

Cambridge International AS & A Level

FURTHER MATHEMATICS**9231/43**

Paper 4 Further Probability & Statistics

May/June 2026**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme contains the instructions and marking guidance that our examiners used to mark candidate responses for this component.

Before they start marking, examiners complete a standardisation process. They review a range of candidate responses and discuss the mark scheme in detail to agree which answers can and cannot be accepted. Examiners award marks where it is clear that candidate responses have met the requirements of the mark scheme. These requirements may vary between different components or different versions of the same component, depending on the exact requirements of the question and the candidate responses seen during the standardisation process.

Sometimes, our mark schemes may allow marks to be awarded to responses which contain elements that are factually incorrect or for content not covered by the syllabus. We do this in response to the demands of a specific question to support candidates and make sure that our assessments are fair. However, these allowances should not be used as a guide for teaching and learning.

We cannot discuss the mark scheme with schools, and we recommend that schools read the mark scheme together with the assessment material and the Principal Examiner Report for Teachers.

This document consists of **15** printed pages.

General marking principles

All examiners must apply these marking principles when marking candidate responses.

1	Examiners must award marks for each response in line with: - the specific content defined in the mark scheme - the specific skills defined in the mark scheme or in the level descriptions - the standard of response required as exemplified by the standardisation scripts.
2	Examiners must award whole marks (not half marks, or other fractions).
3	Examiners must award marks positively . This means that: - marks are not deducted for errors or omissions - marks are awarded for correct/valid answers as defined in the mark scheme and also for other valid answers which go beyond the scope of the syllabus and mark scheme referring to your team leader as appropriate - the quality of spelling, punctuation and grammar are judged only when the mark scheme indicates that these features are specifically assessed in the question. However, the meaning of all responses must be unambiguous.
4	Examiners must consistently apply the requirements of the mark scheme and any rules for what to do when candidates have not followed instructions correctly.
5	Examiners must use the full range of marks defined in the mark scheme, unless limited by the quality of the candidate responses seen.
6	Examiners must award marks according to the mark scheme and must not award marks with grade thresholds or grade descriptions in mind.

Mathematics Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number or sign does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case

Annotation	Meaning
	Indicates that work/page has been seen
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

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The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$\bar{x} = \frac{192.2}{9}$	B1	OE, for example, 21.356 or $\frac{961}{45}$. Accuracy must be 3 significant figures or better.
	$s^2 = \frac{1}{8} \left(4127.64 - \frac{192.2^2}{9} \right)$	M1	Correct form with <i>their</i> values. May be implied by sight of 2.887... or $\frac{2599}{900}$ for s^2 or 1.69934... for s .
	$t = \frac{\frac{192.2}{9} - 20}{\sqrt{\frac{s^2}{9}}}$	M1	Correct form with <i>their</i> values.
	2.39	A1	AWRT 2.39 or 2.40. Accept $\pm$ here.
	'2.39' > 2.306, reject H_0 / significant.	M1	Compare <i>their</i> 2.39 with 2.306 and appropriate conclusion (may be in terms of H_1). Test result can be implied by an attempt at an appropriate conclusion in context. <i>Their</i> 2.39 must come from an attempt at standardising.
	There is sufficient evidence to support that $\mu > 20$.	A1	Correct conclusion from correct working. No contradictions. Level of uncertainty in language used (for example, not 'prove'). Allow μ , or (population) mean. Do not accept statements such as 'there is insufficient evidence to suggest...'.
		6	

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Question	Answer	Marks	Guidance
2(a)	Population of differences may not be normally distributed.	B1	Allow underlying/parent distribution for population. B0 for population mean may not be normal. B0 for (underlying) data may not be normal.
		1	
2(b)	H ₀ : difference in population medians = 0 H ₁ : difference in population medians ≠ 0	B1	Allow use of m_d or $m_1 - m_2$ OE. If m defined, must include reference to population . Median not mean or average.
	Differences: 4, 8, 15, 3, -1, -2, 5, -7, 11, 6	M1	Attempt at differences (at most 2 errors).
	Signed ranks: 4, 8, 10, 3, -1, -2, 5, -7, 9, 6	M1	Attempt at signed ranks from an attempt at differences.
	[P = 45, Q = 10] T = 10	A1	
	Critical value = 8	B1	
	'10' > '8', accept H ₀ / do not reject H ₀ / not significant.	M1	Compare <i>their</i> 10 with <i>their</i> 8 (<i>their</i> 8 may be 13, 10, 8, 5 or 3) and appropriate conclusion (may be in terms of H ₁). <i>Their</i> 10 must come from a valid attempt at ranking. Test result can be implied by an attempt at an appropriate conclusion in context.
	Insufficient evidence to support that there is (on average) a difference between the marks (awarded by the teachers).	A1	Correct conclusion from correct working ignoring hypotheses. In context. Level of uncertainty in language used (for example, not 'prove'). Do not accept 'No evidence...'. Do not accept statements such as 'there is sufficient evidence to suggest...'. 7

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Question	Answer	Marks	Guidance
3(a)	$G'_X(t) = 0.3 + 0.8t + 0.6t^2$	M1	Attempt at differentiating $G_X(t)$. May be implied by expression for $G'_X(1)$ seen.
	$E(X) = 1.7$	A1	CWO
	$G''_X(t) = 0.8 + 1.2t$	M1	Attempt at differentiating $G'_X(t)$. May be implied by expression for $G''_X(1)$ seen.
	$\text{Var}(X) = 2 + 1.7 - 1.7^2$	M1	Use correct formula using <i>their</i> $G'_X(1)$ and $G''_X(1)$.
	0.81	A1	CWO
		5	
3(b)	$G_Y(t) = 0.1t^2 + 0.3t^5 + 0.4t^8 + 0.2t^{11}$	B1	Correct coefficients in a 4-term polynomial.
		B1	Correct powers of t .
		2	

Question	Answer	Marks	Guidance
4(a)	$\int_{1.5}^2 \frac{2}{x^3} dx \quad \left(= \left[-\frac{1}{x^2} \right]_{1.5}^2 = \frac{7}{36} \right)$	M1	Attempt at integral with correct limits.
	$a = \frac{7}{36} \times 200 = 38.89$	A1	AG, convincingly obtained with no errors seen.
	5.89 or 5.90	B1	Summing to 200 gives 5.89, integration gives 5.90.
		3	

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Question	Answer	Marks	Guidance
4(b)	H_0 : f is a good fit for the data. H_1 : f is not a good fit for the data.	B1	Allow ‘satisfactory model for the data’. Must refer to f and data.
	Combine 5th and 6th columns: (16, 9.72) OR Combine final two columns: (20, 16.33)	B1	SOI 5.90 gives 9.73 instead of 9.72.
	5th and 6th columns combined: $\frac{(97 - 111.11)^2}{111.11} + \dots + \frac{(16 - 9.72)^2}{9.72} + \frac{(14 - 12.50)^2}{12.50}$ $[= 1.792 + 0.389 + 1.389 + 2.786 + 4.057 + 0.18]$ Final 2 columns combined: $\frac{(97 - 111.11)^2}{111.11} + \dots + \frac{(10 - 5.89)^2}{5.89} + \frac{(20 - 16.33)^2}{16.33}$ $[= 1.792 + 0.389 + 1.389 + 2.786 + 2.868(2.849) + 0.825]$	M1	At least 2 terms correct. OE, for example, $\frac{97^2}{111.11} + \dots + \frac{16^2}{9.72} + \frac{14^2}{12.50} - 200$. Use of 9.73 instead of 9.72 from using 5.90 for b gives $1.792 + 0.389 + 1.389 + 2.786 + 4.040 + 0.18$. OE, for example, $\frac{97^2}{111.11} + \dots + \frac{10^2}{5.89} + \frac{20^2}{16.33} - 200$. Using 5.90 instead of 5.89 for b gives $1.792 + 0.389 + 1.389 + 2.786 + 2.849 + 0.825$. M1 possible if no columns combined, may be implied by answer in the range 10.61 to 10.63.
	5th and 6th columns combined: 10.6 Final two columns combined: 10.0 or 10.1	A1	SC Unsupported answer of 10.6 or 10.0 or 10.1 scores B1. Must combine columns to earn this mark.
	'10.6' > 9.236, reject H_0 / significant.	M1	Compare <i>their</i> 10.6 with 9.236 and appropriate conclusion (may be in terms of H_1). If no evidence of columns combined, compare <i>their</i> 10.6 with 10.64. Test result can be implied by appropriate conclusion that is consistent with <i>their</i> 10.6.
	Sufficient evidence to suggest that f is not a good fit (for the data).	A1	Correct conclusion from correct working, ignoring hypotheses. Level of uncertainty in language used (for example, not ‘prove’). Do not accept statements such as ‘there is insufficient evidence to suggest...’.
		6	

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Question	Answer	Marks	Guidance
5(a)	$H_0: \mu_x = \mu_y \quad H_1: \mu_x < \mu_y$	B1	If μ not used, must have population means.
	$s_x^2 = \frac{1}{43} \left(1.21 - \frac{6.63^2}{44} \right) [= 4.9065 \times 10^{-3}]$ or $s_y^2 = \frac{1}{51} \left(1.65 - \frac{8.56^2}{52} \right) [= 4.7234 \times 10^{-3}]$	M1	Either expression, may be embedded. Implied by 2.023×10^{-4} .
	$\frac{s_x^2}{44} + \frac{s_y^2}{52} = \frac{4.9065 \times 10^{-3}}{44} + \frac{4.7234 \times 10^{-3}}{52}$	M1	Correct form with <i>their</i> sample variances. For pooled s , award this M1 for correct form of pooled estimate.
	2.023×10^{-4}	A1	May be implied by $s = 0.01422$. Accept 3 significant figure accuracy or better. A0 for pooled s .
	$z = \frac{\frac{6.63}{44} - \frac{8.56}{52}}{\sqrt{\frac{s_x^2}{44} + \frac{s_y^2}{52}}} = -0.98(0)$	M1	Value of z obtained using correct form with <i>their</i> sample variances. Order of numerator may be reversed.
		A1	Accept -0.977 to -0.980 . May see 0.980 for opposite order of numerator. Use of pooled s scores M0 A0.
	' -0.980 ' > -1.282 , accept H_0 / do not reject H_0 / not significant. OR $P(Z < -0.980) = 0.1635 > 0.1$, accept H_0 / do not reject H_0 / not significant.	M1	Compare <i>their</i> z with ± 1.282 (consistent signs) and appropriate conclusion (may be in terms of H_1). Accept 0.1637 . Test result can be implied by a conclusion that is consistent with <i>their</i> -0.980 or 0.1635 .
	Insufficient evidence to support Rhiannon's claim.	A1	Correct conclusion from correct working ignoring hypotheses. In context. Level of uncertainty in language used (for example, not 'prove'). Condone 'no sufficient evidence'. Do not accept statements such as 'there is sufficient evidence to suggest...'. 8 Use of pooled s scores max 3/8 B1 M1 M0 A0 M0 A0 M1 A0.

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Question	Answer	Marks	Guidance
5(b)	$\left(\frac{6.63}{44} - \frac{8.56}{52}\right) \pm 1.645 \sqrt{\frac{s_x^2}{44} + \frac{s_y^2}{52}}$	M1	Accept <i>their s</i> (may be pooled). Use correct formula with any <i>z</i> -value. May subtract the means in opposite order.
		B1	1.645 SOI. Allow 1.64 or 1.65.
	$[-0.0373, 0.00947]$	A1	Accept 0.00947 to 0.0095. Accept with inequality signs or open brackets or $[-9.47 \times 10^{-3}, 0.0373]$ from $\bar{Y} - \bar{X}$. Condone $[9.47 \times 10^{-3}, -0.0373]$. Do not accept -0.0139 ± 0.0234 .
		3	

Question	Answer	Marks	Guidance
6(a)	$F(L) = 0.25$, so $\frac{1}{12}(x^2 + 4x + 4) = \frac{1}{4}$	M1	May be implied by correct answer.
	$L = \sqrt{3} - 2$	A1	
		2	

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Question	Answer	Marks	Guidance
6(b)	$f(x) = \begin{cases} \frac{1}{6}x + \frac{1}{3} & [-2 \leq x < 0] \\ \frac{1}{5}x^2 + \frac{1}{15} & [0 \leq x \leq 2] \\ 0 & \text{otherwise} \end{cases}$	B1	Both expressions seen.
	$E(X^2) = \int_0^{-2} x^2 \left(\frac{1}{6}x + \frac{1}{3} \right) dx + \int_0^2 x^2 \left(\frac{1}{5}x^2 + \frac{1}{15} \right) dx$ $= \left[\frac{1}{24}x^4 + \frac{1}{9}x^3 \right]_{-2}^0 + \left[\frac{1}{25}x^5 + \frac{1}{45}x^3 \right]_0^2$	M1	Attempt to integrate <i>their</i> $x^2 f(x)$. For reference: $\frac{2}{9} + \frac{328}{225} = \frac{42}{25} = 1.68$
	$E(X^4) = \int_{-2}^0 x^4 \left(\frac{1}{6}x + \frac{1}{3} \right) dx + \int_0^2 x^4 \left(\frac{1}{5}x^2 + \frac{1}{15} \right) dx$ $= \left[\frac{1}{36}x^6 + \frac{1}{15}x^5 \right]_{-2}^0 + \left[\frac{1}{35}x^7 + \frac{1}{75}x^5 \right]_0^2$	M1	Attempt to integrate <i>their</i> $x^4 f(x)$. For reference: $\frac{16}{45} + \frac{2144}{525} = \frac{6992}{1575} = 4.439365\dots$
	$\text{Var}(X^2) = \frac{6992}{1575} - \left(\frac{42}{25} \right)^2$	M1	Using correct formula with <i>their</i> values.
	1.62	A1	
		5	
6(c)	$P(X^2 < 1) = P(-1 < X < 1) = F(1) - F(-1) = \left[\frac{7}{15} - \frac{1}{12} \right]$	M1	For either $F(1)$ or $F(-1)$ evaluated. Could be implied by $\frac{7}{15}, \frac{1}{12}, \frac{2}{15}$ or $\frac{1}{4}$ seen (NFWW).
	$\frac{23}{60}$	A1	AWRT 0.383
		2	