



Cambridge International AS & A Level

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FURTHER MATHEMATICS

9231/21

Paper 2 Further Pure Mathematics 2

May/June 2026
2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen. Do **not** use correction fluid or tape.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined pages at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

 This document has **16** pages.



1 Find the first three terms in the Maclaurin's series for $\sinh^{-1}(x+1)$ in the form $\ln a + bx + cx^2$, giving the exact values of the constants a , b and c . [5]

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2 Let $I_n = \int_0^1 (1+t)^n \sin \pi t \, dt$, where n is a non-negative integer.

(a) Show that, for $n \geq 2$, $\pi^2 I_n = \pi(2^n + 1) - n(n-1)I_{n-2}$. [4]

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(b) Find the exact value of I_2 . [3]

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3 (a) Using the substitution $u = \cosh x$, find $\int \sinh^3 x \, dx$. [4]

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(b) Find the particular solution of the differential equation

$$\frac{dy}{dx} + y \coth x = \sinh^2 x,$$

given that $y = 0$ when $x = \ln \frac{1}{3}$. Give your answer in the form $y = f(x)$.

[7]





4 The curve C has equation

$$(x-y)^5 + \ln(xy) = 1 + \ln 2.$$

(a) Show that, at the point $(-1, -2)$ on C , $\frac{dy}{dx} = \frac{8}{11}$. [3]

This image shows a full page of a worksheet designed for handwriting practice. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for writing. The lines are evenly spaced across the entire page, providing a guide for letter height and placement. There is no text or other markings on the page.

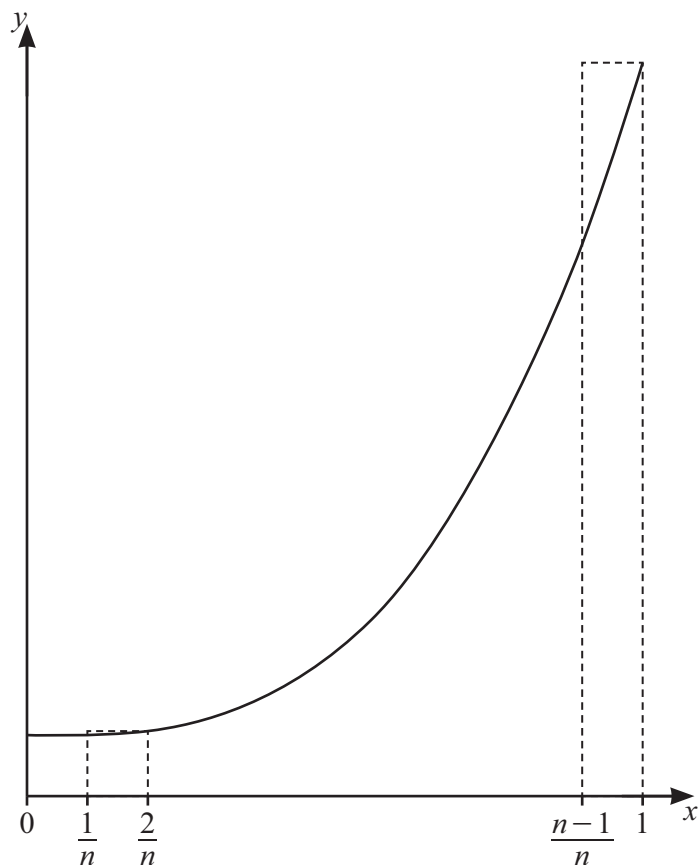


(b) Find the value of $\frac{d^2y}{dx^2}$ at the point $(-1, -2)$.

[5]



5



The diagram shows the curve with equation $y = \frac{11}{10}x^3 + \frac{1}{10}$ for $0 \leq x \leq 1$, together with a set of n rectangles of width $\frac{1}{n}$.

- (a) By considering the sum of the areas of these rectangles, show that $\int_0^1 \left(\frac{11}{10}x^3 + \frac{1}{10} \right) dx < U_n$, where

$$U_n = \frac{11}{40} \left(1 + \frac{1}{n} \right)^2 + \frac{1}{10}. \quad [4]$$

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- (b) Use a similar method to find, in terms of n , a lower bound L_n for $\int_0^1 \left(\frac{11}{10}x^3 + \frac{1}{10} \right) dx$. [4]

- (c) Show that $\lim_{n \rightarrow \infty} (U_n - L_n) = 0$. [2]





6 Find the particular solution of the differential equation

$$5 \frac{d^2 y}{dx^2} + 4 \frac{dy}{dx} + y = x^2 + 7x + 1,$$

given that, when $x = 0, y = 0$ and $\frac{dy}{dx} = 0$. [10]

[illegible]



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7 (a) State the sum of the series $z+z^2+z^3+\dots+z^n$, for $z \neq 1$. [1]

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(b) Find all roots of the equation $1 + z + z^2 + \dots + z^6 = 0$ in the form $e^{iq\pi}$, where q is rational. [2]

(c) Given instead that $z = e^{-1+i\theta}$, use de Moivre's theorem to show that

$$\sum_{m=1}^{\infty} e^{-m} \cos m\theta = \frac{e \cos \theta - 1}{e^2 - 2e \cos \theta + 1}. \quad [7]$$

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



[illegible]



- 8 (a)** The constants a and b are such that the system of equations

$$-x-2y+z=1,$$

$$ax + y - z = 2,$$

$$by + 2z = 3$$

does not have a unique solution. Find a in terms of b .

[3]

[illegible]

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} -1 & -2 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 2 \end{pmatrix}.$$

- (b) Use the characteristic equation of $\mathbf{A}$ to show that $\mathbf{A}^4 = p\mathbf{A}^2 + q\mathbf{I}$, where p and q are integers to be determined. [4]

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[7]

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Additional page

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