

Cambridge International A Level

MATHEMATICS**9709/35**

Paper 3 Pure Mathematics 3

May/June 2026**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme contains the instructions and marking guidance that our examiners used to mark candidate responses for this component.

Before they start marking, examiners complete a standardisation process. They review a range of candidate responses and discuss the mark scheme in detail to agree which answers can and cannot be accepted. Examiners award marks where it is clear that candidate responses have met the requirements of the mark scheme. These requirements may vary between different components or different versions of the same component, depending on the exact requirements of the question and the candidate responses seen during the standardisation process.

Sometimes, our mark schemes may allow marks to be awarded to responses which contain elements that are factually incorrect or for content not covered by the syllabus. We do this in response to the demands of a specific question to support candidates and make sure that our assessments are fair. However, these allowances should not be used as a guide for teaching and learning.

We cannot discuss the mark scheme with schools, and we recommend that schools read the mark scheme together with the assessment material and the Principal Examiner Report for Teachers.

This document consists of **24** printed pages.

General marking principles

All examiners must apply these marking principles when marking candidate responses.

1	Examiners must award marks for each response in line with: - the specific content defined in the mark scheme - the specific skills defined in the mark scheme or in the level descriptions - the standard of response required as exemplified by the standardisation scripts.
2	Examiners must award whole marks (not half marks, or other fractions).
3	Examiners must award marks positively . This means that: - marks are not deducted for errors or omissions - marks are awarded for correct/valid answers as defined in the mark scheme and also for other valid answers which go beyond the scope of the syllabus and mark scheme referring to your team leader as appropriate - the quality of spelling, punctuation and grammar are judged only when the mark scheme indicates that these features are specifically assessed in the question. However, the meaning of all responses must be unambiguous.
4	Examiners must consistently apply the requirements of the mark scheme and any rules for what to do when candidates have not followed instructions correctly.
5	Examiners must use the full range of marks defined in the mark scheme, unless limited by the quality of the candidate responses seen.
6	Examiners must award marks according to the mark scheme and must not award marks with grade thresholds or grade descriptions in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

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The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	Expand $(2+x)^{-3}$ to obtain $\frac{1}{8} \times -3 \times \frac{x}{2}$	B1	OE Allow $(\frac{1}{2})^3$ or $(2)^{-3}$.
	Obtain $\frac{1}{8} \times \frac{-3 \times -4}{2} \times \left(\frac{x}{2}\right)^2$	B1	OE Allow $(\frac{1}{2})^3$ or $(2)^{-3}$. SC allow B0 B1 if $\frac{1}{8}$ wrong for both terms or missing for both terms (even if seen earlier).
	Multiply <i>their</i> $(2+x)^{-3}$ by $(2+3x)$ to find the two x^2 terms which may be unsimplified. <i>Their</i> binomial must include $px + qx^2$.	M1	If correct, expect $2 \times \frac{1}{8} \times \frac{12}{2} \times \frac{1}{4} - 3 \times \frac{1}{8} \times \frac{3}{2}$. Allow M1 if one x^2 term fully “correct” (may be unsimplified) following <i>their</i> binomial expansion and one x^2 term (seen) incorrect. Allow even if $(2+x)^3$ found. Ignore other terms if found, even if extra wrong x^2 terms found.
	Obtain $-\frac{3}{16} [x^2]$	A1	ISW Accept, e.g., -0.1875 but not, e.g., -0.188 . Must be alone or identified if part of a fuller expansion.
		4	
1(b)	State $ x < 2$	B1	Or $-2 < x < 2$. Ignore working.
		1	

Question	Answer	Marks	Guidance
2	Seen, state or imply $\frac{d}{dx} y^3 = 3y^2 \frac{dy}{dx}$	B1	May see $12y^2 \frac{dy}{dx}$.
	Seen, state or imply $\frac{d}{dx} x \ln y = \frac{x}{y} \frac{dy}{dx} + \ln y$	B1	May see $\frac{3x}{y} \frac{dy}{dx} + 3 \ln y$.
	Differentiation must include an attempt at all 3 terms, attempt to find x when $y = 1$ and substitute (<i>their</i> x , 1) Dependent on at least $py^2 \frac{dy}{dx}$ or $q \frac{x}{y} \frac{dy}{dx}$	M1	If correct, expect $3 \ln y + 3 \frac{x}{y} \times \frac{dy}{dx} = 2x - 12y^2 \times \frac{dy}{dx}$, point is (2, 1) and $6 \times \frac{dy}{dx} = 4 - 12 \times \frac{dy}{dx}$.
	Obtain $\frac{dy}{dx} = \frac{2}{9}, \frac{4}{18}$ or 0.222[...] only	A1	Do not ISW. WWW – may see $\frac{dy}{dx} = \frac{2x - 3 \ln y}{12y^2 + \frac{3x}{y}}$ OE. A0 for extra solutions, e.g. $\frac{dy}{dx}$ found at $x = -2$ and not rejected.
		4	

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Question	Answer	Marks	Guidance
3(a)	Take logarithms of both sides to obtain $\ln y + a \ln x = \ln k$	*B1	Or three-term equivalent with no indices.
	Followed by a comparison with $Y = mX + c$ or $pY + qX + r = 0$ or equivalent equation to confirm straight line Equation must be seen (not words or a diagram)	DB1	AG Allow to compare with confused notation, e.g. $y = mx + c$ $py + qx + r = 0$ $\ln y = m \ln x + c$ $\ln y = -a \ln x + k$ $y = kx + c$ $y = ax + b$ $Y = -aX + \ln k$
		2	
3(b)	Form equations in a and k and use a correct method to obtain an equation in one unknown (may be implied by a correct answer for a or k)	M1	$\ln 0.72 + a \ln 5 = \ln k$ $0.72 \times 5^a = k$ or $\ln 5 + a \ln 2.3 = \ln k$ $5 \times 2.3^a = k$ No working needed after simultaneous equations seen.
	Obtain $a = 2.5$ or $k = 40$	A1	Allow to more than 2 sf.
	Obtain $a = 2.5$ and $k = 40$	A1	Must be to 2 sf. Do not allow 2.50 or 40.0. SCB1 only for both answers correct but no simultaneous equations seen.
		3	

Question	Answer	Marks	Guidance
4(a)	State or imply non-modular inequality $(x+4)^2 > (3x-1)^2$, or corresponding quadratic equation, or pair of linear equations or inequalities (treat inequalities as equations) e.g. $x+4 = \pm(3x-1)$	M1	Or by deduction from a correct sketch. Allow $<, \leq, =, \geq, >$.
	Obtain critical values $x = \frac{5}{2}$ and $x = -\frac{3}{4}$	A1	
	State final answer $-\frac{3}{4} < x < \frac{5}{2}$	A1	Strict inequality required. Allow $x < \frac{5}{2}$ and $x > -\frac{3}{4}$. Do not allow $x < \frac{5}{2}$ or $x > -\frac{3}{4}$. Do not allow $x < \frac{5}{2}, x > -\frac{3}{4}$.
		3	
4(b)	Use a correct method to solve $3^{0.05n} < \frac{5}{2}$ as far as $pn < q, pq \neq 0$	M1	I.e. correct use of <i>their</i> upper limit 4(a) . Power law for logs must be used for M1 , e.g. $0.05n \ln 3 < \ln \frac{5}{2}, 0.05n < \log_3 \frac{5}{2}, 0.05n < 0.834$ or $n \ln 3^{0.05} < \ln \frac{5}{2}$. Allow $<, \leq, =, \geq, >$.
	Obtain $n = 16$	A1	CAO $3^{0.05n} < \frac{5}{2}$ followed by 16 with no working is not worthy of credit. Largest integer $< 16.68, 16.6, 16.69, 16.7$. As question says ‘hence’, trial and improvement, or no working shown, is 0/2.
		2	

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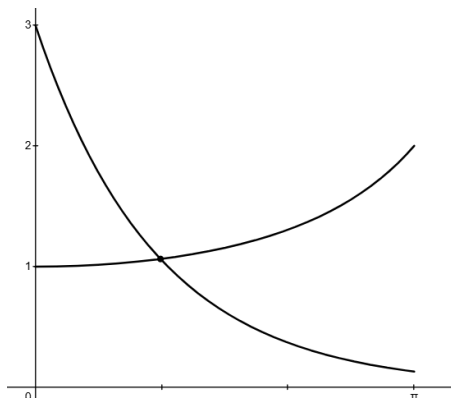
Question	Answer	Marks	Guidance
5(a)	Use correct quotient rule (or correct product rule)	*M1	Allow with a sign error between the terms in the numerator. Allow incorrect differentiation of any of $\sin 2x$, $3 + \cos x$ (quotient rule) and $(3 + \cos x)^{-1}$ (product rule).
	Obtain $\frac{2(3 + \cos x)\cos 2x + \sin 2x \sin x}{(3 + \cos x)^2}$	A1	OE
	Use correct double angle formulae and correct Pythagoras to obtain expression in $\cos x$ only	DM1	$\frac{2(3 + \cos x)(2\cos^2 x - 1) + 2\cos x(1 - \cos^2 x)}{(3 + \cos x)^2}$
	Obtain $\frac{2\cos^3 x + 12\cos^2 x - 6}{(3 + \cos x)^2}$	A1	OE ISW Allow correct answer to imply correct Pythagoras on $\sin^2 x$ for M1A1. Numerator must be simplified, but may have a factor of 2 taken out and denominator may be expanded.
		4	
5(b)	Correct method to find the normal through $(\frac{1}{2}\pi, 0)$ using $\frac{-1}{\text{their } \frac{dy}{dx}}$	M1	$\frac{y-0}{x-\frac{1}{2}\pi} = \frac{3}{2}$ Available if finding m and c .
	Obtain $2y = 3x - \frac{3}{2}\pi$	A1FT	ISW Or exact three-term equivalent, e.g. $y + \frac{3}{4}\pi = \frac{3}{2}x$. Follow through from <i>their</i> 5(a) .
		2	

Question	Answer	Marks	Guidance
6	State or imply $\frac{dy}{dx} = [k]y$	B1	Throughout the question, k and C may be represented by different letters.
	Separate variables correctly: $\int \frac{1}{y} dy = \int [k] dx$	B1	
	Integrate to obtain $\ln y = [k]x(+C)$	B1	
	Substitute $(2, e^2)$ and $(5, e^3)$ into an equation of the form $\ln y = kx + C$ to form simultaneous equations in k and C and solve for k or C	M1	$2 = 2k + C$ $3 = 5k + C$ Allow M1 even if they substitute incorrectly to form their simultaneous equations, provided working is seen to solve them. No working seen and incorrect values for <i>their</i> simultaneous equations, e.g. using calculator incorrectly, M0.
	Obtain $k = \frac{1}{3}$ or $C = \frac{4}{3}$	A1	Or $C = 4$ if $\frac{1}{k} \frac{dy}{dx} = y$ used.
	Obtain $k = \frac{1}{3}$ and $C = \frac{4}{3}$	A1	
	Obtain $y = e^{\frac{1}{3}(x+4)}$	A1FT	OE FT <i>their</i> k and <i>their</i> C values $y = e^{kx+C}$ (depending upon where <i>their</i> C and k are), provided $k \neq 1$.
		7	

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Question	Answer	Marks	Guidance
7(a)	EITHER: Multiply both sides by $a - 3i$ and attempt expansion of right-hand side	(*M1)	At least three of four unsimplified terms correct.
	Use of $i^2 = -1$ seen at least once (or implied)	DM1	E.g. $2 + ai = \lambda(a - 15) + \lambda i(-5a - 3)$.
	Compare real and imaginary parts, each containing λ , to obtain an equation in a only $[2 = \lambda(a - 15), a = \lambda(-5a - 3)]$	M1	E.g. $\frac{a}{2} = \frac{-5a - 3}{a - 15}$. Any equivalent form. SCB1: Compare real and imaginary parts, each containing λ , to obtain an equation in λ only, solve for λ and a , and verify $a^2 - 5a + 6 = 0$ from $a = 2$ and $a = 3$. E.g. $-\frac{2}{\lambda} - 15 = 78\lambda + 10 \Rightarrow$ $\lambda = -\frac{1}{6}, \lambda = -\frac{2}{13}$ and $a = 2, a = 3$. Note: max 3 out of 4 marks for this approach as verifying the equation.
	Obtain $a^2 - 5a + 6 = 0$ from correct working	A1)	AG
	OR: Multiply top and bottom of the left-hand side by $a + 3i$ and attempt both expansions	(*M1)	Do not need the right-hand side at this stage. At least six of unsimplified terms correct.
	Use of $i^2 = -1$ seen at least once or implied	DM1	E.g. $[\lambda(1 - 5i) =] \frac{-a + i(a^2 + 6)}{a^2 + 9}$.
	Compare real and imaginary parts, each containing λ , to obtain an equation in a only	M1	$\frac{5a}{a^2 + 9} = \frac{a^2 + 6}{a^2 + 9}$ or $5a = a^2 + 6$. Any equivalent form.
	Obtain $a^2 - 5a + 6 = 0$ from correct working	A1)	AG

Question	Answer	Marks	Guidance
7(a)	OR: Divide both sides by $1 - 5i$ and attempt expansion of denominator of left-hand side	(*M1)	At least three of four unsimplified terms correct.
	Use of $i^2 = -1$ seen at least once (or implied)	DM1	E.g. $\frac{2 + ai}{a - 15 - 3i - 5ai}$.
	Complete realising of denominator of left-hand side, state λ is real and set imaginary part equal to zero	M1	E.g. $\frac{(-30 - a - 5a^2) + i(a^2 - 5a + 6)}{(a - 15)^2 + (3 + 5a)^2}$ or $\frac{(-30 - a - 5a^2) + i(a^2 - 5a + 6)}{26a^2 + 234}$.
	Obtain $a^2 - 5a + 6 = 0$ from correct working	A1)	AG
		4	
7(b)	Solve given quadratic to obtain a value of a and use this to form a real equation in λ only. Can use their expression for λ from (a) or can make restart but must get to a real equation	M1	Can be implied by relevant working seen or a correct value for λ seen without working, i.e. may use a calculator.
	Obtain $a = 2, \lambda = -\frac{2}{13}$ or $a = 3, \lambda = -\frac{1}{6}$ or $-\frac{3}{18}$	A1	Allow -0.15 and -0.17 .
	Obtain second correct pair of values	A1	For those who solve for λ first in (a) , award marks if stated in (b) as value of a and λ M1 A1, second pair of values A1.
		3	

Question	Answer	Marks	Guidance
8(a)	Sketch a relevant graph with an indication of scale on the x-axis, condone 180 for π on the diagram only For $y = \sec\left(\frac{1}{3}x\right)$: correct y-intercept, must label 1 No incorrect curvature Need $1 \leq \sec\left(\frac{1}{3}\pi\right) \leq 3$ Ignore anything outside $0 < x < \pi$ For $y = 3e^{-x}$: correct y-intercept, must label 3 not touching the x-axis Need $3e^{-\pi} < 1$ Ignore anything outside $0 < x < \pi$	M1	
	Sketch a second relevant graph and justify the given statement, e.g. by a dot, a line down to the x-axis, a comment such as ‘the graphs intersect once’ Both graphs must satisfy the conditions set out in the first M1 mark	A1	
		2	
8(b)	Calculate the value of a correct relevant expression or values of a pair of expressions at $x = 1$ and $x = 1.1$. Must be working in radians. Values correct to at least 2 sf when rounded or truncated. Need all relevant values but only one (pair) needs to be correct to score M1 If not comparing with 0 or 1 then the pairing must be clear, not just embedded values.	M1	E.g. comparing $3e^{-x}$ with $\sec\left(\frac{1}{3}x\right)$ at 1, $1.1036 > 1.058$ at 1.1, $0.9986 < 1.07$. $\sec\left(\frac{1}{3}x\right) - 3e^{-x} = 0$ $-0.04539 < 0 < 0.07259$. $\frac{\sec\left(\frac{1}{3}x\right)}{3e^{-x}} = 1$ or $\frac{e^x}{3\cos\left(\frac{1}{3}x\right)} = 1$ $0.9589 < 1 < 1.0727$.
	Complete the argument correctly with correct calculated values (AWRT 2 sf)	A1	Allow work on a smaller interval.
		2	

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Question	Answer	Marks	Guidance
8(c)	Starting with iterative formula: write without suffixes and rearrange to $e^x = 3\cos\left(\frac{1}{3}x\right)$	*B1	No slips allowed throughout. Allow x or α used consistently throughout. Starting with the equation in 8(a) : Take reciprocal and obtain $e^x = 3\cos\left(\frac{1}{3}x\right)$, $x = -\ln\frac{1}{3\cos\frac{1}{3}x}$ or $\ln\left(\sec\frac{1}{3}x\right) = \ln 3 - x$ or better.
	Take reciprocal and obtain $\sec\left(\frac{1}{3}x\right) = 3e^{-x}$ Need at least one line of working between $e^x = 3\cos\left(\frac{1}{3}x\right)$ and $\sec\left(\frac{1}{3}x\right) = 3e^{-x}$	DB1	Take logs and arrange in iterative form $x_{n+1} = \ln\left(3\cos\left(\frac{1}{3}x_n\right)\right)$. Need at least one line of working between $e^x = 3\cos\left(\frac{1}{3}x\right)$ and $x_{n+1} = \ln\left(3\cos\left(\frac{1}{3}x_n\right)\right)$.
		2	
8(d)	Use the iterative process correctly at least once Must be working in radians	M1	Need not be 5 d.p. here, but values correct to at least 3 d.p. when rounded or truncated.
	Obtain final answer 1.038	A1	Do not allow x_n, x_∞ etc.
	Show sufficient iterations to at least 5 dp to justify 1.038 to 3 dp Allow recovery	A1	1, 1.04200, 1.03704, 1.03764, 1.03756 Allow full marks even if a different initial value used (may need to check on calculator for the M1).
		3	

Question	Answer	Marks	Guidance
9(a)	State $R = \sqrt{34}$	B1	Do not allow decimal equivalent.
	Correct use of trigonometry to find $\tan \alpha$ [$R \sin \alpha = 3$, $R \cos \alpha = 5$]	M1	M0 if from $\cos \alpha = 5$, $\sin \alpha = 3$ or from $R \sin \alpha = -3$.
	$\alpha = \tan^{-1}\left(\frac{3}{5}\right)$	A1	ISW (no working needed). Allow 0.6 for $\frac{3}{5}$.
		3	
9(b)	Use result from 9(a) to rewrite the integral as $\int \left(\frac{68}{(\sqrt{34} \cos(\theta + \alpha))^2} \right) d\theta$	M1	For <i>their</i> R and <i>their</i> α
	Obtain $2 \tan(\theta + \alpha) \quad (+C)$	A1FT	Follow <i>their</i> R and <i>their</i> α .
	Use correct limits correctly in $p \tan(\theta + \alpha)$ and correct formula for $\tan 2\alpha$	M1	E.g. $2 \tan 2\alpha - 2 \tan \alpha = 2 \frac{2 \tan \alpha}{1 - \tan^2 \alpha} - 2 \tan \alpha$ or $2 \tan\left(2 \tan^{-1}\left(\frac{3}{5}\right)\right) - 2 \tan\left(\tan^{-1}\left(\frac{3}{5}\right)\right)$ $= 2 \frac{2 \tan\left(\tan^{-1}\left(\frac{3}{5}\right)\right)}{1 - \tan^2\left(\tan^{-1}\left(\frac{3}{5}\right)\right)} - 2 \tan\left(\tan^{-1}\left(\frac{3}{5}\right)\right).$
	Obtain $\frac{51}{20}$ from full and correct working Need to see $\frac{4 \times \frac{3}{5}}{1 - \left(\frac{3}{5}\right)^2} - 2 \times \frac{3}{5}$ or equivalent M0 A0 for equating to $\frac{51}{20}$ and solving for $\tan \alpha$	A1	AG SCB1: calculator gives $\tan\left(2 \tan^{-1}\left(\frac{3}{5}\right)\right) = \frac{15}{8}$, so B1 for $2 \times \frac{15}{8} - 2 \times \frac{3}{5} = \frac{51}{20}$ in place of final M0 A0.
		4	

Question	Answer	Marks	Guidance
10(a)	State or imply $\frac{dx}{du} = 2 \cos u$	B1	Needs du
	Obtain $a = 0, b = \frac{1}{6}\pi$ May be seen on integral	B1	Accept $x = 0, u = 0$ and $x = 1, u = \frac{1}{6}\pi$. If using degrees, then B0.
	Substitute u for x throughout Allow omission of du	M1	
	Obtain $\int 24 \sin^3 u \, du$ from correct working	A1	AG Need to see $\cos u$ in denominator before cancelling (allow, e.g., $\sqrt{4 + 4 \cos^2 u - 4}$). Needs du . Ignore limits.
		4	
10(b)	Rewrite as $\int 24 \sin u (1 - \cos^2 u) \, du$ and integrate to obtain $p \cos u + q \cos^3 u$	M1	
	Obtain $-24 \cos u + 8 \cos^3 u$	A1	
	Use correct limits in an expression containing $p \cos u$ and $q \cos^3 u$	M1	$-24 \frac{\sqrt{3}}{2} + 8 \times \frac{3}{4} \times \frac{\sqrt{3}}{2} + 24 - 8$ Do not allow slips.
	Obtain $16 - 9\sqrt{3}$	A1	If using degrees, can score 4/4.
		4	

Question	Answer	Marks	Guidance
11(a)	Obtain any one of $\overrightarrow{AB} = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix} -4 \\ -1 \\ -5 \end{pmatrix}$ $\overrightarrow{DC} = \begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix}$ $\overrightarrow{AD} = \begin{pmatrix} -5 \\ 1 \\ -4 \end{pmatrix}$	*B1	OE, e.g. $\overrightarrow{BA} = \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}$. If misread, or an error, then max B1 for one of these vectors correct together with possible final *B1. Allow all marks following this misread in (b) and (c) .
	Obtain correct $\overrightarrow{DC}$ and $\overrightarrow{AB}$	*B1	Allow $\overrightarrow{DC}$ and $\overrightarrow{BA}$ or $\overrightarrow{CD}$ and $\overrightarrow{AB}$.
	Show that $\overrightarrow{DC}$ and $\overrightarrow{AB}$ are parallel, e.g. $2\overrightarrow{AB} = \overrightarrow{DC}$ Do not accept $k\overrightarrow{AB} = \overrightarrow{DC}$; correct value for k must be found	DB1	Dependent on both previous B1 marks. Allow $\overrightarrow{DC}$ and $\overrightarrow{BA}$ or $\overrightarrow{CD}$ and $\overrightarrow{AB}$. Must be using vector notation.
	EITHER: Attempt to obtain $\overrightarrow{BC}$ and $\overrightarrow{AD}$ and state it is not possible to write $\overrightarrow{BC}$ as a multiple of $\overrightarrow{AD}$ so the sides are not parallel and $ABCD$ is a trapezium (accept not a parallelogram)	(DB1)	Dependent only on first *B1. Allow $\overrightarrow{BC}$ and $\overrightarrow{DA}$, or $\overrightarrow{CB}$ and $\overrightarrow{AD}$. Allow slips in finding $\overrightarrow{BC}$ and $\overrightarrow{AD}$.
	OR: Alternative Method for final DB1 mark: Attempt to show that $\overrightarrow{AB} \neq \overrightarrow{DC}$ or $ \overrightarrow{DC} \neq \overrightarrow{AB} $ and state $ABCD$ is a trapezium (accept not a parallelogram)	(DB1)	For $ \overrightarrow{DC} \neq \overrightarrow{AB} $ accept $DC \neq AB$ Allow $\overrightarrow{DC}$ and $\overrightarrow{BA}$ or $\overrightarrow{CD}$ and $\overrightarrow{AB}$. Allow slips in finding the vectors and/or lengths. (Must not use BC and AD as these are of equal length).
		4	

Question	Answer	Marks	Guidance
11(b)	EITHER: Method 1 Find $\overrightarrow{AP}$ for a general point P on CD Using $\overrightarrow{AC}$ or $\overrightarrow{AD}$	(*M1)	E.g. $\begin{pmatrix} -5+\lambda \\ 1-2\lambda \\ -4-\lambda \end{pmatrix}$ or $\begin{pmatrix} -3+2\mu \\ -3-4\mu \\ -6-2\mu \end{pmatrix}$, as may be using $\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$ or $\begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix}$.
	Equate the scalar product of <i>their</i> $\overrightarrow{AP}$ and $\pm\overrightarrow{CD}$ to zero and obtain an equation in λ	DM1	E.g. $(-5+\lambda)+(-2+4\lambda)+(4+\lambda)=0$ or $(-6+4\mu)+(12+16\mu)+(12+4\mu)=0$. Note scalar product may be with $\begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$ or $\begin{pmatrix} 2 \\ -4 \\ -2 \end{pmatrix}$.
	Solve to obtain a value for λ or μ	DM1	OE, if using a multiple of the direction vector. If correct, expect e.g. $\lambda = \pm\frac{1}{2}$ or $\pm\frac{1}{4}$ or $\mu = \pm\frac{3}{4}$ or $\pm\frac{3}{2}$.
	Carry out a method to the distance	M1	If correct, expect e.g. $4.5^2 + 4.5^2$.
	Obtain $\frac{9}{2}\sqrt{2}$ from correct working	A1)	AG

Question	Answer	Marks	Guidance
11(b)	OR: Method 2 Attempt to find $\cos \angle BAD$ from $\frac{\overrightarrow{AD} \cdot \overrightarrow{AB}}{ \overrightarrow{AD} \overrightarrow{AB} }$	(*M1)	Or if finding $\angle ACD$: $\cos \angle ACD = \frac{\overrightarrow{CA} \cdot \overrightarrow{CD}}{ \overrightarrow{CA} \overrightarrow{CD} }$.
	Obtain $\cos \angle BAD = -\frac{1}{14}\sqrt{7}$	A1	Or exact equivalent. $\cos \angle ACD = \frac{1}{2}$
	Attempt to find the exact value of $\sin \angle BAD$ from <i>their</i> $\cos \angle BAD$	*DM1	If correct, $\sin \angle BAD = \frac{3}{14}\sqrt{21}$ or $\sin \angle ACD = \frac{1}{2}\sqrt{3}$.
	Find $ \overrightarrow{AD} \sin \angle BAD$ for <i>their</i> $ \overrightarrow{AD} $ and <i>their</i> $\sin \angle BAD$, must be working in exact form	DM1	Dependent on both previous M marks. If correct, expect $\sqrt{42} \times \frac{3}{14}\sqrt{21}$ or $\frac{1}{2}\sqrt{3} \times \sqrt{54}$.
	Obtain $\frac{9}{2}\sqrt{2}$ from correct working	A1)	AG
		5	

Question	Answer	Marks	Guidance
11(c)	Use correct method for area of a trapezium	M1	E.g. use $\frac{a+b}{2}h$ with $h = \frac{9}{2}\sqrt{2}$.
	Obtain $\frac{\text{length of their } AB + \text{length of their } DC}{2} \times \frac{9}{2}\sqrt{2}$	A1FT	FT <i>their</i> $\overrightarrow{AB}$ and <i>their</i> $\overrightarrow{DC}$. If correct, expect $\frac{\sqrt{6} + \sqrt{24}}{2} \times \frac{9\sqrt{2}}{2}$.
	Obtain $\frac{27}{2}\sqrt{3}$	A1	Or simplified equivalent, e.g. $\frac{81}{2\sqrt{3}}$. Allow full credit even if <i>their</i> $\overrightarrow{DC}$ and <i>their</i> $\overrightarrow{AB}$ have sign errors.
		3	