 A1 M1 A1 [4]	For equality or inequality involving width or equivalent and term in $1/\sqrt{n}$ and a z-value For correct inequality For solving a relevant equation For correct answer two
2 $\lambda = 4.5$ $P(X = 2, 3, 4) = e^{-4.5} \left(\frac{4.5^2}{2!} + \frac{4.5^3}{3!} + \frac{4.5^4}{4!} \right)$ $= 0.471$	M1 B1 M1 A1 A1 [5]	For using Poisson approximation any mean For correct mean used For calculating $P(2, 3, 4)$ their mean For correct numerical expression For correct answer NB Use of Normal can score B1 M1 SR Correct Bin scores M1 A1 A1 only
3 $SU \sim N(19, 12)$ $P(T - SU > 0) \text{ or } P(T - S > 5) = 1 - \Phi\left(\frac{0-1}{\sqrt{21}}\right)$ $= \Phi(0.2182)$ $= 0.586$	B1 M1 M1 M1 A1 [5]	For correct mean and variance. Can be implied if using $P(T - S > 5)$ in next part For consideration of $P(T - SU > 0)$ For summing their two variances For normalising and finding correct area from their values For correct answer
4 (i) $\lambda = \frac{20}{80} = 0.25$ $P(X \geq 3) = 1 - P(X \leq 2)$ $= 1 - e^{-0.25} \left(1 + 0.25 + \frac{0.25^2}{2} \right)$ $= 0.00216$ (ii) $e^{\frac{-k}{80}} = 0.9$ $\frac{-k}{80} = -0.10536$ $k = 8.43$	B1 M1 M1 A1 [4] M1 M1 M1 A1 [4]	For $\lambda = 0.25$ For calculating a relevant Poisson prob(any λ) For calculating expression for $P(X \geq 3)$ their λ For correct answer For using $\lambda = -t/80$ in an expression for $P(0)$ For equating their expression to 0.9 For solving the associated equation For correct answer two
5 (i) $P(\bar{X} > 1800) = 1 - \Phi\left(\frac{1800 - 1850}{117/\sqrt{26}}\right)$ $= \Phi(2.179)$ $= 0.985$	B1 M1 A1 [3]	For $117/\sqrt{26}$ (or equiv) For standardising and use of tables For correct answer two

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(ii) $H_0: \mu = 1850$ $H_1: \mu \neq 1850$ Test statistic = $\frac{1833 - 1850}{117/\sqrt{26}}$ $= -0.7409$ Critical value $z = \pm 1.645$ Accept H_0, no significant change	B1 M1 A1 M1 A1ft [5]	Both hypotheses correct Standardising attempt including standard error Correct test statistic (+/-) Comparing with $z = \pm 1.645$, + with + or – with – (or equiv area comparison) ft 1 tail test $z = 1.282$ For correct conclusion on their test statistic and their z. No contradictions.
6 (i) (a) Rejecting H_0 when it is true (b) Accepting H_0 when it is false (ii) (a) $P(\text{NNNNN})$ under $H_0 = (0.94)^5$ $= 0.7339$ $P(\text{Type I error}) = 1 - 0.7339$ $= 0.266$ (b) $P(\text{NNNNN})$ under $H_1 = (0.7)^5$ $= 0.168$ $P(\text{Type II error}) = 0.168$	B1 B1 [2] M1* A1 M1* A1ft dep* [4] M1 M1 A1 [3]	Or equivalent For evaluating $P(\text{NNNNN})$ under H_0 For correct answer (could be implied) For identifying the Type I error outcome For correct final answer SR If M0M0 allow B1 for $\text{Bin}(5, 0.94)$ used For $\text{Bin}(5, 0.7)$ used For $P(\text{NNNNN})$ under H_1 For correct final answer
7 (i) $\int_0^{\infty} k e^{-3x} dx = 1$ $0 - \frac{-k}{3} = 1 \Rightarrow k = 3$ (ii) $\int_0^{q_1} 3e^{-3x} dx = 0.25$ $\left[-e^{-3x} \right]_0^{q_1} = 0.25$ $-e^{-3q_1} + 1 = 0.25$ $0.75 = e^{-3q_1}$ $q_1 = 0.0959$	M1 A1 [2] M1 M1 A1 [3]	For attempting to integrate from 0 to ∞ and putting the integral = 1 For obtaining given answer correctly For equating $\int 3e^{-3x} dx$ to 0.25 (no limits needed) For attempting to integrate and substituting (sensible) limits and rearranging For correct answer

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(iii) Mean = $\int_0^{\infty} 3xe^{-3x} dx$ $= \left[-xe^{-3x} \right]_0^{\infty} - \int_0^{\infty} -e^{-3x} dx$ $= \left[\frac{e^{-3x}}{-3} \right]_0^{\infty}$ $= 0.333 \text{ or } 1/3$	B1 M1 A1 M1 A1 A1 [6]	For correct statement for mean For attempting to integrate $3xe^{-3x}$ (no limits needed) For $-xe^{-3x}$ or $-xe^{-3x}/3$ For attempt $\int -e^{-3x} dx$ (their integral) For $0 + \left[\frac{e^{-3x}}{-3} \right]_0^{\infty}$ For correct answer
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